ELECTROSTATIC SOLITARY WAVES IN MULTICOMPONENT NONTHERMAL PLASMA

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A systematic investigation has been made of ion-acoustic solitary waves in plasma consisting of cold positively and negatively charged ions, as well as nonthermal electrons. The basic features of the solitary waves are investigated by using the reductive perturbation method, which is valid for the small but finite amplitude waves. Computational investigations have been performed to examine the effects of the negative ion density and charge, as well as nonthermal electron parameter. The present results are useful in understanding the nonlinear dynamics of the electrostatic solitary waves in the Titan atmosphere.
Keywords: ion-acoustic solitons, negative ions, KdV equation

I. Introduction

Recently, Cassini spacecraft have confirmed the presence of heavy negative ions in the upper regions of Titan atmosphere [1]. These particles may act as organic building blocks for even more complicated molecules. Also, the observations of nonthermal ions/electrons in space physics have been a proved by the Vela satellite and ASPERA satellite [2]. Due to the weak magnetic field, the impacting of the solar wind with the planetary atmosphere (such as Titan's atmosphere) results in nonthermal ion/electron fluxes. Therefore, it is of interest to investigate the properties of electrostatic excitations in the presence of both negative ions and nonthermal electrons in the Titan atmosphere.

II. Fluid Theory

We investigate the propagation of small but finite amplitude low-frequency electrostatic excitations in a three-component plasma, consisting of two distinct ion species of opposite polarity ($i+, i-$), as well as nonthermal electrons ($e-$). A two-fluid plasma model is employed, modeling the two ion species. The propagation of the ion-acoustic waves of interest is governed by a system of fluid equations for the positive and negative ion fluids, respectively distinguished by using the index $+$ " and $-$ ". We consider the (2) continuity equation(s)

$$\frac{\partial n_{+,-}}{\partial t} + \nabla.(n_{+,-}u_{+,-}) = 0, \qquad (1)$$

and the (2) momentum equations,

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_+ \cdot \nabla\right)\mathbf{u}_+ = -Z_+ \nabla\phi, \qquad (2)$$

and

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_- \cdot \nabla\right)\mathbf{u}_- = Z_- Q \nabla\phi. \qquad (3)$$

The nonthermal electrons is described by the following distribution [3]

$$n_e = \mu(1 - \beta\phi + \beta\phi^2)\exp\phi. \qquad (4)$$

Equations (1)-(4) are coupled through Poisson equation

$$\nabla^2\phi = Z_- n_- + n_e - Z_+ n_+. \qquad (5)$$

In Eqs. (1)-(5), $n_{+,-}$ is the positive (negative) ion number density, $u_{+,-}$ is the positive (negative) ion fluid velocity, ϕ is the electrostatic wave potential. The variables appearing in Eqs. (1)-(5) have been scaled by appropriate quantities. Thus, the density n_j (for $j = +,-$) is normalized by the unperturbed ion density n_{+0} , u_j is scaled by the ion sound speed $C_s = (T_e/m_+)^{1/2}$, the potential ϕ by (T_e/e) , the time by the ion plasma period $\omega_{pi} = (4\pi e^2 n_{+0}/m_+)^{1/2}$, and the space variable is in units of the ion Debye radius $\lambda_{Di} = (T_e/4\pi e^2 n_{+0})^{1/2}$. We have defined the mass ratio $Q = m_+/m_-$ (where m_+ and m_- are the positive and negative ion fluid mass, respectively). The neutrality condition implies

$$Z_+ = Z_- \nu + \mu, \qquad (6)$$

where $\mu = n_{e0}/n_{+0}$ and $\nu = n_{-0}/n_{+0}$ (the index `0' denotes the unperturbed density states).

