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Studying the Formation of the Pre-Sheath in an Oblique Magnetic Field using a Fluid Model and PIC Simulation

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Pre-sheath formation in front of a floating electrode immersed in a magnetized plasma with the magnetic field at an oblique angle to the wall is studied with a one-dimensional fluid model. The model equations are integrated numerically in order to find the space profiles of the potential, density and ion velocities for various densities and angles of the magnetic field. We also obtain comparable spacial profiles from a BIT1 particle-in-cell simulation. A method is developed to compare both sets of results. The qualitative agreement of model results to simulation results is quite good.

PACS. 52.25.Xz Magnetized plasmas – 52.65.Rr Particle-in-cell method – 52.40.Kh Plasma sheaths

1. Introduction

In this paper we investigate the structure of the magnetic pre-sheath using a simple fluid model proposed by Riemann [1, 2]. A magnetized plasma boundary layer is of key interest to many fields of research, for example a scrape-off layer in a tokamak, where we have a relatively strong magnetic field applied at various angles. We tried to emulate similar conditions with this model, which would allow a quick, yet comprehensive study of the pre-sheath. In second chapter the analytical and the simulation model are described. In third chapter a part of the extensive study with the analytical model is presented. Also some comparable PIC simulations results are shown. There is a comparison made between both sets of results with the comparison method also being explained. The fourth chapter draws some conclusions and an evaluation of the model is made.

2. Model

We considered a typical boundary layer problem, where a quasineutral plasma is shielded from a negative absorbing wall by a thin positive space charge layer (sheath) with a thickness of several electron Debye lengths λ_D . $L >> \lambda_D$ where L is a characteristic extension of the boundary layer. A stationary sheath can exist only if the Bohm criterion [3] is fulfilled.

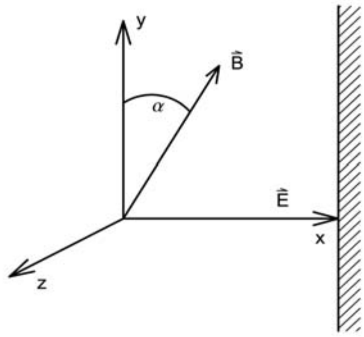

Fig. 1 Schematic of the model

In order to fulfill the Bohm criterion the ions must be pre-accelerated by a finite electric field that exists in the pre-sheath, that is the layer preceding the sheath still disturbed by the presence of the wall. If in addition a magnetic field at an oblique angle to the wall is present in the system, Chodura [8] has identified a third

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layer that sets in between the sheath and the ordinary pre-sheath, sometimes also called the "Chodura layer". In this region there is already a noticeable electric field as opposed to the ordinary pre-sheath, though much weaker than the electric field in the sheath. In this region ions are diverted from the direction of motion parallel to the magnetic field into the direction perpendicular to the wall. The characteristic length scale of the Chodura layer is of the order of a few Larmor radii of ions moving with the ion acoustic velocity $r_{Li} = \left(m_i c_s\right) / \left(e_0 B\right)$. An additional condition was later found [2] to complement the Bohm condition under magnetic field the Bohm-Chodura-Riemann condition [9]. We used a one-dimensional, collisionless system with a uniform magnetic field of different magnitudes applied at various angles similar to Zimmermann et al [10]. There are two species of singly charged particles in the plasma, positive ions and electrons. The model geometry is shown in Fig. 1. Magnetic field lies only in the *x-y* plane. The self-consistent electric field has the direction perpendicular to the wall. The equations for ion momentum and continuity are:

$$m_i \left(\vec{v} \cdot \nabla\right) \vec{v} = e_0 \left(\vec{E} + \vec{v} \times \vec{B}\right) - \frac{1}{n_i} \nabla p_i - m_i \nu_t \vec{v}$$
$$\nabla \cdot \left(n_i \vec{v}\right) = n_e \nu_i \tag{1}$$

Here ν_i is the ionization rate while ν_t is the total collision rate. The electrons are assumed to be Boltzmann distributed while both, electrons and ions are isothermal. Assuming the pre-sheath quasineutrality we can write:

$$n_e = n_i = n_s \exp\left(\frac{eU}{k_B T_e}\right), \tag{2}$$

where n_s is the charged particle density in the bulk plasma and U is the electrostatic potential. The ion pressure p_i and density n_i are related by:

$$\nabla p_i = \gamma k_B T_i \nabla n_i \,. \tag{3}$$

We assume the ion flow to be isothermal ($\gamma = 1$), despite the recent discovery [5, 7], that γ is a spatially varying quantity. In this way we avoid the variation of ion sound speed c_s with T_i.

