Transit-Time Effects on Cyclotron-Resonance Heating

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(Received: 25 August 2008 / Accepted: 19 December 2008)

Particle accelerations by an Alfvén-wave pulse in comparison with the usual cyclotron-resonance acceleration/heating theory are investigated analytically and numerically. Usual linear theory is based on the following two obvious approximations: 1. sinusoidal wave is introduced, and 2. nonlinear effects are ignored. Consequently, the theory breaks down during initial as well as later times of resonance. To overcome such difficulties a simple-minded theory based on transit-time acceleration due to a square-wave pulse is introduced. It demonstrates that in realistic plasmas the basic process that prevails during linear cyclotron-resonance heating is transit-time acceleration due to finite-sized pulses.

Keywords: pulse-particle interaction, wave-particle interaction, cyclotron-resonance, transit-time acceleration, cosmic rays, heating, damping

1. Introduction

Thus far particle acceleration mechanisms due to electromagnetic waves have extensively been investigated for various ends [1]. If we could classify waves into sinusoidal waves and pulses, then corresponding acceleration mechanisms of charged particles by such waves become either resonance-type acceleration [1] or transit-time acceleration [2-4]. The former is well known; the latter had been applied to various problems such as Fermi acceleration of cosmic rays [3] and plasma heating for nuclear fusion [4]. Here, in this research acceleration of beam-like protons initially traveling along an external magnetic field by an Alfvén wave-pulse, including a sinusoidal wave, will be studied with particular emphasis on transit-time acceleration and its transition to the usual cyclotron-resonance acceleration. The pulse to be studied has a square (or rectangular) envelope with zero rise- and fall-times except in subsection 2.3. Inside the pulse is a carrier wave with a left circularly polarized electric field (and its induced magnetic field) such as

$$E_0 \left\{\hat{x}\cos(k\ z-\omega\ t+\theta)-\hat{y}\sin(k\ z-\omega\ t+\theta)\right\},$$

which is directed perpendicular to the z-axis, along which the pulse propagates at group velocity v_g; here, E_0, z, t, $\hat{x}$, $\hat{y}$ are the amplitude, the longitudinal position, and the time, unit vectors in the x and y directions, respectively, and θ is the phase constant; furthermore, k (ω) is the wave number (angular frequency) of the carrier wave. In the following we will focus on proton/ion acceleration.

2. Theory

2.1 Review of the usual cyclotron heating theory

If the wave amplitude is sufficiently small such that nonlinear effects may be ignored, one can utilize the perturbation method to the equation of motion of a proton/ion [2] to obtain the perpendicular components v_x, v_y at time t of initially beam-like particles with $v_z = v_0$ and $v_x = v_y = 0$ as below.

$$v_x = \frac{eE_0}{m}\frac{1-kv_0/\omega}{kv_0-\omega+\Omega_p}\times\left[\sin\{(kv_0-\omega)t+kz_0\}-\sin(\Omega_p t+kz_0)\right]$$
$$v_y = \frac{eE_0}{m}\frac{1-kv_0/\omega}{kv_0-\omega+\Omega_p}\times\left[\cos\{(kv_0-\omega)t+kz_0\}-\cos(\Omega_p t+kz_0)\right] \quad (1)$$

Here, Ω_p is the proton/ion cyclotron frequency.

It is quite significant to note the fact that Eqs.(1) depict perpendicular velocity shifts of a proton/ion that enters a square "pulse" of Alfvén wave at t=0, and exits after interaction time t. In this sense the acceleration mechanism is the transit-time acceleration that is valid for pulses [2] rather than the usual cyclotron resonance that is valid for sinusoidal waves with infinite extent [1]. In what follows let us interpret the forthcoming results, emphasizing this point of view.

Next, one can calculate from Eqs. (1) above and their parallel counterpart the gain in kinetic energy of protons/ions as follows.

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$$\Delta\left(\frac{1}{2}mv^2\right)=\frac{e^2E_0^2}{m}\left(1-\frac{kv_0}{\omega}\right)\frac{\sin\left\{kv_0-\omega+\Omega_p\right\}t}{kv_0-\omega+\Omega_p}$$

The power absorbed by protons/ions may be obtained by first multiplying this kinetic energy increase by a one-dimensional longitudinal distribution function f (v).

$$P(t)=\int f(v_0)\left\{\frac{d}{dt}\Delta\left(\frac{1}{2}mv^2\right)\right\}dv_0$$
$$=\frac{e^2E_0^2}{m}\int f(v_0)\left(1-\frac{kv_0}{\omega}\right)\frac{\sin\left\{kv_0-\omega+\Omega_p\right\}t}{kv_0-\omega+\Omega_p}dv_0 \qquad (2)$$

