Dust dynamics in cryogenic environment

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Theoretical study on dust dynamics under cryogenic environment in a complex plasma is presented. Expansion process and cooling process of highly charged dust particles in a plasma containing background ultracold neutral particles are studied. The processes are described by kinetic theory with Fokker-Planck collision term of the interaction between neutral particles and dust particles. Time evolution of kinetic dust temperature and spatial spread of dust particles are revealed. As dust particles spread in space, the dust temperature decreases drastically in time scale of dust plasma period and cooled to background cryogenic temperature. The dust cloud spreads to some extent but mostly suppressed by the presence of background plasma.

Keywords: Complex plasma, cryogenic environment, diffusion process, kinetic theory, Fokker-Planck collision term

1. Introduction

Dusty plasma includes ions electrons, neutrals, and micron-sized dust particles and is found in laboratory as well as in space. Dusty plasma is often called a "complex plasma" because of its novel nature of an interaction between dust particles and plasma particles in a collective way [1]. Dust particles are negatively charged due to high mobility of fast electrons absorbed onto the dust surface. Such charged dust particles show novel features such as Coulomb crystals in certain background pressure [2-4] and wake potential in the ion flow [5-7]. A crystal state of dust particles is related to strong correlation and is characterized by a structure of dust cluster. Especially, strength of correlation is described by a coupling parameter Γ_d,

$$\Gamma_d = \frac{{Z_d}^2 e^2}{4\pi\varepsilon_0 d k_B T_d}, \tag{1}$$

where Z_d is the charge state of a dust particle, e is an elementary charge, ε_0 is a permittivity of free space, d is an interparticle distance, k_B is a Boltzmann constant, and T_d is a dust temperature. Generally, Γ_d could become larger than 1 in a complex plasma, and strong coupling system could be formed.

In this paper, we study a new feature of a complex plasma in cryogenic environment. Our model is prompted by a cryogenic complex plasma experiment at Yokohama National University [8], where dynamics of dust particles under cryogenic environment, especially in liquid helium vapor and on the liquid helium surface has been studied. From Eq. (1), a value of coupling parameter is expected to increase if T_d decreases. It will lead to stronger correlation among dust particles. Therefore, it is expected that dust particles could form Coulomb crystals on the liquid helium surface as was shown for macroscopic dimple crystals due to electrons [9,10]. It is interesting to note that the number of electrons in each dimple is 10^5~10^7, while the charge on a dust particle of a micron size could be in the range 10^3~10^5. A complex plasma experiment in cryogenic environment was also reported by Antipov *et al.* [11]. On the other hand, Rosenberg and Kalman suggested that two-dimensional structure of dust particles could be formed on the surface of liquid helium [12].

Here, we consider expansion process of dust particles and study kinetic dust temperature and potential structure in a cryogenic complex plasma.

2. Theoretical model

2.1 Theoretical description

We consider a dust cloud, a collection of charged dust particles, in a plasma with neutrals at a cryogenic temperature. Such a plasma may be generated by electric discharge in a vapor of liquid helium as in the experiment [8]. Neutral helium atoms are kept in a cryogenic temperature by the presence of liquid helium. The dust cloud, characterized by the dust number density n_d and T_d, is assumed to be located at the origin with a spherical spatial spread with a radius σ. We take spherical symmetry in space. Free expansion process of dust cloud with the cooling process through the interaction between

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dust particles and background plasma particles including neutrals is considered.

As for background plasma, electron density n_e and ion density n_i are assumed to be in a local thermal equilibrium and the ion density is given by

$$n_i = n_{i0} \exp\left(-\frac{e\phi}{k_B T_i}\right), \quad (2)$$

where ϕ is electrostatic potential and T_i is ion temperature. Charge neutrality condition in a complex plasma is written by

$$Z_d n_d e + n_i e - n_e e = 0. \quad (3)$$

Dust particles are charged negatively in a plasma, or $Q = Z_d e\ (Z_d < 0)$. All the dust particles are assumed to be equally charged and no temporal variation is considered in the model. The number density of dust particles is given by

$$n_d = \int f_d(\mathbf{r},\mathbf{v},t)d\mathbf{v} \propto \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad (4)$$

where the dust distribution function and f_d is assumed to have a form

$$f_d \propto \exp\left(-\frac{r^2}{2\sigma(t)^2} - \frac{m_d(\mathbf{v}-\gamma(t)\mathbf{r})^2}{2k_B T_d(t)}\right). \quad (5)$$

