Electric Potential in a Magnetized Plasma with Magnetic Field Increasing toward a Wall

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Electric potential in cusp magnetic field in a negative ion source is investigated analytically. A magnetic field that is symmetric about an axis and increases monotonically toward the wall is considered. The energy space is divided into three regions, that is, the reflection region that ions are reflected at the turning point toward the wall, the passing region, and the reciprocation region that ions are reciprocated between the two turning points. The potential profile is analyzed by solving the plasma-sheath equation that gives the electric potential in the plasma region and the sheath region near the wall self-consistently. The potential in the plasma region depends on the profile of the magnetic field and the ion temperature. As the increase of the magnetic field becomes large and the ion energy decreases, the potential drop in the plasma region decreases. On the other hand, dependence of the potential in the sheath region on the profile of the magnetic field and the ion temperature is small.

Keywords: negative ion source, cusp magnetic field, cusp loss width, heat transmission coefficient, electric potential, magnetized plasma, plasma region, sheath region, plasma-sheath equation,

1. Introduction

Neutral beam injection (NBI) using negative ion source is one of the most promising method of heating plasma confined magnetically in Tokamak. Plasma in a negative ion source is confined by a cusp magnetic field in order to reduce plasma loss on the wall. However, plasma particles that arrive to the cusp magnetic field move along the magnetic field and are lost through the cusp magnet region on the wall. Therefore, in plasma confinement it is important to investigate the plasma loss through the cusp magnetic field, especially the width of the plasma loss region, the so-called ‘cusp loss width’. It has been described that the cusp loss width of electron energy depends on a heat transmission coefficient defined by the ratio of the heat flux to the particle flux multiplied by the electron temperature along the magnetic field [1]. The heat transmission coefficient is related to the sheath potential near the wall surface.

Emmert *et al.* investigated formation of the potential considering both the plasma region and the sheath region self-consistently by using a plasma-sheath equation [2]. Electric sheath between a metal surface and magnetized plasma was studied by considering with the particle orbits by U. Daybelge and B. Beion [3]. They assumed that the magnetic field was uniform and shown that the sheath potential was essentially independent of the angle of magnetic field. Sato *et al*. extended the method of Emmert *et al.* to a case of magnetized plasma. In their analysis, the magnetic field of which strength decreases monotonically toward the wall was considered [4]. However, the potential formation in the plasma and the sheath regions for the case of the magnetic field of which strength increases toward the wall such as the cusp magnetic in the negative ion sources has not been clearly understood.

In this paper, we will investigate the potential profile near the wall in a magnetic field increasing monotonically toward the wall. The plasma-sheath equation is derived and solved self-consistently over the whole region from the plasma to the wall.

2. Model

The geometry of analytical model is shown in Fig. 1. The electric potential $\phi(z)$ is assumed to be symmetric about z=0 and decreases monotonically for z>0 and zero at z=0. The magnetic field is assumed to be symmetric about z=0 and increases monotonically for z>0 and B_0 at z=0.

3. Analysis of Electric Potential

Constant energy E of an ion in the z-direction is

$$E = \frac{1}{2} M(\upsilon_{\perp}^2 + \upsilon_{//}^2) + q\phi(z), \qquad (1)$$

where M is the ion mass, $\upsilon_{\perp}$ and $\upsilon_{//}$ are the velocities perpendicular and parallel to the magnetic field, and q is the charge of the ion, respectively. The magnetic moment is

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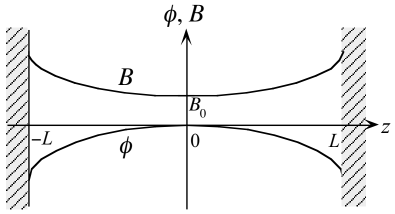

Fig.1 The geometry of the potential and the magnetic field in the analysis.

given by

$$\mu = (1/2)\, M\upsilon_\perp^2 / B(z), \qquad (2)$$

where $B(z)$ is the magnetic field at the position z. The kinetic equation in the phase space (z,E,μ) is described by

$$\sigma\upsilon_{//}(z,E,\mu)\frac{\partial f(z,E,\mu,\sigma)}{\partial z} = S(z,E,\mu), \qquad (3)$$

where $\sigma(=\pm1)$ is the direction of the particle motion, $f(z,E,\mu,\sigma)$ is the distribution function, and $S(z,E,\mu)$ is the source function. The velocity parallel to the magnetic field is given by

$$\upsilon_{//} = [(2/M)\{E - \mu B(z) - q\phi(z)\}]^{1/2}. \qquad (4)$$

We assume a symmetry about z=0, that is, $\phi(-z)=\phi(z)$, $B(-z)=B(z)$, and $S(-z,E,\mu)=S(z,E,\mu)$. Furthermore, we assume that particles are not reflected at the wall, then the boundary condition of the distribution function is $f(-L,E,\mu,+1)=f(L,E,\mu,-1)=0$.