III. Derivation of Kortweg-de Vries equation

Using the reductive perturbation technique [4], we shall derive Kortweg de Vries (KdV) equation to describe the evolution of the system. According to this method the independent variables can be stretched as:

$$X = \varepsilon^{1/2}(x - \lambda t), \text{and } T = \varepsilon^{3/2}t, \tag{7}$$

where ϵ is a small (real) parameter and λ is the wave propagation speed. The dependent variables are expanded as

$$\Psi = \Psi^{(0)} + \sum_{n=1}^{\infty} \varepsilon^n \Psi^{(n)}, \tag{8}$$

where

$$\Psi = \{n_+, n_-, u_+, u_-, \phi\}^T, \tag{9}$$

and

$$\Psi^{(0)} = \{1, \nu, 0, 0, 0\}^T. \tag{10}$$

Employing the variable stretching (7) and the expansions (8)-(10) into Eqs. (1)-(5), we may now isolate distinct orders in ϵ and derive the corresponding variable contributions. The lowest-order equations in ϵ read

$$n_+^{(1)} = \frac{Z_+}{\lambda^2}\phi^{(1)}, \; u_+^{(1)} = \frac{Z_+}{\lambda}\phi^{(1)}, \tag{11}$$

and

$$n_-^{(1)} = -\frac{Z_- Q\nu}{\lambda^2}\phi^{(1)}, \; u_-^{(1)} = \frac{Z_- Q}{\lambda}\phi^{(1)}. \tag{12}$$

The Poisson equation provides the compatibility condition

$$\lambda^2 = \frac{Z_+^2 + Z_-^2 Q\nu}{(Z_+ - Z_-\nu)(1-\beta)}. \tag{13}$$

The next order in ϵ yields a set of equations which, after using Eqs. (11)-(13), can be reduced to a Kortweg-de Vries (KdV) equation

$$\frac{\partial\varphi}{\partial T} + A\varphi\frac{\partial\varphi}{\partial X} + B\frac{\partial^3\varphi}{\partial X^3} = 0, \tag{14}$$

where $\varphi \equiv \phi^{(1)}$ for simplicity and the coefficients A and B are read

$$A = B\left(\frac{3Z_+^3}{\lambda^4} - \frac{3\nu Q^2 Z_-^3}{\lambda^4} - Z_+ + Z_-\nu\right), \tag{15}$$

and

$$B = \frac{\lambda}{2(Z_+ - Z_-\nu)(1-\beta)}. \tag{16}$$

IV. Traveling wave analysis – Pulse shaped localized solutions

We shall use the traveling wave transformation $\zeta = X - \vartheta T$, where ϑ is a real variable (representing a constant speed, scaled by the ion sound speed). Equation (14) is thus reduced to the ordinary partial differential equation:

$$-\vartheta\varphi' + A\varphi\varphi' + B\varphi''' = 0, \tag{17}$$

The prime in Eq. (17) denotes the derivative with respect to ζ. Integrating Eq. (17) once, by assuming the boundary conditions φ, φ' and $\varphi'' \to 0$ for $\zeta \to \pm\infty$, we obtain the pseudo-energy-balance equation

$$\frac{1}{2}\varphi'^2 + S(\varphi) = 0. \tag{18}$$

This relation suggests that the evolution of a solitary excitation is analogous to the problem of motion of a unit mass in a (Sagdeev-like) pseudopotential, given by

$$S(\varphi) = \frac{1}{B}\left(\frac{-\vartheta}{2}\varphi^2 + \frac{A}{6}\varphi^3\right). \tag{19}$$

The solitary wave solution of Eq. (17) exists if $d^2S/d\varphi^2 < 0$ at $\varphi = 0$. A value of $d^2S/d\varphi^2$ smaller than zero predicts the formation of solitary structure in the system. Therefore, we have

$$d^2S/d\varphi^2 = -\vartheta/B < 0. \tag{20}$$

It is noted that Eq. (20) is always satisfied, except for $Z_+ = Z_-\nu = Z_c$ or for $\beta = 1$ (implying vanishing the nonthermality in the system), where Z_c is a critical charge number. One can thus conclude that the solitary waves can always propagate in this plasma system except for $Z_c = Z_-\nu$; where solitons do not exist.