The Bohm criterion now becomes:

$$c_s = \sqrt{\frac{k_B \left(T_i + T_e\right)}{m_i}} \tag{4}$$

We introduce the following dimensionless variables:

$$V = \frac{v}{c_s},\ X = \frac{x}{L},\ \Phi = -\frac{e_0 U}{k_B T_e},$$
$$K = \frac{L}{c_s},\ \omega K = \frac{e_0 B}{m_i}\frac{L}{c_s} = \frac{L}{r_{Li}}. \tag{5}$$

Assuming a collisionless pre-sheath $\left(\nu_i = \nu_t = 0\right)$ we can separate the ion momentum equations by its components and add a fourth equation for potential:

$$\left(V_x - \frac{1}{V_x}\right)\frac{\partial V_x}{\partial X} = -1 - \omega K \cos\left(\alpha\right) V_z,$$
$$V_x \frac{\partial V_y}{\partial X} = \omega K \sin\left(\alpha\right) V_z,$$
$$V_x \frac{\partial V_z}{\partial X} = \omega K \cos\left(\alpha\right) V_x - \omega K \sin\left(\alpha\right) V, \tag{6}$$
$$V_x \frac{\partial \Phi}{\partial X} = 1 - \frac{\partial V_x}{\partial X}$$

We now have a system of 4 equations for 3 unknown velocity components V_x(x), V_y(x), $V_z(x)$ and potential $\Phi(X)$.

In our model we can change to independent parameters: ωK which is a measure of the applied magnetic field density and the angle of the applied field α. We introduced a quantity ωK, which gives the number of the Larmor radii of an ion moving with the ion acoustic velocity. The equations (as) are then integrated using an adaptive step Runge-Kutta method. A very small value of 10^{-9} was chosen for the initial value of V_x at a large distance from the wall. The integration was stopped when unity was reached, defining the sheath edge.

We used a BIT1 [11, 12] PIC code to

simulate a similar system. The length of the system was 9cm, and was divided into 24000 cells. The temperatures of the injected particles were the same $k_B T_e = k_B T_i = 1eV$. Hydrogen ions (protons) were selected. We used the time step $5 \cdot 10^{-12} s$. The code was run in the fast mode until the steady state was reached. This state was saved, and then the code was run in the slow mode with full diagnostics. One computer run usually took around 4 days.

3. Results

The model presented is used to study the effects of the magnetic field on a collisionless pre-sheath. Apart from the three velocities and the potential aforementioned, we can also calculate two velocities with reference to the magnetic field direction: V_{par} – parallel to the magnetic field and V_{perp} – perpendicular to the magnetic field. We can also calculate

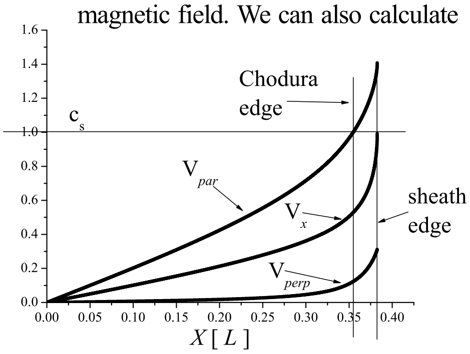

Fig. 2 Spatial dependence of $V_x(X)$, $V_{par}(X)$ and $V_{perp}(X)$

the angle of incidence for ions β. In Fig. 2. we show spatial dependence of $V_x(X)$, $V_{par}(X)$ and $V_{perp}(X)$ for $\alpha = 30°$ and $\omega K = 50$, which means a somewhat elevated magnetic field density. Formation of the magnetized pre-sheath (Chodura layer) can be well observed. If the sheath edge is defined at the position where $V_x = 1$ and the magnetized pre-sheath as the position where $V_{par} = 1$, the thickness of the Chodura layer can be determined. In this Fig.2 for example it is approximately *0.026 L*. With $\omega K = 50$ the

