# DNLS Solitons in Collisional Dusty Plasma

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Alfven waves are possible sources of turbulence in the interstellar medium. These waves are responsible for the transport of angular momentum and energy in the accretion discs. Such waves have been studied for last several decades in space plasmas. Most of the space plasmas are dusty. They contain charged grains and coupling of the grains to the magnetic field determines the wave propagation in the planetary and interstellar medium. In the present work, the nonlinear wave propagation in collisonal dusty plasma is considered. When the electrons and ions are highly magnetized and charged dust remains weakly magnetized, the relative drift between the dust and the plasma particles may significantly modify the wave characteristics in such a medium. It is shown that the collisonal dusty medium is inherently dispersive in nature and the balance between the dispersion and nonlinearity leads to derivative nonlinear Schrödinger (DNLS) equation. The relative drift between the charged grains and the plasma particles (electrons and ions) gives rise to the Hall diffusion in the medium. This Hall diffusion causes the wave dispersion. In many dusty plasma situations, Hall MHD is the only proper description of the plasma dynamics.

Keywords: Magnetized dusty plasma, plasma waves, nonlinear waves, DNLS equation, collision effects

## 1. Introduction

Alfvén waves are responsible for the transport of angular momentum and energy in the accretion disks and possible sources of turbulence in the interstellar medium [1]. When the amplitude of the waves is large, a nonlinear effect plays an important role in their propagation. It is known that in a homogeneous, uniform background medium, the interplay between dispersion and nonlinearity can give rise to solitary wave structures [2]. The standard magnetohydrodynamic (MHD) waves have degeneracy; i.e., two wave propagation velocities coincide when the wave normal direction is parallel to the ambient background magnetic field. When the wave degeneracy is retained and the wave normal is quasi-parallel to the ambient magnetic field, two coupled equations for the confluent MHD modes can be combined to give derivative nonlinear Schrödinger (DNLS) equation [3].

Most of the space plasma such as those in cometary tails, interstellar molecular clouds, and planetary nebulae is dusty [4]. It contains charged grains and, the coupling of the grains to the magnetic field determines the wave propagation in the planetary and interstellar medium. The plasma-grain collision can causes not only the damping of the high frequency waves but also assist the excitation and propagation of the low frequency fluctuations in the medium. For example, if the plasma-dust collision frequency is higher than the dynamical frequency then collision will be responsible for dragging the dust along the plasma fluctuations. In such a scenario collision will always cause the propagation of the waves in the medium without any damping. In the opposite case, for high frequency oscillations, the dust-plasma collision will cause the damping of the waves.

The dusty plasma dynamics can be studied in either of the two limits: (1) The heavy dust particles provide a stationary background and couples to the plasma fluid through the charge neutrality condition, and (2) the perturbations are on the order of or less than the typical plasma frequencies of the dusty fluid. It is known that the collisional processes in a weakly ionized dusty plasma can strongly influence the nonlinear ambipolar diffusion and the ensuing current sheet formation [5]. Further collision between plasma particles and the grain is responsible for some of the novel features in dusty plasma [6,7].

Here, we investigate the nonlinear wave properties of collisional dusty plasmas. It is shown that the collisional dusty medium is inherently dispersive in nature and the balance between the dispersion and nonlinearity leads to DNLS equation.

## 2. Model Equations

The simplest description of dusty plasma, consisting of the electrons, ions, and charged grains is given in terms of continuity and momentum equations for respective species with a suitable closure model; viz., an equation of state. The model equations are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left( \rho \mathbf{v} \right) = 0, \qquad (1)$$

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$$\rho\frac{d\mathbf{v}}{dt} = -\nabla P + \frac{\mathbf{J}\times\mathbf{B}}{c}, \tag{2}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times\left(\mathbf{v}_d\times\mathbf{B} - \frac{\mathbf{J}\times\mathbf{B}}{Zen_d}\right), \tag{3}$$

and

$$P\rho^{\gamma} = \text{Const.} \tag{4}$$

Here, $\rho = \rho_e + \rho_i + \rho_d \approx \rho_d$ is the mass density of the bulk fluid, $\mathbf{v} = \left(\rho_e\mathbf{v}_e + \rho_i\mathbf{v}_i + \rho_d\mathbf{v}_d\right)/\rho$ is the bulk velocity, $P = P_e + P_i + P_d$ is the total plasma pressure, $\mathbf{J} = -en_e\mathbf{v}_e + en_i\mathbf{v}_i$ is the current density in the dust frame, and $\gamma$ is the adiabatic constant.

