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Current Sheet Thickness in the Plasma Focus Snowplow Model

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It is well known that the snowplow model approximates the discharge in a plasma focus device, with great accuracy and a considerable lower complexity than the full MHD model. However, one of the disadvantages of the snowplow model relies in the fact that it assumes an infinitely thin current sheet moving first axially along the electrodes. Then, in order to study the current sheet thickness, an analysis based on the full MHD model is considered (i.e. a gridless method, very useful for problems involving large deformations). The aim of this work is to perform, from a theoretical rather than a statistical point of view, an analysis of the current sheet thickness, incorporating it in a consistent way as an additional dynamic characteristic to the snowplow model. Simulations over the 2D modified snowplow model are considered and confronted with other models currently available.

Keywords: Plasma focus discharge, current sheet, plasma focus snowplow model.

1. Introduction

The plasma focus devices were developed in the early 1960s independently by Mather and Filipov in the USA and the former Soviet Union respectively. The discharge chamber consists of two coaxial electrodes separated at one end by an insulator and opened at the other end (Fig. 1). To briefly describe the operation of a plasma focus, we can tell that the discharge begins (breakdown) at the base of the gun along the insulator surface, and a radial current sheet develops, giving rise to an azimuthal magnetic field, which drives the sheet towards the top end of the electrodes. The Lorentz force sweeps up the ionized gas and leaves behind a vacuum region. This current sheet layer, which carries the high-current discharge, undergoes first an axial acceleration, and after reaching the electrodes end is subjected to a radial collapse toward the system axis. As a global result, the sheet sweeps and ionizes the neutral and undisturbed gas that finds on its way. Finally, it focuses into a plasma column at the tip of the inner electrode and a confinement is achieved by the pinch effect due to the axial current.

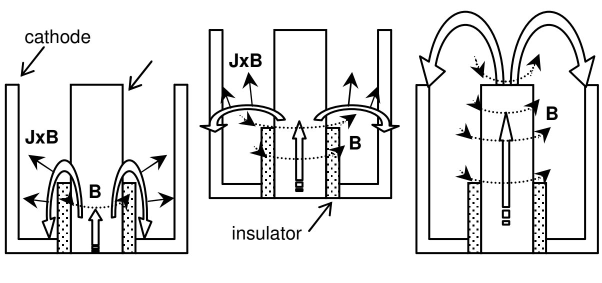

Fig.1 Scheme of a plasma focus coaxial system discharge (axial revolution) with the current sheet at three different stages.

When the current sheet flows between the electrodes and evolves towards the electrode end side, it drags with her an amount of plasma that finally it will be compressed in the final stage. At any time the current sheet moves with a certain thickness which represents the ionized gas volume. Obviously the current sheet is not an infinitely thin surface.

A simple 2 phase numerical model was developed by S. Lee in 1985 [1] to design new plasma focus machines, and to simulate the dynamics of the discharge among others interesting features. The first phase (axial phase) is currently simulated by using the called snowplow model where the magnetic field (acting like a magnetic piston) pushes the current carrying plasma toward the undisturbed gas. "The simplified two-dimensional snowplow model assumes that all the mass swept up is compressed into an infinitely thin layer immediately behind the shock, so that the magnetic piston edge, the current sheet and the shock from the same interface. Thus, the intermediate plasma region is reduced into an infinitely thin surface" [2].

In this work we show our first results using the snowplow model when the current sheet is not an infinitely thin surface. We have introduced a vector parameter which gives a certain thickness to the current sheet and we observe its evolution in time. We have used a simple

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snowplow code running under MATLAB software.

2. The model

We have used the most used snowplow model [1,4] to simulate the movement of the plasma current sheet from the basis of the electrode system to the open end side (axial phase). Principally, the current sheet is simulated using an infinitely thin thickness.

The coupled system of equations is as follow:

$$L = L_0 + \frac{\mu_0}{2\pi}\left(z_{end} \ln\left(\frac{r_{ext}}{r_{\text{int}}}\right) + \int_{\Lambda} \cos\theta \ln\left(\frac{r_{\max}}{r_{\min}}\right) d\lambda \right), \qquad (1)$$

$$\frac{\partial \vec{v}}{\partial t} = \frac{1}{\delta m}\left(\frac{\mu_0}{4\pi}\frac{I^2}{r}\delta\lambda\hat{n} - 2\pi\rho_0 r\delta\lambda(\vec{v}\cdot\vec{n})\vec{v} \right), \qquad (2)$$

$$\frac{\partial \vec{r}}{\partial t} = \vec{v}\,, \qquad (3)$$

$$\frac{\partial(\delta m)}{\partial t} = 2\pi\rho_0 r\delta\lambda(\vec{v}\cdot\hat{n}) \qquad (4)$$