Here, to simplify the velocity integral the sinc function, which is present on the right-hand side of Eq. (2) is usually approximated in the limit $t\to\infty$ as below.

$$\lim_{t\to\infty}\frac{\sin\left\{kv_0-\omega+\Omega_p\right\}t}{kv_0-\omega+\Omega_p}=\frac{\pi}{k}\delta\left(v_0-\frac{\omega-\Omega_p}{k}\right) \qquad (3)$$

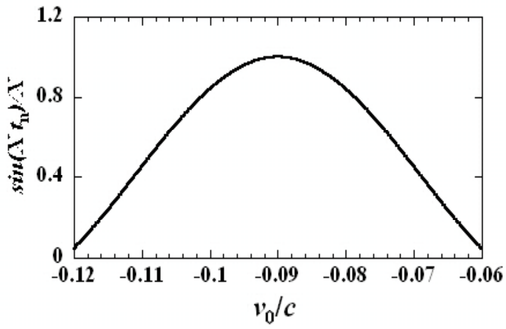
In this limit the square-wave pulse is transformed into a sinusoidal wave, which is infinitely long. As a result of this approximation the integral is completed to yield the following power.

$$P_{old}=\int f(v_0)\left\{\frac{d}{dt}\Delta\left(\frac{1}{2}mv^2\right)\right\}dv_0$$
$$=\frac{\pi e^2E_0^2}{mk}\left(-\frac{\Omega_p}{\omega}\right)f\left(\frac{\omega-\Omega_p}{k}\right) \qquad (4)$$

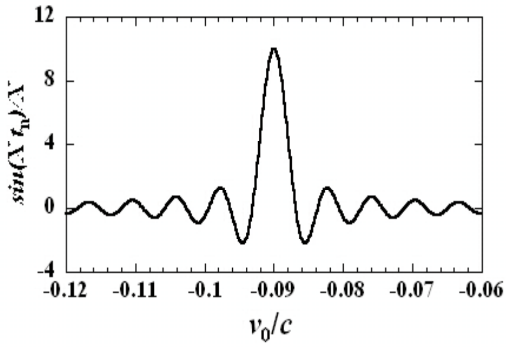
This is a rather well-known result. Up until this point we have made two approximations. One is introduction of a sinusoidal wave, which is represented by approximation (3). However, sinusoidal waves are ideal entities that never exist in reality. When they were introduced the time t in Eq. (3) was made as large as infinity. However, before reaching the long limit one enters the nonlinear regime where particle trapping becomes crucial, and the theory faces the breaking down of the linear approximation. These dilemmas are brought by the introduction of a sinusoidal wave into the above theory.

2.2 emergence of lower limit and validity of sinusoidal wave approximation

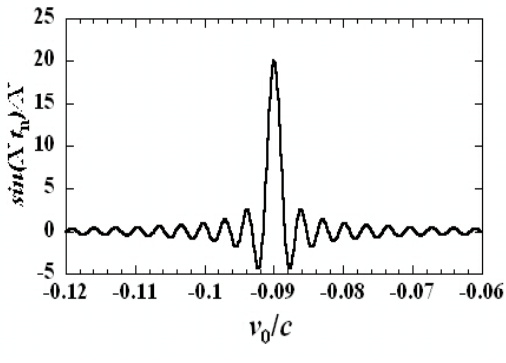
Here, let us evaluate the validity of introducing a sinusoidal wave by plotting the sinc function as a function of $v_z=v_0$ at various times. This way it will be revealed that after what time the sinc function may be effectively approximated by a delta function like Eq. (3). Figs.1(a), (b), and (c) depict the sinc functions at three separate times $t_n=\omega t=1$, 10, and 20 respectively. Here, we assume the phase speed of the wave equals $v_p=0.01c$. Since throughout the calculation the magnetic field strength will be held constant at $\Omega_p=10\,\omega$, the protons with longitudinal velocities such as $v_0\approx(\omega-\Omega_p)/k=-0.09c$ are cyclotron-resonant.



(a) t_n=1



(b) t_n=10



(c) t_n=20

Fig. 1 The sinc function is plotted at three different times.

At around the resonance velocity the sinc function yields a broad peak as in Fig. 1(a), and it is, being too broad in width and low in height, clearly premature to be approximated by a delta function for $f(v)$ that extends from v=0 to the velocity range shown in Fig. 1. The peak is somewhat too broad even at t_n =10 to regard it as a delta-function. However, at t_n =20, the peak appears sufficiently tall and narrow to satisfy the approximation (3). Actually, the peak height is proportional to t_n as can easily be deduced from Eq. (3). Thus, the sinusoidal-wave approximation is valid at t_n >10.