In the magnetic field of which strength increases toward the wall, dependence of $-\mu B(z)-q\phi(z)$ on z is divided into two cases as shown in Fig. 2. The case (i) is increase of $\mu B(z)$ is larger than the decrease of $q\phi(z)$, that is, $\mu(B(z)-B_0)>-q\phi(z)$. In this case, $-\mu B(z)-q\phi(z)$ becomes upwards convex as shown by the solid line. The case (ii) is increase of $\mu B(z)$ is smaller than the decrease of $q\phi(z)$, that is, $\mu(B(z)-B_0)<-q\phi(z)$. In this case, $-\mu B(z)-q\phi(z)$ becomes downwards convex as shown by the dotted line. Where monotonic decrease of the electric potential and monotonic increase of the magnetic field are assumed.

In each case, particle motion is divided into some regions depending on it's energy, which comes from the conditions that $\upsilon_{//}$ must be real number, that is, $E-\mu B(z)-q\phi(z)\geq 0$ as shown in Fig. 3. For the case of upwards convex, ion in the energy region of $E>\mu B(\pm L)+q\phi(\pm L)$ can reach and pass thorough the center of the plasma. Ion in the energy region of $E_{min}<E<\mu B(\pm L)+q\phi(\pm L)$ cannot reach the wall and repeats the reciprocation between the turning points $z=-z_t(E)$ and $z_t(E)$, where $E_{min}=\mu B(z)+q\phi(z)$. Ion in the energy region of $E<E_{min}$ cannot exist. On the other hand, for the case of downwards convex, ion in the energy region of $E>\mu B_0$ can reach and pass thorough the center of the plasma. Ion in the energy region of $E_{min}<E<\mu B_0$ cannot reach the center of the plasma and reflected at the turning point $z=z_t(E)$ for $\sigma=-1$ and $z=-z_t(E)$ for $\sigma=1$. Ion in the energy region of $E<E_{min}$ cannot exist.

The distribution functions $f(z,E,\mu,\sigma)$ for the cases of (i) and (ii) are obtained for $\sigma=\pm1$ by integrating Eq. (3) for particle trajectory on the boundary conditions. The sum of the distribution functions about $\sigma=\pm1$ for each energy region of the cases (i) and (ii) becomes

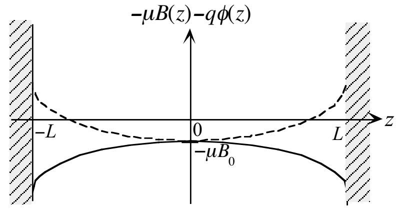

Fig.2 Dependence of $-\mu B(z)-q\phi(z)$ on z. Solid line is for the case of $\mu(B(z)-B_0)>-q\phi(z)$ and dotted line is the case of $\mu(B(z)-B_0)<-q\phi(z)$.

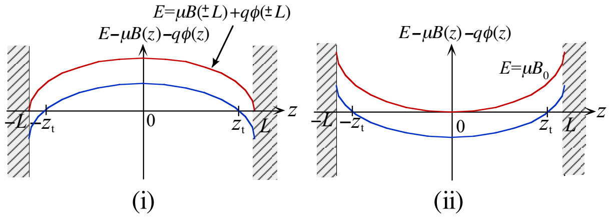

Fig.3 Energy spaces of the ion of (i) upwards convex and (ii) downwards convex.