$$\frac{dE}{dt} = R_P'\left(1 - \frac{E}{E_{ion}}\right) I^2\,, \qquad (5)$$

$$\frac{d(LI)}{dt} = V_C - \left(R_0 + R_{SG} + R_p'\left(1 - \frac{E}{E_{ion}}\right)\right) I\,, \qquad (6)$$

$$\frac{dV_C}{dt} = -\frac{I}{C}\,, \qquad (7)$$

where R_0 and L_0 are the resistance and inductance of the transmission line respectively; R_p is the plasma resistance, R_{SG} is the spark gap resistance; L is the addition inductance between the circuit inductance and the plasma inductance; V_C is the charging voltage in the capacitor bank; I and C are the current and the capacitance of the system. The ρ_0 parameter is the gas density at the initial time. The equations (2) and (3) are the equations for the gas layer motion, and the equation (4) represents the temporal mass evolution of the gas and following plasma layer. The time evolution of the gas layer internal energy E is given for the equation (5) where E_{ion} is the necessary energy to ionize the 80% of the gas. The r_{ext}, r_{int}, r_{max}, and r_{min} are the schematized in the figure 2; normally r_{int} and r_{ext} are known as anode and cathode radius respectively. The radius r_{max} and r_{min} are used to establish the radial evolution of the current sheet, and z_{end} is the z-coordinate in which the sheet is in contact with the cathode. The θ angle represents the slope of the current sheet respect the z axis. The feature of the plasma current sheet is described in the plane (r, z) as a function of a local coordinate λ along the profile (Fig.2).

The main task involved concerns reshaping of the plasma current sheet after a temporal iteration. The objective of this reshaping is to preserve a plasma current sheet composed by elements of the same length. After this geometrical modification, the physical properties of each element are recalculated from a vector average of the previous sheet.

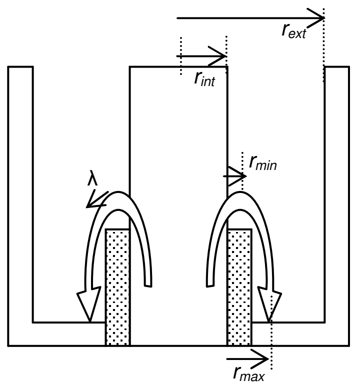

Fig.2 Scheme of the radius and local coordinate λ in the plane (r, z) involved in the equation (1).

To incorporate the sheet thickness to this scheme, as a simplest primary approach, we have considered that all the sheet elements have the same constant density ρ. After successive iterations, a new mass sheet is linked to each sheet element to obtain a dynamical value for the current sheet thickness (s parameter). The equations (1) to (7) were discretized for the necessary space and time iterations. The included s current sheet thickness parameter and the total mass of the plasma sheet (eq. 8) were also discretized to follow their space and time evolution.

We have chosen the time zero just before the discharge evolve, and the respective initial parameters for the simulation are:

$$E = 0$$
$$I = 0$$
$$L = L_0 = 1.7929\times10^{-9}\,[Hn]$$
$$V = V_0 = 2.3\times10^{4}\,[V]$$
$$R = R_0 = 0.03[\Omega]$$
$$\vec{v} = 0$$
$$\vec{r} = \vec{r}_{ins} = r_{anode} + 1.56\times10^{-3}\,[m]$$

where $\vec{r}_{ins}$ is a function of (r, z) which define the boundary of the insulator. For the other initial values, we have used experimental ones corresponding to our SPEED4 plasma focus device [5]. The mass evolution is given by the equation (4) and its initial condition is given by

$$m = m_0 = (2\pi)s\int_{\Lambda} r\rho_0 d\lambda.\,, \qquad (8)$$

where the initial current sheet thickness is given by $s = s_0$; this parameter evolve over time.

We have considered the plasma sheet as the union of a finite number of elements and they evolve in time according to a dynamic discretization using an Euler's scheme [6].

3. Results and discussion

Our first step was to test the simulation program to obtain, firstly, the full time evolution of the plasma current sheet in the axial phase with our experimental operating parameters corresponding to the SPEED4 plasma focus device.

The simulations of the full time evolution of the current sheet in the axial phase are shown, figures 3 and 4, for two anode radius at different filling gas pressures. In the figure 3 (15 mm anode radius) the sheet lines are

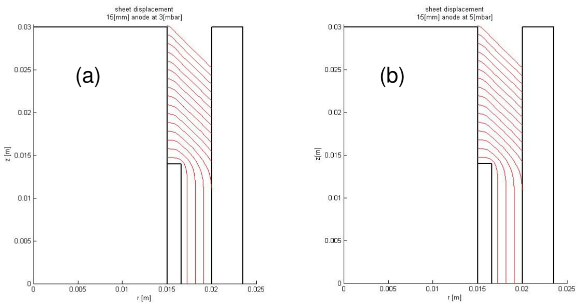

Fig.3 Time evolution of the current sheet for 15mm of anode radius at 3 mbar (a) and 5 mbar (b) of filling gas pressure.

separated by around 2,5 μsec, in the figure 4 (7 mm anode radius) the step time is around 2,0 μsec. So, we are testing the program with a current sheet axial velocity of around 0,2 cm/μsec like a first approach to obtain the representation of the sheet thickness at some given moments.