A solitary wave solution of Eq. (18) given as

$$\varphi = \varphi_0 \text{sech}^2(\zeta/W), \tag{21}$$

where $\phi_0 = 3\vartheta/A$ is the maximum amplitude of the potential perturbation and $W = \sqrt{4B/\vartheta}$ measures its spatial extension (width). The localized pulses predicted via this form may be either positive or negative, depending entirely on the sign of the nonlinearity coefficient A. The characteristics of these pulses will be discussed below.

V. Numerical Results and Discussion

The effect of the plasma parameters on the propagation speed λ is depicted in Figs. 1 and 2. It is clear that increasing Z_-, Q, ν and β increase the propagation speed λ. Also, the nonthermality parameter β increases the propagation speed λ. Figure 3 and 4 represents a contour plot of the amplitude φ_0 and the width W viruses Z_-, Q, ν and β. Notice that lighter regions show higher values of the amplitude and the width. From Fig. 3, it is clear that the amplitude and the width increase with increasing Z_- and Q, but the amplitude becomes infinity for certain values of Z_- and Q. This situation is out the scope of the present paper, which will investigate in the future. The effect of ν and β are depicted in Fig. 4. It is clear that both the amplitude and the width increase with increasing ν and β, and again there is a region (represented by white color) where the amplitude vanishes, it is seen for small ν and β. Finally, this investigation may be helpful to understand the properties of the electrostatic excitations in the Titan's atmosphere.

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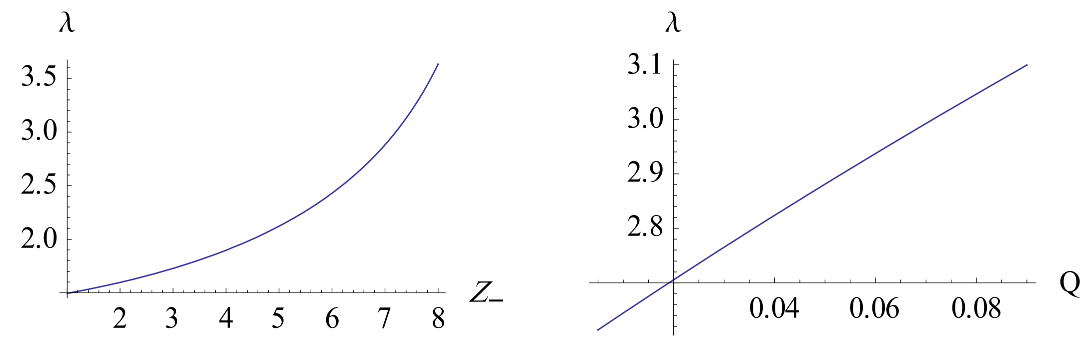

Figure 1: The variation of phase velocity λ [given by Eq. (13)] with Z_- (left) and Q (right), for Z_+ =1, β=0.5 and ν=0.1

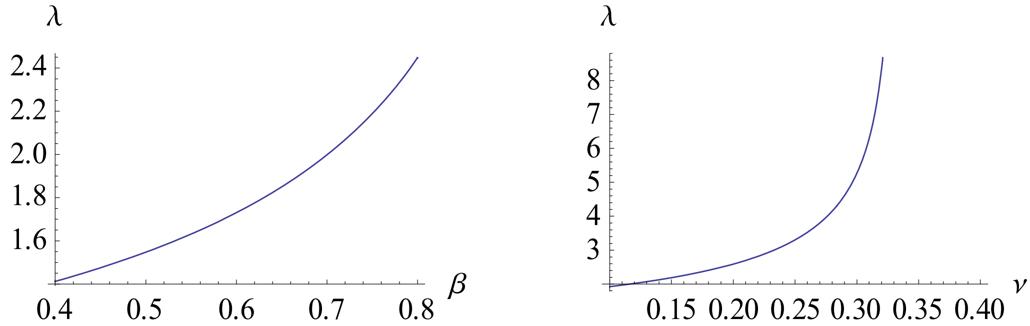

Figure 2: The variation of phase velocity λ [given by Eq. (13)] with β (left) and ν (right), for Z_+ =1, Z_- =3 and Q=0.4.