$$\sum_\sigma f(z,E,\mu,\sigma) = \begin{cases} 2\int_0^L \dfrac{S(z',E,\mu)}{\upsilon_{//}(z,E,\mu)}\mathrm{d}z', & (E > \mu B(\pm L) + q\phi(\pm L)), \\ 2\int_0^{z_t(E,\mu)} \dfrac{S(z',E,\mu)}{\upsilon_{//}(z,E,\mu)}\mathrm{d}z', & (E_{\min} < E < \mu B(\pm L) + q\phi(\pm L)), \end{cases} \qquad (5)$$

and

$$\sum_\sigma f(z,E,\mu,\sigma) = \begin{cases} 2\int_0^L \dfrac{S(z',E,\mu)}{\upsilon_{//}(z,E,\mu)}\mathrm{d}z', & (E > \mu B_0), \\ 2\int_{z_t(E,\mu)}^L \dfrac{S(z',E,\mu)}{\upsilon_{//}(z,E,\mu)}\mathrm{d}z', & (E_{\min} < E < \mu B_0), \end{cases} \qquad (6)$$

respectively, where z' is the position of ion generation.

The ion density is obtained by integrating $f(z,E,\mu,\sigma)$ over the E-μ space as [4]

$$n_i(z) = \frac{2\pi B(z)}{M^2}\sum_\sigma \int dE \int d\mu \frac{f(z,E,\mu,\sigma)}{\upsilon_{//}(z,E,\mu)}. \qquad (7)$$

Substituting Eq. (5) into Eq. (7), the ion density n_i for the case (i) becomes

$$\begin{aligned} n_i(z) = \frac{4\pi B(z)}{M^2}\Bigg(& \int_{\frac{-q\phi(z)B(\pm L)}{B(z)-B_0}+q\phi(\pm L)}^{\infty} dE \\ & \times \int_{\frac{-q\phi(z)}{B(z)-B_0}}^{\frac{1}{B(\pm L)}\{E-q\phi(\pm L)\}} d\mu \frac{1}{\upsilon_{//}(z,E,\mu)} \int_0^L \frac{S(z',E,\mu)}{\upsilon_{//}(z',E,\mu)}dz' \\ & + \int_{\frac{-q\phi(z)B(z)}{B(z)-B_0}+q\phi(z)}^{\frac{-q\phi(z)B(\pm L)}{B(z)-B_0}+q\phi(\pm L)} dE \int_{\frac{-q\phi(z)}{B(z)-B_0}}^{\frac{1}{B(z)}\{E-q\phi(z)\}} d\mu \frac{1}{\upsilon_{//}(z,E,\mu)} \\ & \times \int_0^{z_t(E,\mu)} \frac{S(z',E,\mu)}{\upsilon_{//}(z',E,\mu)}dz' + \int_{\frac{-q\phi(z)B(\pm L)}{B(z)-B_0}+q\phi(\pm L)}^{\infty} dE \\ & \times \int_{\frac{1}{B(\pm L)}\{E-q\phi(\pm L)\}}^{\frac{1}{B(z)}\{E-q\phi(z)\}} d\mu \frac{1}{\upsilon_{//}(z,E,\mu)} \int_0^{z_t(E,\mu)} \frac{S(z',E,\mu)}{\upsilon_{//}(z',E,\mu)}dz' \Bigg), \end{aligned} \qquad (8)$$

and Eq. (6) into Eq. (7), n_i for the case (ii) becomes

$$n_i(z) = \frac{4\pi B(z)}{M^2}\left(\int_0^{\frac{-q\phi(z)B_0}{B(z)-B_0}} dE \int_0^{\frac{E}{B_0}} d\mu \frac{1}{\upsilon_{//}(z,E,\mu)} \int_0^L \frac{S(z',E,\mu)}{\upsilon_{//}(z',E,\mu)} dz' \right.$$
$$+ \int_{\frac{-q\phi(z)B_0}{B(z)-B_0}}^{\infty} dE \int_0^{\frac{-q\phi(z)}{B(z)-B_0}} d\mu \frac{1}{\upsilon_{//}(z,E,\mu)} \int_0^L \frac{S(z',E,\mu)}{\upsilon_{//}(z',E,\mu)} dz'$$
$$+ \int_0^{\frac{-q\phi(z)B_0}{B(z)-B_0}} dE \int_{\frac{E}{B_0}}^{\frac{1}{B(z)}\{E-q\phi(z)\}} d\mu \frac{1}{\upsilon_{//}(z,E,\mu)} \int_{z_t}^L \frac{S(z',E,\mu)}{\upsilon_{//}(z',E,\mu)} dz'$$
$$\left. + \int_{q\phi(z)}^0 dE \int_0^{\frac{1}{B(z)}\{E-q\phi(z)\}} d\mu \frac{1}{\upsilon_{//}(z,E,\mu)} \int_{z_t}^L \frac{S(z',E,\mu)}{\upsilon_{//}(z',E,\mu)} dz' \right). \quad (9)$$