We see from the induction equation that the ideal-MHD limit corresponds to the absence of the **J**×**B** term. Note that the ideal MHD description of the two component plasma assumes that the relative drift between electrons and ions is absent. The reason for the Hall effect in such a plasma is attributed to the finite ion inertia which breaks the symmetry between electrons and ions with respect to the magnetic field. This symmetry breaking results in the finite relative drift between the electrons and ions. However, in a dusty plasma, or for that matter in any multi-component plasma, since not only electrons and ions are the charge carriers, even when relative drift between the plasma particles are absent, owing to the presence of a third, charged component, the Hall effect will always be present.

## 3. The DNLS Equation

Assuming uniform field $B = B_z$ and one-dimensional perturbations, the above set of equation in the stretched variables $\xi = \varepsilon(z-V_A t)$, $\zeta = \varepsilon^2 t$, where $V_A = B_z^2/4\pi\rho$ is the Alfvén velocity, becomes

$$D\rho + \frac{\partial}{\partial\zeta}\left(\rho v_z\right) = 0, \tag{5}$$

$$\rho\left(D + v_z\frac{\partial}{\partial\zeta}\right)v_z = -\frac{\partial}{\partial\zeta}\left(p + \frac{\mathbf{B}_\perp^2}{8\pi}\right), \tag{6}$$

$$\rho\left(D + v_z\frac{\partial}{\partial\zeta}\right)\mathbf{v}_\perp = \frac{B_z}{4\pi}\frac{\partial\mathbf{B}_\perp}{\partial\zeta}, \tag{7}$$

and

$$D\mathbf{B}_\perp = B_z\frac{\partial\mathbf{v}_\perp}{\partial\zeta} - \frac{\partial}{\partial\zeta}\left(v_z\mathbf{B}_\perp\right) + \varepsilon\sigma v_A\delta_d z\times\frac{\partial}{\partial\zeta}\left(\frac{\rho_0}{\rho}\frac{\partial\mathbf{B}_\perp}{\partial\zeta}\right). \tag{8}$$

Here, $D = \varepsilon\partial/\partial\zeta - V_A\partial/\partial\xi$, $\delta_d = V_A/\omega_{cd}$ is the dust skin depth, $\sigma = \pm 1$, and $\mathbf{v}_\perp$ and $\mathbf{B}_\perp$ are the transverse (to $z$) component of the velocity and magnetic field, respectively. The above set of equations admits monochromatic circularly polarized waves $b = B_x + iB_y = B_0 \exp(i\omega t - ikz)$, $v_z = 0$, $\rho = \rho_0$, as an exact solution.

The DNLS equation can be derived using reductive perturbation method (RPM). We follow RPM in the present work and assume that the perturbations to physical quantities vanish at infinity, i.e. $\rho\to\rho_0$, $p\to p_0$, $v_z\to 0$, and $\mathbf{B}\to\mathbf{B}_{\perp,0}$. Then the DNLS equation is given by

$$\frac{\partial b}{\partial\zeta} - \alpha\frac{\partial}{\partial\zeta}\left[b\left(|b|^2 - |b_0|^2\right)\right] + i\sigma D_1\frac{\partial^2 b}{\partial\zeta^2} = 0. \tag{9}$$

Here, $\alpha = v_A/16\pi\rho_0 c_s^2$, $D_1 = v_A\delta_d/2$, and $c_s^2 = \varepsilon^2\gamma p_0/\rho_0$. The DNLS equation describes the evolution of weakly nonlinear, weakly dispersive waves propagating either exactly parallel to the ambient magnetic field $\mathbf{B} = B_z$ (corresponding to the boundary condition $b_0 \to 0$ as $\xi \to \infty$) or slightly oblique propagation ($b_0 \neq 0$ at infinity). Dispersion in the DNLS equation is caused by the dust inertia and nonlinear term arises due to coupling between the transverse magnetic field and plasma pressure perturbations. We note that the derivation of the above DNLS equation is very similar to the MHD case although we have considered a set of dissipative multi-component dusty plasma equations. In the low-frequency limit (i.e. frequencies much lower than the plasma-dust collision frequencies), the dust-plasma collision causes the dust and the plasma fluids to stick together as a single fluid. This results in a MHD like set of equation with dispersion due to Hall term – a manifestation of the multi-component nature of the plasma.