Here m_d is a dust mass, γ is a hydrodynamic expansion parameter, and the magnitude of displacement vector is written as $r = |\mathbf{r}|$. Equation (5) is a formalism used for free plasma expansion [13]. The distribution function is governed by

$$\frac{\partial f_d}{\partial t} + \mathbf{v}\cdot\frac{\partial f_d}{\partial \mathbf{r}} + \frac{Z_d e}{m_d}\mathbf{E}\cdot\frac{\partial f_d}{\partial \mathbf{v}} = \left(\frac{\partial f_d}{\partial t}\right)_{coll}, \quad (6)$$

with a collision term,

$$\left(\frac{\partial f_d}{\partial t}\right)_{coll} = \beta\left(\frac{\partial}{\partial \mathbf{v}}\cdot\mathbf{v}f_d + \frac{k_B T_c}{m_d}\frac{\partial^2}{\partial v^2}f_d\right), \quad (7)$$

where β is a transport coefficient (collision frequency) $\beta = \nu_{nd} \cong 10 r_d^2 p/(m_d v_{Tn})$ (r_d=dust radius, p=pressure of neutral particles (helium atoms), v_{Tn}=neutral thermal velocity) [14,15] and T_c is a cryogenic neutral temperature. The electric field may be obtained by substituting Eqs. (2) and (4) into Eq. (3) with assumption of $n_e << n_i$ as

$$\mathbf{E} = -\nabla\phi = -(k_B T_i/\sigma^2 e)\mathbf{r}. \quad (8)$$

Temporal evolution of physical quantities $T_d(t)$, $\gamma(t)$, and $\sigma(t)$ associated with dust cloud expansion is expressed by a differential equations which can be obtained by substituting Eqs. (5) and (8) into Eq. (6). Here we introduce characteristic frequency; ω_{pd}, dust plasma frequency, a characteristic length; λ_{Di}, ion Debye length, and a characteristic temperature $|Z_d|T_i$. Normalized equations are given by

$$\dot{\tilde{T}}_d = -2\tilde{\gamma}\tilde{T}_d - 2\tilde{\beta}\tilde{T}_d + 2\tilde{\beta}\tilde{T}_c, \quad (9)$$

$$\dot{\tilde{\gamma}} = -\tilde{\gamma}^2 - \tilde{\gamma}\tilde{\beta} + \tilde{T}_d/\tilde{\sigma}^2 + 1/\tilde{\sigma}^2, \quad (10)$$

$$\dot{\tilde{\sigma}} = \tilde{\gamma}\tilde{\sigma}, \quad (11)$$

where $\tilde{t} = \omega_{pd}t$, $\tilde{\gamma} = \omega_{pd}^{-1}\gamma$, $\tilde{\beta} = \omega_{pd}^{-1}\beta$, $\tilde{\sigma} = \lambda_{Di}^{-1}\sigma$, $\tilde{T}_d = T_d/(|Z_d|T_i)$, $\tilde{T}_c = T_c/(|Z_d|T_i)$. Dust plasma frequency and ion Debye length are described as $\omega_{pd} = (n_{d0}Z_d^2 e^2/\varepsilon_0 m_d)^{1/2}$ with n_{d0} the initial dust density and $\lambda_{Di} = (\varepsilon_0 k_B T_i/n_{i0}e^2)^{1/2}$. The last term in the RHD (right-hand-side) of Eq. (10) is the effect of electric field, while the second and third terms in the RHD of Eq. (9) and the second term in the RHD of Eq. (10) are the effect of damping due to neutral particles.

2.2 Steady state solutions

Steady state solutions of Eqs. (9), (10) and (11) are given in a dimensional form

$$T_d = \beta T_c/(\gamma+\beta), \quad (12)$$

$$k_B T_d = Z_d k_B T_i + m_d\sigma^2\gamma(\gamma+\beta), \quad (13)$$

$$\gamma\sigma = 0. \quad (14)$$

Here we note that the term $Z_d k_B T_i$ originates from the approximation $Z_d k_B T_i \approx -n_i k_B T_i/n_d$. In collisional limit $(\beta >> \gamma)$, Eq. (12) predicts that the dust temperature approaches $T_d = T_c$, which gives the restriction on the dust cloud expansion, from Eq. (13), as $\sigma^2 = (k_B T_d - Z_d k_B T_i)/(m_d\gamma\beta)$ and vanishing hydrodynamic expansion parameter as $\gamma \to 0$ from Eq. (14).