Here, we assume the magnetic field B of which increase rate is smaller than the decrease rate of $q\phi$. By interchanging the order of integrations of Eqs. (8) and (9), respectively and taking sum of them, the ion density can be written as

$$n_i(z) = \frac{4\pi B(z)}{M^2}\left(\int_0^L dz' \int_{\frac{-E_pB_0}{B_p-B_0}}^{\infty} dE \right.$$
$$\times \int_0^{\frac{1}{B(z)}\{E-q\phi(z)\}} d\mu \frac{1}{\upsilon_{//}(z,E,\mu)} \frac{S(z',E,\mu)}{\upsilon_{//}(z',E,\mu)}$$
$$\left. + \int_0^L dz' \int_{E_p}^{\frac{-E_pB_0}{B_p-B_0}} dE \int_0^{\frac{1}{B_p}(E-E_p)} d\mu \frac{1}{\upsilon_{//}(z,E,\mu)} \frac{S(z',E,\mu)}{\upsilon_{//}(z',E,\mu)} \right), \quad (10)$$

where $E_p=q\phi(z')$ and $B_p=B(z')$ for $z'<z$ and $E_p=q\phi(z)$ and $B_p=B(z)$ for $z'>z$ according to the conditions that $\upsilon_{//}$ must be real number.

As the source function $S(z,E,\mu)$, we use the expression same as Emmert *et al.* [2]

$$S(z,E,\mu) = S_0 h(z) \frac{M^2}{4\pi(kT_i)^2} \upsilon_{//}(z,E,\mu) \exp\left\{ -\frac{E-e\phi(z)}{kT_i} \right\}, (11)$$

where T_i is the temperatures, $h(z)$ is the source strength, and S_0 is the average source strength of the ion, respectively. Substituting Eq. (11) into Eq. (10) and integrating it for μ and E, the ion density becomes

$$n_i(z) = S_0 \left(\frac{\pi M}{2kT_i} \right)^{1/2} \int_0^L dz' I(z,z') h(z'), \quad (12)$$

where

$$I(z,z') = \begin{cases} \exp\left\{ \frac{q\phi(z')-q\phi(z)}{kT_i} \right\} \mathrm{erfc}\left[\left\{ \frac{q\phi(z')-q\phi(z)}{kT_i} \right\}^{1/2} \right] \\ + \frac{2}{\sqrt{\pi}} \left\{ \frac{(B(z)-B_0)q\phi(z')-(B(z')-B_0)q\phi(z)}{kT_i(B(z')-B_0)} \right\}^{1/2} \\ \times \exp\left\{ \frac{q\phi(z')B(z')}{kT_i(B(z')-B_0)} \right\} + \left\{ \frac{B(z)-B(z')}{B(z')} \right\}^{1/2} \frac{2}{\sqrt{\pi}} \\ \times \exp\left\{ \frac{(q\phi(z')-q\phi(z))B(z')}{kT_i(B(z')-B(z))} \right\} \left[D\left[\left\{ -\frac{(q\phi(z')-q\phi(z))B(z')}{kT_i(B(z')-B(z))} \right\}^{1/2} \right] \right. \\ \left. -D\left[\left\{ -\frac{\{(B(z)-B_0)q\phi(z')-(B(z')-B_0)q\phi(z)\}B(z')}{kT_i(B(z')-B(z))(B(z')-B_0)} \right\}^{1/2} \right] \right], \quad z'<z \\ \exp\left\{ \frac{q\phi(z')-q\phi(z)}{kT_i} \right\}, \quad z'>z \end{cases} \quad (13)$$

where $D(z)$ is the Dawson function [5]

$$D(x) = \int_0^x \exp(t^2) dt. \quad (14)$$

For the electron density n_e, we use a Maxwell–Boltzmann distribution for simplicity

$$n_e(z) = n_0 \exp\{e\phi(z)/kT_e\}, \quad (15)$$

where n_0 is the density at z=0, $-e$ is the electron charge, k is the Boltzmann's constant, and T_e is the electron temperature.