To determine a starting point for the initial thickness we have fixed a maximal limit of to the *s* parameter, this is because we use a constant value for density; the figure 5 shows three example of a real ICCD image (visible range)

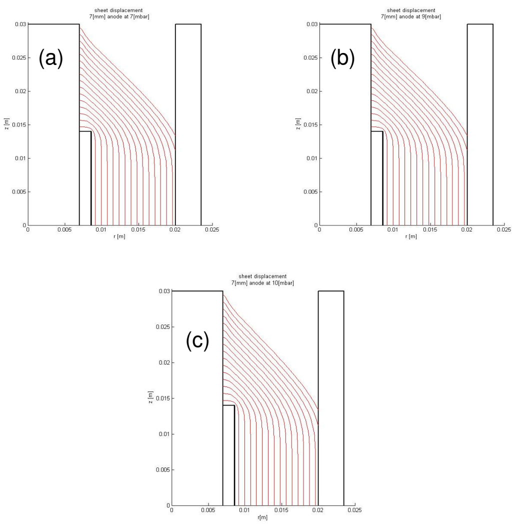

Fig.4 Time evolution of the current sheet for 7mm of anode radius at 7 mbar (a), 9 mbar (b) and 10 mbar (c) of filling gas pressure.

obtained from three different shots in the SPEED4 plasma focus device at early times, around 30 nsec after electrical breakdown.

For a total time of simulation of 6,4 μsec for an anode radius of 7 mm at 7 mbar as filling gas pressure, the feature

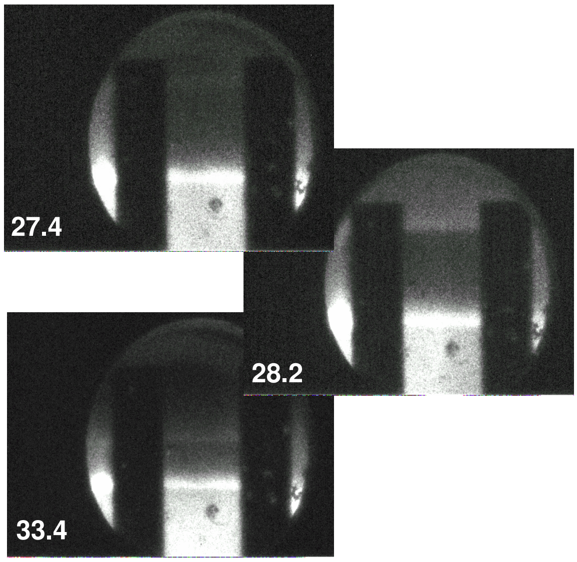

Fig.5 ICCD image obtained in the SPEED4 plasma focus device. The cathode rods, the black column shadows in the present figure, have a diameter of 9 mm; so, we can infere that the plasma sheet current thickness is not more than 1 mm after 30 nsec of electrical breakdown.

of the current sheet thickness for three given times are shown in the figure 6. We have followed the *s* parameter keeping constant the gas density and with an initial thickness of 0,1 μm.

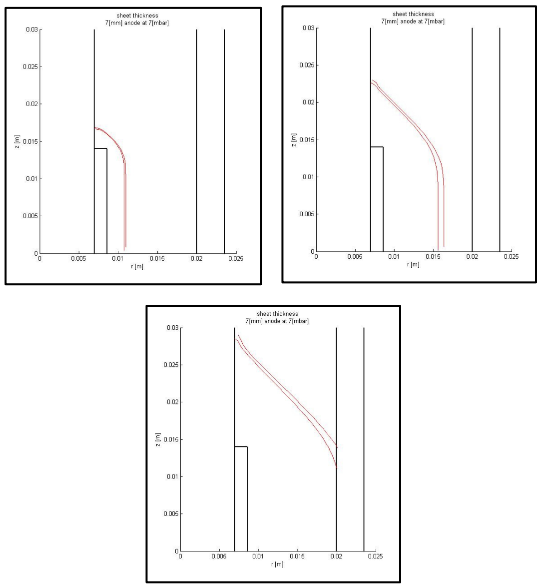

Fig.6 Time evolution of the current sheet thickness for 7 mm of anode radius at 7 mbar at 1,2 μsec, 3,6 μsec and 6,0 μsec respectively.