Now numerical solution is needed to find the time change of kinetic dust temperature and spatial spread. Let us consider the time evolution of them in short time range well before reaching steady state.

3. Numerical solutions

Here we solve Eqs. (9), (10) and (11) numerically. For numerical solutions, we consider typical mass density $\rho \sim 1.0$ gcm^{-3}, dust radius $r_d \sim 1\,\mu$m, charge number $|Z_d| = 10^2 - 10^3$, transport coefficient $\beta = 1 - 10^2$ s^{-1}, neutral temperature $T_c \sim 10$ K, ion temperature $T_i \sim 10^2$ K, ion number density $n_{i0} \sim 10^8$ cm^{-3}, dust number density $n_{d0} \sim 10^5$ cm^{-3}, ion Debye length $\lambda_{Di} \sim 10^{-1}$ mm and dust plasma frequency $\omega_{pd} \sim 10^2$ s^{-1}.

Initial conditions are chosen as to $\sigma(0) = 5$mm, $\gamma(0) = 0$ s^{-1}, $T_d(0) = 300$ K or in dimensionless values $\tilde{\sigma}(0) = 82$, $\tilde{\gamma}(0) = 0$, $\tilde{T}_d(0) = 3.8\times10^{-3}$ and $\tilde{T}_c(0) = 1.3\times10^{-5}$.

Figure 1 shows temporal evolution of dust temperature for different transport coefficients. Dust temperature is shown to decrease exponentially with time. With higher collision frequency of dust particles with neutrals, the dust kinetic temperature decreases more

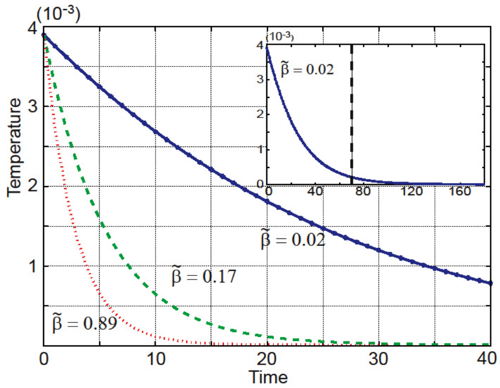

Fig.1 Dust temperature vs time for various transport coefficients $\tilde{\beta}$.

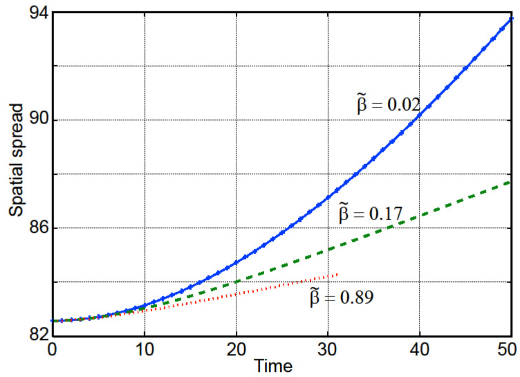

Fig.2 Spatial spread vs time for various transport coefficients $\tilde{\beta}$.

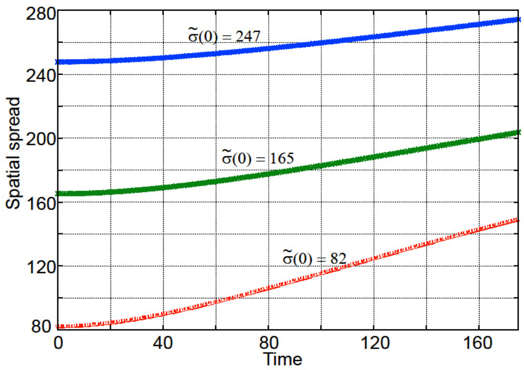

Fig.3 Spatial spread vs time for various initial spatial spreads at $\tilde{\beta} = 0.02$.