2.3 emergence of upper limit and validity of sinusoidal wave approximation

In this subsection we will compare analytical and numerical solutions of the equation of motion of protons driven by an Alfvén-wave pulse with a Gaussian profile,

$$E_y(z,t) = E_0\, e^{-\{(z-v_g t)/L\}^2 + i(k\ z-\omega\ t+\theta)};$$

which is more general than the square pulses investigated so far. Here L determines the pulse length(width) and θ is the phase constant. Starting from the relativistic equation of motion, and applying the perturbation method, analytical solution for the maximum perpendicular velocity shift of a proton/ion with initial longitudinal velocity v_o throughout the pulse-particle interaction can be obtained as follows.

$$(\Delta v_\perp)_{\max} = \frac{\sqrt{\pi}}{2}\frac{eE_0\,\Delta t}{m\gamma_o}\times \left\{ e^{-(\omega_o+\Omega/\alpha\gamma_o)^2\Delta t^2/4} + e^{-(\omega_o-\Omega/\alpha\gamma_o)^2\Delta t^2/4} \right\}$$

Here, $\alpha = 1 - v_0/v_p$, $\beta = (v_0 - v_g)/(v_0 - v_p)$, and $\Delta t = L/(\beta v_p)$, with v_p being the phase velocity of the Alfvén wave. Also, the symbol γ_o denotes the relativistic factor. This analytic expression will be compared to its numerical counterpart. To obtain numerical solutions the relativistic equation of motion is solved with the use of the 4th order Runge-Kutta scheme. However, due to the nature of this problem relativistic effects do not play a crucial role.

Plotted in Fig. 2 are final perpendicular velocity shifts of initially beam-like protons propagating at initial velocity v_0 along the external magnetic field after interacting with an Alfvén wave-pulse with $E_n = eE_0/mc\,\omega$=0.0001, $v_p = v_g$ = 0.01c and $L_n = L/(c/\omega)$ = 0.1 and 0.5, computed and plotted as a function of v_0/c, where c denotes the speed of light. Since the magnetic field strength is held constant such that Ω_p=10 ω, the protons with $v_o \approx -0.09c$ are cyclotron-resonant, as mentioned previously.

Overall agreement between the theory (circles) and numerical results (solid curves) is excellent. Dominant features of linear cyclotron-resonance acceleration by an Alfvén-wave pulse are, as demonstrated in Fig.2, that the

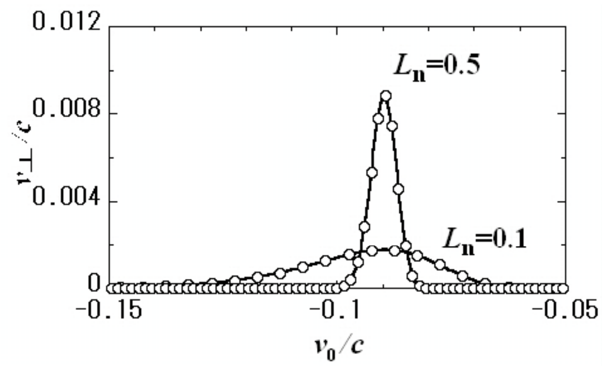


Fig. 2 Initial velocity dependence of cyclotron resonance accelerations with various pulses

resultant velocity shifts form a single peak at the cyclotron resonance velocity $v_0 \approx -0.09$, and the peak velocities are greater than the quiver velocity cE_n=0.0001c by a factor of 20-100, which is a major effect of resonance. In addition, as the shifts in longitudinal velocity in the linear regime are negligible and the perpendicular velocity shifts are dominant, the entire process is similar to transit-time accelerations, in which particles penetrate an entire electromagnetic pulse [2].

As the pulse length is increased from L_n = 0.1, the curve shows a sharper peak approaching a delta function. However, when L_n = 2.0 the resonance velocity deviates significantly from the value of -0.09c [5]. As the pulse is further elongated the resonance becomes multiple owing to particle trapping. The multiple resonances become more pronounced as the pulse is made longer than L_n = 8 (not shown). And with somewhat longer pulses the heights as well as width of resonances remain almost fixed no matter how long the pulses are made [5]. Therefore, well inside this nonlinear time scale it is next to impossible to apply the delta-function approximation since the linear results are not valid anymore. A similar conclusion may be obtained for square pulses. This nonlinear trapping effect yields an upper limit in the validity of Eq. (3).