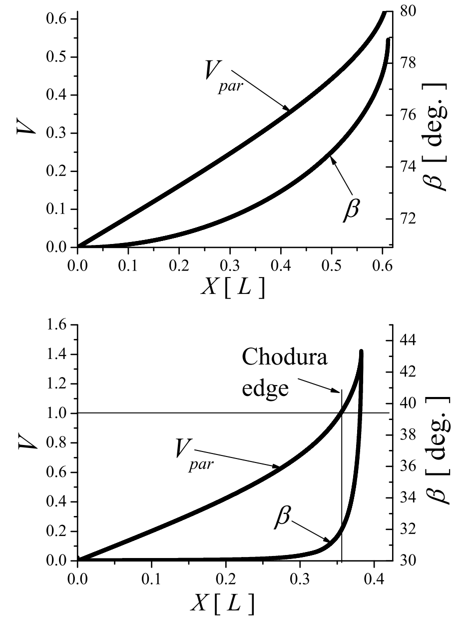

Fig. 3 Spatial dependence of $V_{par}(X)$ and the angle of incidence β of the ion flow

thickness of the Chodura layer can be estimated to approximately $1.3 r_{Li}$. Note how V_{perp} gains the largest part of its increase in the Chodura layer where the effects of the electric and magnetic field become comparable. In Fig. 3

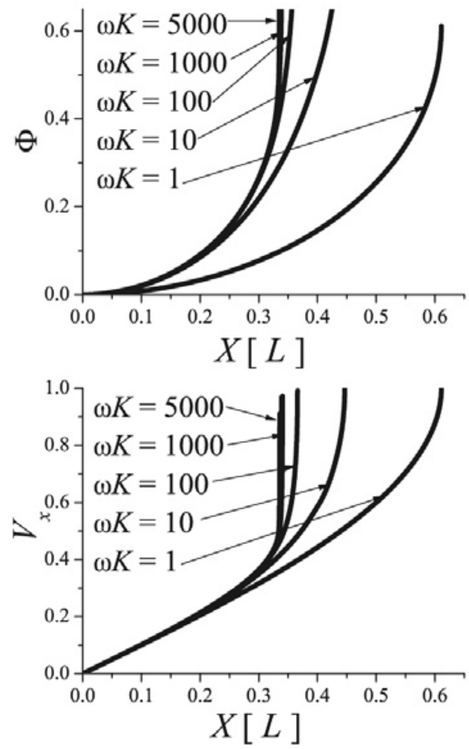

Fig. 4 Axial profiles of $\Phi(X)$ and $V_x(X)$ for various values of ωK

we show a comparison between the weak and the strong magnetic field. The parallel velocity V_{par} and the angle of incidence of the ion flow are shown versus X. For the top figure we select $\alpha = 30°$ and $\omega K = 1$, while for the bottom figure we select $\alpha = 30°$ and $\omega K = 50$. As we can see in the top picture, weak magnetic field only has little effect on the direction of the ion flow and no Chodura layer is formed. When the strong magnetic field is applied (bottom figure) the Chodura edge can be well identified. In the pre-sheath the ion flow is practically in line with the magnetic field. Note that β is almost exactly equal to the selected angle $(\alpha = 30°)$.

Further illustration of the effect of the increasing magnetic field density is shown in Fig. 4, where the axial profile of the velocity $V_x(X)$ and the potential $\Phi(X)$ are plotted for several values of ωK, while α is kept constant $\alpha = 30°$. As ωK is increased the length of the pre-sheath decreases, while the gradient of the potential increases by absolute value. Larger electric field in the Chodura layer is required for stronger acceleration of the ions in the direction perpendicular to the wall. One also notes that there is a limit in the pre-sheath length. Above a certain value of ωK the length of the pre-sheath becomes insensitive to further increase of ωK.