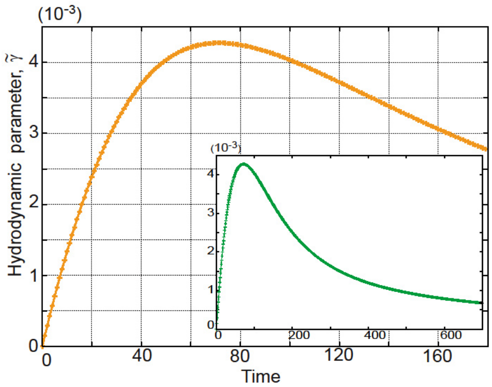

Fig.4 Temporal evolution of hydrodynamic velocity parameter $\tilde{\gamma}$. $\tilde{T}_d = 3.8\times10^{-3}$, $\tilde{\sigma}(0) = 82$, $\tilde{\beta} = 0.02$.

sharply with time. It means that collisions of background neutrals have strong effects on dust temperature. In cryogenic environment, collisions between dust particles and neutrals occur frequently, so collision is considered as one of dominant effects in cryogenic environment. With due time, the dust temperature reaches to cryogenic temperature, which corresponds to the results in steady state solution.

In Fig. 2, time evolution of spatial spread of dust particles is shown. Each line shows spatial spread for various transport coefficients as given in Fig. 1. Spatial spread increases with time and the increasing rate is inversely proportional to the strength of transport coefficient β , indicating that collisions with neutrals suppress expansion of dust particles.

Next we change the initial spread of dust cloud and see the change in the evolution of spatial spread. Figure 3 represents time change of spatial spread for $\tilde{\sigma}(0) = 82$, $\tilde{\sigma}(0) = 165$, and $\tilde{\sigma}(0) = 247$. Spatial spread increases with time as was seen in Fig. 2. It is also found that the increasing rate of spread is inversely proportional to magnitude of initial spatial spread.

In Fig. 4, temporal evolution of hydrodynamic parameter γ is plotted for initial conditions, $\tilde{\gamma}(0) = 0$, $\tilde{\sigma}(0) = 82$, $\tilde{T}_d(0) = 3.8\times10^{-3}$ and $\tilde{\beta} = 0.02$. The γ increases for $\tilde{t} < \tilde{t}_c \approx 70$ and saturates at a critical value followed by the slow decrease in the value, which means dust particles expand exponentially in space in early stage followed by a linear expansion as shown in Fig. 3. After hitting the maximum value γ decreases gradually, that is, expansion speed gets slow although the change rate of speed is small in short time range. The dust temperature decreases rapidly within time $\tilde{t} < \tilde{t}_c$ followed by the slower decrease rate as shown in an inset in Fig. 1, where $\tilde{t} = \tilde{t}_c$ is shown by a dashed line. Temporal evolution of γ in longer time scale is shown in the inset in Fig. 4.

4. Discussion

4.1 Spatial spread σ

Initially spatial spread increases rapidly with time as seen in Fig. 2 and Fig. 3, but the expansion of dust particles will be suppressed in later time. Dust particles initially placed at the origin expand as they follow electrostatic force as shown in Eq. (8). Spatial spread σ has time dependence so if σ becomes large, electrostatic force acting on dust particles becomes weak gradually.

So the diffusion rate will be suppressed. This means although ions move as motive force to make dust particles expand initially, but they work as confining force with time because of quasi neutrality condition.

4.2 Relation among γ , σ and T_d

Consider the kinetic energy $(K.E)_d$ for all dust particles,

$$(K.E)_d = \sum_{i=1}^{N_d} \frac{1}{2} m_d v_i^2 = (m_d/2)\int v^2 f_d d\mathbf{v} d\mathbf{r}$$
$$= \frac{3}{2} N_d k_B T_d + \frac{3}{2} N_d m_d (\gamma\sigma)^2, \qquad (15)$$

where N_d is the number of dust particles in a system given by $N_d = \int f_d d\mathbf{v} d\mathbf{r}$. The first term in the RHD of Eq. (15) is thermal energy and the second term is kinetic energy for hydrodynamic expansion. Total energy in the system is given by

$$E = (3/2)N_i k_B T_i + (3/2)N_n k_B T_c + (3/2)N_d k_B T_d + (3/2)m_d N_d \gamma^2 \sigma^2 \qquad (16)$$