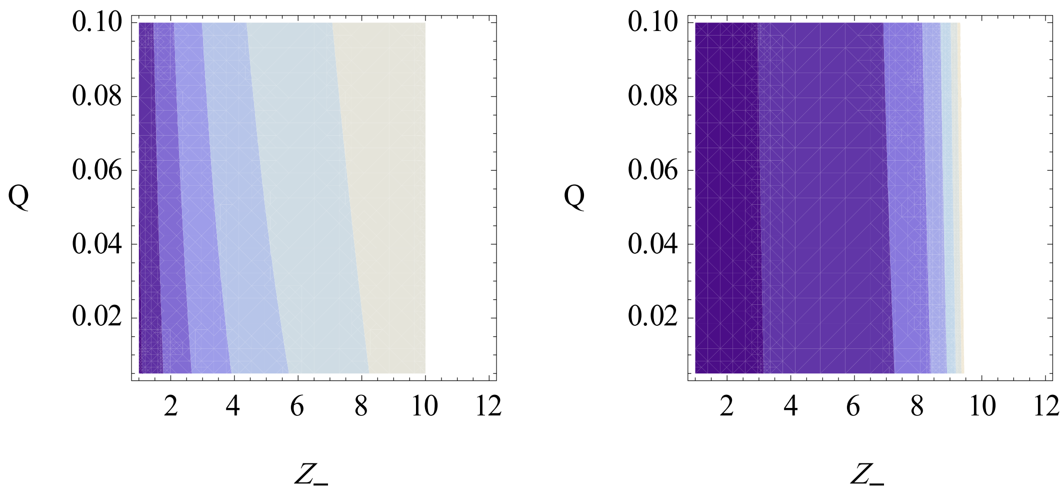

Figure 3: The contour plot of the soliton amplitude φ_0 (left) and the soliton width W (right) [defined in Eq. (21)] with Z_- and Q. Light-colored regions correspond to higher values of φ_0 and W. Here Z_+ =1, β=0.5 and ν=0.1.

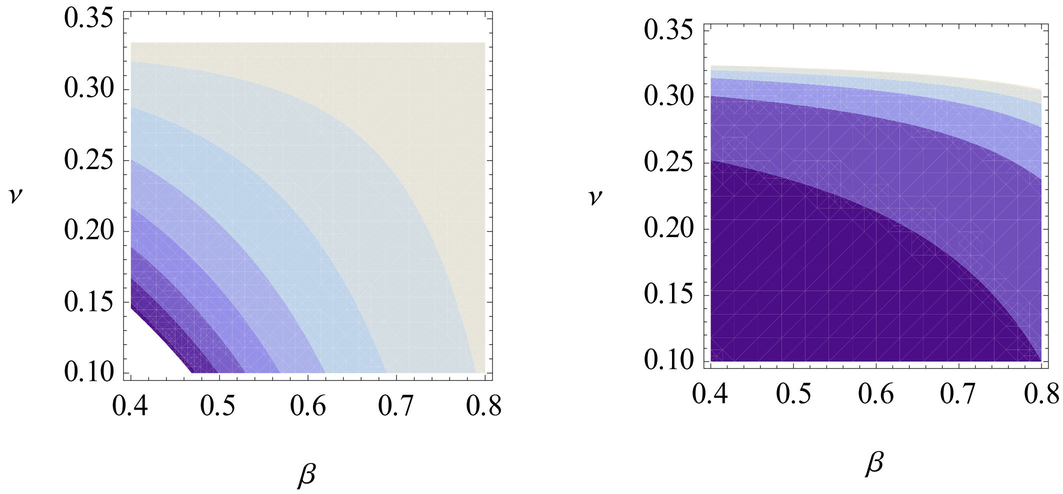

Figure 4: The contour plot of the soliton amplitude φ_0 (left) and the soliton width W (right) [defined in Eq. (21)] with β and ν. Light-colored regions correspond to higher values of φ_0 and W. Here Z_+ =1, Z_- =3 and Q=0.1.