Substituting Eqs. (12) and (15) into Poisson's equation, the plasma-sheath equation is derived as

$$\lambda_D^2 \frac{e}{kT_e} \frac{\mathrm{d}^2\phi}{\mathrm{d}z^2} = \exp\left(\frac{e\phi(z)}{kT_e} \right) - \frac{q}{e} \frac{S_0}{n_0} \left(\frac{\pi M}{2kT_i} \right)^{1/2} \int_0^L \mathrm{d}z' I(z,z') h(z'), (16)$$

where $\lambda_D=(\varepsilon_0 kT_e/n_0e^2)^{1/2}$ is the Debye length. The average source strength S_0 is decided by the equilibrium of the fluxes of the plasma particles at the wall. We consider that $j_{iw}+j_{ew}=0$, where j_{iw} is the ion current density and j_{ew} is the electron current density at the wall, then

$$S_0 \frac{q}{e} L = n_0 \left(\frac{kT_e}{2\pi m} \right)^{1/2} \exp\left(\frac{e\phi_w}{kT_e} \right), \quad (17)$$

where m is the electron mass and ϕ_w is the wall potential. Substituting S_0 given by Eq. (17) into Eq. (16), we obtain

$$\lambda_D^2 \frac{e}{kT_e} \frac{\mathrm{d}^2\phi(z)}{\mathrm{d}z^2} = \exp\left(\frac{e\phi(z)}{kT_e} \right) - \frac{1}{2L} \left(\frac{M}{m} \frac{T_e}{T_i} \right)^{1/2} \exp\left(\frac{e\phi_w}{kT_e} \right)$$
$$\times \int_0^L \mathrm{d}z' I(z,z') h(z'). \quad (18)$$

4. Numerical Solution of the Plasma-Sheath Equation

Here, we introduce the normalized variables: $\eta=(e/kT_e)(\phi_w-\phi)$, $s=z/L$, $\tau=T_e/T_i$, $Z=q/e$, $R=B/B_0$, where R is the mirror ratio. The mirror ratio R can be expressed by the function of η because that the electrical potential and the magnetic field are assumed to vary monotonically and the coordinate s corresponds to the value of η at each position. We assume the mirror ratio to be given by

$$R(\eta) = \exp\left[\alpha \left\{ \eta - e\phi_w/(kT_e) \right\}^{1/2} \right]. \quad (19)$$

The normalized plasma-sheath equation is solved numerically by transforming it into a set of finite difference equations and using a Newton method and a successive over–relaxation method [6,7]. The boundary conditions are $\mathrm{d}\eta/\mathrm{d}s|_{s=0}=0$ and $\eta(s=1)=0$. We assume that the ion source is uniform, that is, $h(z)=1$.

The numerically solved profile of the potential for various values of λ_D/L is shown in Fig. 4, where Z=1, $\tau=T_e/T_i$=1 and α=0.4. Where the normalized potential $\psi=\eta-e\phi_w/(kT_e)=-e\phi/(kT_e)$ is shown. The potential drop increases and the sheath width also increases as λ_D/L increases. The profiles of the mirror ratio and the potential for various values of α are shown in Fig. 5 and Fig. 6, respectively, where Z=1, τ=1 and λ_D/L=5x10^{-2}. As the value of α increases, the mirror ratio becomes large, especially near the wall, and the potential drop decreases. This may be because that as the value of α increases, the ions are reflected by the

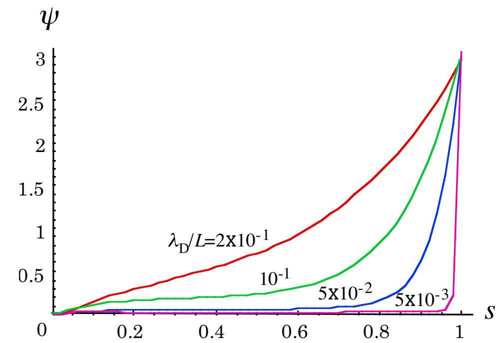

Fig.4 Profile of the normalized potential $\psi=-e\phi/kT_e$ for various values of λ_D/L under the conditions of Z=1, $\tau=T_e/T_i$ =1.0, and α=0.4.