The figure 6 shows the current sheet thickness at 1,2 μsec, 3,6 μsec and 6,0 μsec. A reference snowplow model [2] determines a time evolution of the current sheet in the axial phase of around 7,4 μsec, a time too large to compare with our plasma focus machine. However, we will maintain this time order of magnitude to a possible comparison. For this case, we have obtained an average of 0,2 mm and 0,8 mm for the current sheet thickness at 1,2 μsec and later (3,6 μsec and 6,0 μsec) respectively, which are reasonable values for a first approach.

The figures 7 and 8 show the simulation of the current sheet thickness for an anode radius of 15 mm but at two different pressures (5 mbar and 7 mbar). The three chosen representative times for both representations were 1,5 μsec, 4,5 μsec and 7,5 μsec, and the initial thickness was also 0,1 μm.

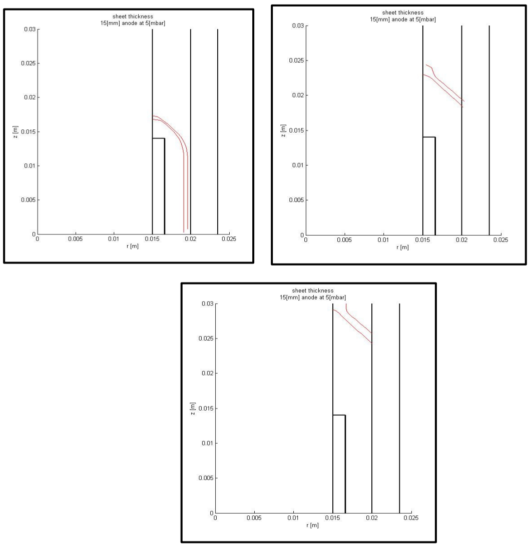

Fig.7 Time evolution of the current sheet thickness for 15 mm of anode radius at 5 mbar at 1,5 μsec, 4,5 μsec and 7,5 μsec respectively.

It can be observed than the global behavior in these cases is very different than at lower anode radius, in average, at early times the current sheet thickness is around 0.6 mm to reach 1,0 mm, for both pressures, to 7,5 μsec. At the upper edge of the current sheet is observed an anomaly for both pressures and that we could attribute to the limitation of our border conditions. However, ICCD images observed at later times, around 400 ns (Fig. 9) it is possible to observe the formation of a different feature at the top end open side of the anode, but this part of the simulation is not included in this work because it corresponds to the radial phase of the plasma current sheet evolution. In our results we can see a little discontinuity at the top of anode, because the numerical error probably produced by the Euler's method.

Qualitatively, it is possible to see that the evolution of the current sheet is close to that it is observed inside

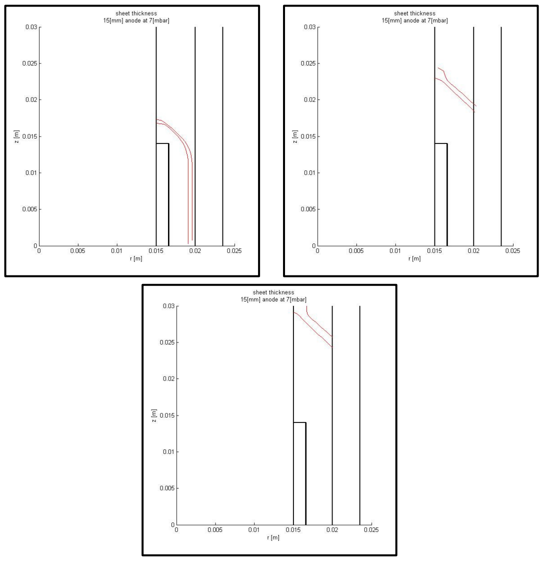

Fig.8 Time evolution of the current sheet thickness for 15 mm of anode radius at 7 mbar at 1,5 μsec, 4,5 μsec and 7,5 μsec respectively.

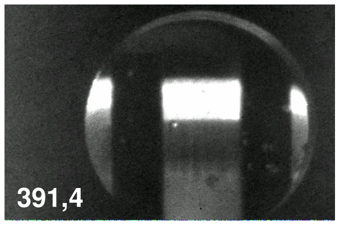

Fig.9 ICCD image at around 400 nsec when the axial phase is close to finish. The thickness of the upper side of the current sheet is around 4,5 mm.

the discharge chamber.

To make only a slight modification to the snowplow model we have been able to reproduce the behavior of the plasma layer in its first phase. We have considered a constant gas density, although there are many tests to be undertaken, the most important is to give a variable plasma density layer with which we will bring what is happening inside the chamber with plasma

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