Next we will present the results of the computer simulations. The example shown in the Fig. 5 was made at an intermediate angle $(\alpha = 20°)$ and for 4 different magnetic (B=0.005 T, B=0.01 T, B=0.1 T and B=1 T). The output of the BIT1 diagnostics only gives two velocity profiles: the velocity component perpendicular to the wall v_x and the velocity parallel to the magnetic field v_{par}. Both are given in absolute units m/s. In Fig. 5 also the potential and density profiles are shown. The potential U is given in volts and the density n in the number of charged particles per cubic meter. The results are as expected and in accordance with theoretical model. When the magnetic field is increased v_{par} increases also. Note that v_{par} exceeds v_x on the entire length of the system. This indicates that v_{par} very probably

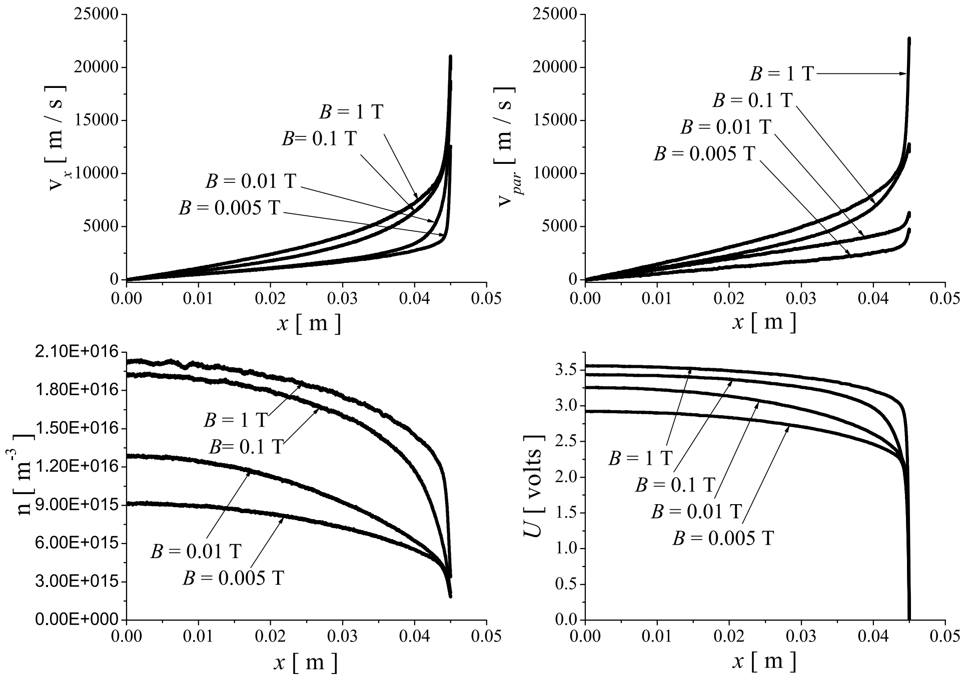

Fig. 5 The velocities v_x and v_{par}, ion density n and potential U obtained from PIC simulation. Only the right half of 9cm long system is shown

exceeds the ion sound speed already at some place before the sheath entrance. The density increase with the increasing magnetic field can be explained by the inhibited transport of charged particles towards the wall across the magnetic field lines.

Defining the length scale L for the PIC results is not unambigous. As the example in Fig. 6 shows the main problem is a correct determination of the length scale L that corresponds to the simulation parameters. This length scale is defined in (5) in such way that the parameter ωK gives the number of the ion Larmor radii r_{Li} that fit into L. The ion Larmor radius is calculated for an ion that moves with ion acoustic velocity.

For the computer simulation shown in Fig. 7 we select $\alpha = 20°$, $B = 1T$ and the temperatures of the ions and electrons at injection are $k_B T_e = k_B T_i = 1eV$. However, by the time the system reaches steady state the temperatures drop to $k_B T_e = 0.9eV$ and $k_B T_i = 0.8eV$ respectively. Also they are space dependent with highest values in the middle of the system and decreasing towards both boundary electrodes. We must take these two temperatures as constants so that we can calculate the ion sound speed according to (4). We get approximately $c_s \approx 13000 m/s$. The next task is to find ωK for the numerical model that corresponds to the magnetic field $B = 1T$ in the simulation. The parameter ωK is given by (5). Because L is rather arbitrarily selected length scale for the pre-sheath, which is not known, we proceed in the following way. We select an initial value for ωK and solve (6). The value of X where V_x reaches unity is identified. With this value of X the length scale L is found from (5) with x=0.045m. This is simply one half of the length of the simulated system. Very small thickness of the sheath is neglected – a visual conformation of this statement can easily be made. If we look at the bottom picture in Fig. 6 which shows V_x, the sheath is represented by the part of the curve (PIC simulation), where $V_x \geq 1$, which is indeed a very small length comparing to the length of the whole system. It would be difficult to establish the precise sheath edge anyhow. After that a new approximation