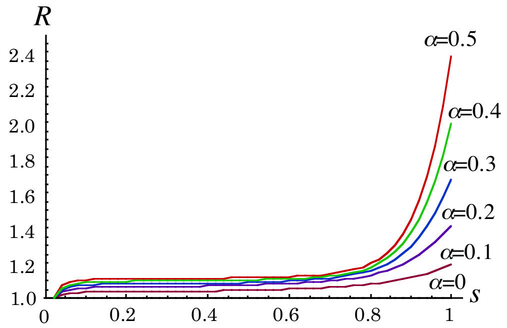

Fig.5 Profile of the mirror ratio R for various values of α under the conditions of Z=1, $\tau=T_e/T_i$ =1.0, and λ_D/L=5x10^{-2}.

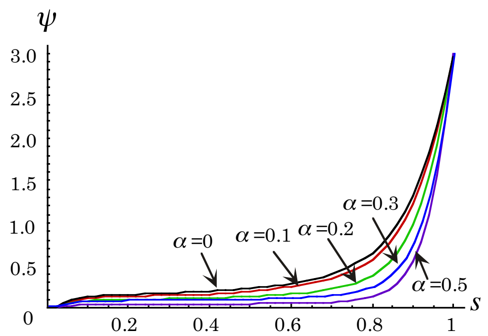

Fig.6 Profile of the normalized potential ψ=-$e\phi/kT_e$ for various values of α under the conditions of Z=1, $\tau=T_e/T_i$ =1.0, and λ_D/L=5x10^{-2}.

increasing magnetic field near the wall and reciprocated between the two turning points. As a result, the ions in the plasma region increase and the potential drop decrease. On the other hand, the potential in the sheath region does not depend greatly on the value of α. The profiles of the mirror ratio and the potential for various values of the temperature ratio τ are shown in Fig. 7 and Fig.8, respectively, where Z=1, α=0.4 and λ_D/L=5x10^{-2}. As the value of $\tau=T_e/T_i$ increases, the mirror ratio and the potential drop decrease. It may be because that the low energy ions are reflected by the strong magnetic field near the wall and reciprocated between the two turning points. Although the magnetic field in the plasma region decreases as the value of τ increases, it does not depend greatly on the value of τ in the sheath

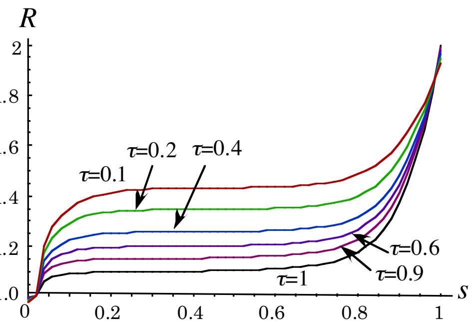

Fig.7 Profile of the mirror ratio R for various values of the temperature rate $t=T_e/T_i$ under the conditions of Z=1, R=exp($\alpha\psi^{1/2}$), α=0.4 and λ_D/L=5x10^{-2}.

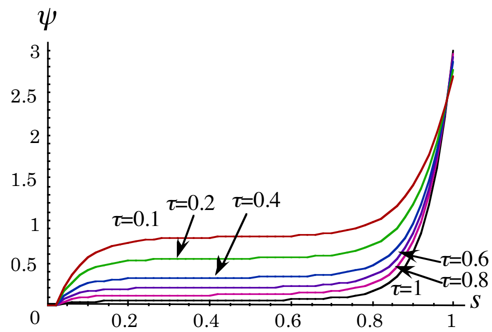

Fig.8 Profile of the normalized potential ψ=-$e\phi/kT_e$ for various values of the temperature rate $t=T_e/T_i$ under the conditions of Z=1, R=exp($\alpha\psi^{1/2}$), α=0.4 and λ_D/L=5x10^{-2}.

region. As a result, the ions in the plasma region increase and the potential drop decreases. The potential in the sheath region also does not depend greatly on the value of τ.

5. Conclusions

The electric potential near the wall has been investigated by considering the magnetic field increasing toward the wall. The profile of the potential has been obtained by solving the plasma-sheath equation. The potential drop in the plasma region depends on the profile of the magnetic field and the ion temperature. On the other hand, dependence of the potential in the sheath region on them is small.

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