where $3N_i k_B T_i/2$ is the thermal energy of ions and $3N_n k_B T_c/2$ is the thermal energy of neutrals with. N_i the number of ions and N_n the number of neutrals. In Eq. (16) the energy in the macroscopic electric fields are neglected. In our model, temporal evolution of ions are neglected and background neutrals are playing the role of a thermal bath to keep the neutral temperature in a cryogenic condition provided by liquid helium. Since our system is an open dissipative system, the total energy is not conserved, but in a limited condition $\gamma t << 1$, the energy is well conserved as

$$(3/2)k_B T_d + (3/2)m_d(\gamma\sigma)^2 = const, \qquad (17)$$

which gives us an insight of heat transfer between expansion energy and the dust temperature although the conservation is not kept in the later time. During the initial diffusion process, hydrodynamic parameter γ and spatial spread σ are both found to increase, and dust temperature decreases. As shown in Fig. 4, the expansion parameter γ is not a monotonous increasing function, but it starts to decrease gradually for $\tilde{t} > \tilde{t}_c \approx 70$, while the spatial spread σ changes its exponential growth to much slower linear growth after $\tilde{t}_c$.

It should be noted that the parameters γ, σ and T_d are affected by transport coefficient, β . The collision between helium atoms and dust particles, indicated by β , is controlling the temporal evolution of dust temperature.

5. Summary

Expansion process and cooling process of dust particles in a complex plasma under cryogenic environment are studied. Analysis of dust dynamics involving dust cloud expansion based on kinetic theory is a novel method. Our kinetic treatment reveals the microscopic nature associated with the expansion process, where the Fokker-Planck collisional term plays an essential part in the energy exchange of dust particles.

Our results reveal that dust particles diffuse in space with time due to the effect of potential determined by background plasma particles and dust particles. When negatively charged dust particles expand, ions will follow by the electric field. But dust particles could not expand freely because of neutral particles. The kinetic temperature decreases to cryogenic temperature through diffusion.

When the kinetic temperature decreases while dust particles keep their charge, strong coupling state may be formed among dust particles. The coupling parameter Γ_d may be on the order of a few hundreds for dust charge $|Z_d| \sim 200$ and interparticle distance $100\mu\mathrm{m}$, density $n_d \sim 10^5 \mathrm{cm}^{-3}$ and dust temperature $T_d \sim 10\,\mathrm{K}$.

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References

[1] O. Ishihara, *J. Physics D* **40**, R121 (2007).

[2] H. Chu and L. I, *Phys. Rev. Lett.* **72**, 4009 (1994).

[3] H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher and D. Mohlmann, *Phys. Rev. Let.* **73**, 652 (1994).

[4] Y. Hayashi and K. Tachibana, *J. Appl. Phys.* **33**, L804 (1994).

[5] M. Nambu, S. V. Vladimirov and P. K. Shukla, *Phys. Lett.* A **203**, 40 (1995).

[6] S. V. Vladimirov and O. Ishihara, *Phys. Plasmas* **3**, 444 (1996).

[7] O. Ishihara and S. V. Vladimirov, *Phys. Rev.* E **57**, 3392 (1998).

[8] C. Kojima, M. Kugue, T. Maezawa, M. Shindo, Y. Nakamura and O. Ishihara, *Proceedings of 13th International Congress on Plasma Physics*, E134p, Kiev, May 22-26 (2006).

[9] C. C. Grimes and G. Adams, *Phys. Rev. Lett.* **42**, 795 (1979).

[10] P. Leiderer and M. Wanner, *Phys. Lett.* **73A**, 189 (1979).

[11] S. N. Antipov, E. I. Asinovskii, V. E. Fortov, A. V. Kirillin, V. V. Markovets, O.F. Petrov and V.I. Platonov, *Phys. Plasmas* **14**, 090701 (2007).

[12] M. Rosenberg and G. J. Kalman, *Europhys. lett.* **75**, 894 (2006).

[13] D. S. Dorozhkina and V. E. Semenov, *Phys. Rev. Lett.* **81**, 2691 (1998).

[14] J. D. Huba, *NRL Plasma Formulary* (2006)

[15] P. S. Epstein, *Phys. Rev.* **23**, 710 (1924).