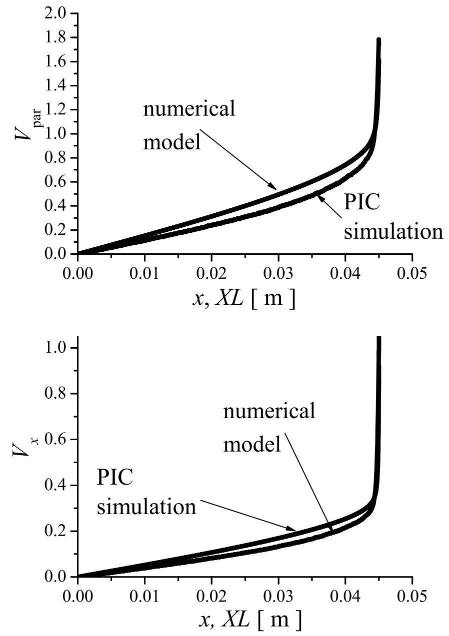

Fig. 6 Comparison of V_{par} and V_x obtained from numerical model and PIC simulation

for ωK is found from (5) with the values of B, m_i and c_s found from the simulation. Then (6) is solved again with the new ωK and a new value of X where V_x reaches unity is found and the procedure is repeated. This procedure converges rather quickly. In the case shown in Fig. 7 the value is $\omega K = 1751.39$. This gives $L = 0.233m$. Such value of L is in very good agreement with the value of X, where V_x reaches unity, which is in the case shown in Fig. 7 $X = 0.193$. We then calculate the profiles of V_x and V_{par} and compare them with the results of the simulation normalized to c_s. The comparison shown in Fig. 6 shows quite good qualitative and to some extent even quantitative agreement.

4. Conclusions

A one-dimensional fluid model was presented for a collisionless magnetized pre-sheath. We have compared the results with the results of the PIC computer simulation made by BIT1 code in order to establish the usefulness of the analytical method. A quantitative comparison was only made possible by the correct determination of the length scale *L*. A simple method for the correct determination of *L* was developed and presented.

The results of the analytical model were in very good agreement with the simulation results. Therefore it was established that an analytical model of this sort, which offers a comprehensive analysis of the magnetic pre-sheath, can, up to a certain point, replace the time-costly PIC simulation.

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References

[1] K. U. Riemann, Contrib. Plasma Phys. **34**, 127 (1994)

[2] K. U. Riemann, Phys. Plasmas, **3**, 552 (1994)

[3] D. Bohm, *The Characteristics of Electrical Dischargesin Magnetic Fiels* (McGraw-Hill, New York, 1949), Chap. 3, p.77

[4] K. U. Riemann, J. Phys. D: Appl. Phys., **24**, 493 (1991)

[5] S. Kuhn, K.U. Riemann, N. Jelić, D. Tskhakaya, M. Stanojević, Phys. Plasmas, **13**, 013503 (2006)

[6] N.Jelić, K.U. Riemann, T. Gyergyek, S. Kuhn, M. Stanojević, J. Duhovnik, Phys. Plasmas, **14**, 103506 (2007)

[7] N. Jelić, T. Gyergyek, S. Kuhn, J.Krek, J. Duhovnik, to be published in the *Proceedings of the SPPT 2008*

[8] R. Chodura, Phys. Fluids, **25**, 1628 (1982)

[9] M. Stanojević *et al*. Plasma Phys. Control Fusion, **47**, 685 (2005)

[10] T.M.G. Zimmermann, M.Coppins, J. E. Allen in the *Proceedings of the 28th ICPIG, July 15-20, 2007, Prague, Czech Republic)*, (Institute of Plasma Physics of the Academy of Sciences of the Czech Republic, Prague, 2007), 335

[11] D. Tskhakaya, R. Schneider, J. Comp. Phys. **225**, 829 (2007)

[12] D. Tskhakaya, S. Kuhn, Y. Tomita, K. Matyash, R. Scneider, F. Taccogna, Contrib. Plasma Phys., **48**, 121 (2008)