## 4. Discussion

For exactly parallel ($b_0 = 0$), circularly polarized, finite amplitude Alfvén waves, $b = B_0 \exp i(\omega\tau - k\xi)$ satisfies the following nonlinear dispersion relation:

$$\omega = \alpha B_0^2 k + D\sigma k^2. \tag{10}$$

We note that when $\alpha = 0$, Eq. (10) is a linear dispersion relation describing the whistler mode. When $\alpha \neq 0$, for the left-hand polarized waves, $\sigma = -1$, k > 0, is unstable to parallel modulation while the right-hand polarized branch $\sigma = 1$, k < 0, is stable to the parallel modulation.

For exactly parallel, circularly polarized, finite amplitude Alfvén waves, Eq. (10) admits localized envelope soliton solutions

$$|b|^2 = \frac{2V}{\alpha}\left[\sqrt{2}\cosh\left(\frac{\xi - V\zeta}{L}\right) - 1\right]^{-1}, \quad (11)$$

where $V$ represents the velocity in the co-moving frame, $L = D/V = (V_A/2V)\delta_d$ is the width of the soliton, and $2V/\alpha$ is the maximum amplitude of the soliton. Here, we have assumed $\sigma = 1$.

It is instructive to calculate the soliton width for the astrophysical plasmas. For example, in dense molecular cloud cores the neutral density $>10^{10}$ $cm^{-3}$, charged grains are more numerous than the electrons and ions. The ionization fraction of the plasma is strongly affected by the abundance and size distribution of the grains through the recombination process on the grain surface. The ratio of dust to neutral mass density is $\rho_d/\rho_n = 0.01$. Although the grain density is small in comparison with the neutral hydrogen density in such clouds, it may significantly affect the dynamics of star formation. The grain size in such clouds may vary from a few micrometers down to a few tens of atoms in the interstellar medium. In fact, the presence of very small grains (polycyclic aromatic hydrocarbons) can play an important role in the plasma dynamics. The small grains are numerous and the absolute value of their charge may vary between 1 and 0 (note that the grains can be negatively as well as positively charged). Thus, if one assumes $m_d = 10^{-15}$ g, then for $m_n = m_p$, $n_d = 10^{-11} n_n$. Assuming a dense molecular cloud with $n_n = 10^6$ $cm^{-3}$, one gets $n_d = 10^{-5}$ $cm^{-3}$. For a typical mGauss field $B = 10^{-3}$ G, one gets $V_A$ of the order of 0.1 km, and dust skin depth $\delta_d \sim 10^5$ km. Thus, $L \sim 10^{-8}$ pc, where 1 pc = $3 \times 10^{13}$ km. Therefore, the width of the nonlinear structure appears to be too small in order to be meaningful in cloud cores of a few pc. However, for a much heavier dust $m_d \sim 10^{-12}$ g, we have $V_A \sim 1$ km and the dust skin depth $\delta_d \sim 10^{11}$ km. This gives L ~ 0.01 pc, which is comparable to the size of the dense cloud cores. Therefore, the soliton may explain some of the observed structures in the astrophysical plasmas. Our results are suggestive in nature as we have ignored the neutral dynamics completely in the present work notwithstanding the fact that neutral density dominates the star forming region. However, it can be shown [7] that the basic set of equation in the presence of neutral can be reduced to the present set. Therefore, although the details of the soliton structure may be affected, broader conclusion drawn in this work will be similar even when the neutral dynamics is included.

To summarize, we have shown that in the low frequency limit, when the electrons and ions are highly magnetized, the multi-fluid set of equations reduces to the set of equations which are very similar to the single-fluid Hall MHD equations. The nonlinear propagation of the waves in the magnetized, collisional dusty medium can be described by the DNLS equation. The soliton solution of DNLS equation could be probably invoked to explain the pc range structures in the molecular cloud.

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