In passing, we note that it is quite interesting to observe increased nonlinearity as a pulse is elongated further, and approaches a sinusoidal wave, which is usually thought to be a linear entity. It happens because as the pulse is made longer the interaction time becomes comparable to the nonlinear trapping period. Although these results were obtained for Gaussian shaped pulses, they hold true for square pulses qualitatively.

Consequently, the usual cyclotron-resonance theory has a lower- as well as an upper-limit for its validity. The former is not known, and is a new result of this research. Occasionally, should the pulse-amplitude is relatively large, it is possible for those two limits to overlap with each other, and the linear theory becomes nowhere valid!

2.4 Transit-time cyclotron acceleration

The presence of the two limits tends to restrict the linear cyclotron resonance theory. Here, therefore, a new but simple-minded theory which is embedded in the usual one is excavated, and presented with some extension. This theory is based on the transit-time acceleration due to a square pulse-wave. We first recall that the perpendicular velocity components (1) are equivalent to those of initially beam-like particles with velocity $v_z = v_0$ which enter a square wave-pulse of length $v_{0p}\,t$, and exits at t, where v_{0p} is the relative pulse-particle speed. Then, the power absorbed by particles is given simply by Eq.(2). Note that Eq.(2) is time-dependent, and does not have the lower limit for its validity. In the limit of infinite t (sinusoidal wave approximation), Eq.(4) is recovered. This property means that the basic process in cyclotron-resonance wave-particle interaction is the transit-time acceleration due to pulses. It may be called *transit-time cyclotron acceleration* utilized as the title of this sub-section. The usual cyclotron resonance is a special case of this process applied to an infinitely long pulse, i.e., a sinusoidal wave.

Finally, Fig. 3 shows the ratio of powers obtained by the transit-time theory and the usual theory as a function of normalized time $t_n = \omega t = \omega_0 t$ in the presence of a flat-top distribution of protons at $- 0.1c < v_0 < 0.1c$. This figure represents a main result of this report. If the ratio is unity, the cyclotron transit-time theory becomes degenerate with the typical theory for cyclotron resonance. Figure 3 reveals that at t_n <20 the usual theory may yield significant errors, oscillating up and down as a function of time. Particularly, at t_n <5 it may break down considerably, and the transit-time theory must be used there.

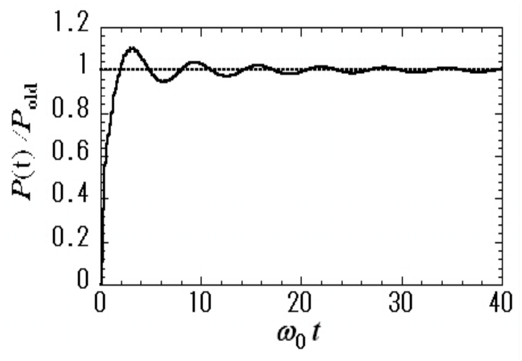


Fig.3 The ratio of $P(t)$ based on transit-time theory normalized by P_{old} based on existing theory is plotted as a function of normalized time.

3. Conclusions

First, a linear perturbation theory is successfully applied to protons (and ions) interacting with a small-amplitude Alfvén-wave square-pulse through transit-time acceleration. Then, based on this formalism a simple theory to obtain the power and, therefore, the damping rate for transit-time cyclotron acceleration is obtained. In the limit of a sinusoidal wave, the theory becomes identical with the usual linear theory for cyclotron-resonance heating. This fact demonstrates that the basic process of cyclotron resonance in realistic plasmas, where numerous pulses are generated, is transit-time acceleration of particles due to pulses.

It is known that as the wave amplitude increases particle trapping becomes important. In the presence of pulses, however, incoming particles tend to be reflected by the pulses instead of being trapped inside them. This is even more so for non-dispersive waves such as Alfvén-waves investigated here in this paper. Hence, a conclusion to be drawn here is that cyclotron-resonance scenario has to be modified significantly both in the linear as well as nonlinear regimes, if one tries to seriously investigate pulse-particle interactions. To short pulses in particular the usual cyclotron-resonance theory may not be applied, and instead transit-time acceleration approach should be utilized. More specifically, if the number of period contained in a pulse becomes less than four or five the usual cyclotron resonance theory starts to break down.

Possible applications of this research include interaction of particles with shock waves in space and lab as well as plasma heating/acceleration by electromagnetic solitons that are generated by intense laser-plasma interactions [6, 7].

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