Dust Cluster Oscillation Spectrum in the Presence of Charge Fluctuations

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The mode spectra of small N=3 clusters of dust particles with fluctuating charges are analysed. Building on previous results investigating the influence of charge variance and correlation, efforts were made to quantify these effects on mode spectra via affected mode properties. This allows a more quantitative assessment of results, which is useful particularly as the number of particles in a cluster increases. The effect of charge fluctuation distribution on mode spectra properties for N=3 clusters was also examined using a selection of different probability distributions.

Keywords: Coulomb clusters, dust in plasma, cluster modes, charge fluctuations, oscillations

1. Introduction

Dust crystals with a small number of particles are called dust or Coulomb clusters [1]. Two-dimensional (2D) dust particle clusters provide useful model systems for the study of properties of dust structures in complex plasmas. Such clusters consist of a small number of micron-sized particles trapped in the sheath of a discharge plasma. For ground experiments, the confinement in the vertical direction is usually due to the the sheath electric field that levitates the particles against the gravity and the ion drag forces. In the horizontal plane, the particles are weakly confined by a shallow (parabolic in the first approximation) potential due to, e.g., trough in the electrode and/or additional ring electrode [1, 2, 3]. Therefore, the dust particles arrange in flat 2D structures forming finite planar clusters [4, 5]. Note that clusters of a finite number of charged particles confined by an external potential [6] are of particular interest as they can serve as classical analog to ion traps [7, 8], quantum dots [9], and artificial atoms [10].

The ground state and metastable structure of 2D dust particle clusters, their dynamical properties, and phase transitions have been studied theoretically [11, 12, 13], and in applications to dusty plasmas [14, 15], by Lozovik et al. Furthermore, Bedanov and Peeters [16] numerically studied ordering of charged particles in a classic 2D system, and Schweigert et al [17, 18] and Peeters et al. [19, 20] studied equilibrium configurations and the spectrum of normal modes of dust particles in clusters; spectral properties of small dusty clusters were also studied in [21]. Experimentall observation of dust clusters in a plasma trap was reported in [22]. Klindworth et al. [23] have excited the so-called intershell rotation mode by focused laser beams, whereas Melzer et al. have extracted [24, 25] the normal modes of various finite clusters. The spectral power density of the modes was measured for Coulomb clusters and several modes were compared such as the breathing mode and intershell rotation as well as the lowest- and highest-frequency modes.

The considered studies usually assume constant dust charges when analyzing Coulomb cluster modes. In reality, the dust charges vary according to the plasma conditions. In the recent work [26] the modes of clusters formed by two and three dust particles were analyzed and a number of new effects, including spatial variations of the particle charge and shielding parameters, was accounted for. On the other hand, variations of dust charges include also the random fluctuating component which through the charging equation are related to plasma fluctuations (always present in laboratory discharges). The fluctuations can in turn influence the kinetic energy of dust particles (see also references therein) as well as the collective modes in dust particle structures.

We report here results of numerical modeling of the cluster oscillation spectra in the presence of dust charge flustuations. The eigenfrequencies of normal modes where calculating for the cluster consisted of 3 particle (N=3 cluster). We pay particular attention to the influence of the nature of the charge fluctuations on the mode spectrum. It has been found that the main effects related to the dust charge fluctuations are the splitting/spreading and the shifting of the spectral lines of the oscillation modes, proportional to the value of the charge fluctuation variance and the type (correlated/uncorrelated) of the fluctuations. Efforts were made to quantify these phenomena using the changing properties of modes. The role of charge distribution in determining the oscillation patterns of the modes was investigated by sampling different probability distributions in a preliminary study, showing the significant influence of charge distribution on mode spectra.

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Fig. 1 Dependence of the mean frequency of the normal modes (see legend on the figure) on the charge variance in the case of uncorrelated, (a), and correlated, (b) fluctuations.

2. Mode Spectra Analysis

There are 6 normal modes in oscillation spectrum of N=3 cluster. The 1st mode is purely rotational, the 2nd and 3rd purely translational, the 6th is a breathing mode (particles moving in radial direction only), while the 4th and 5th modes combine translational and radial motion. The 2nd and 3rd modes as well 4th and 4th are degenerate modes (have the same eigenfrequencies), for the mode pattern and detail description see [25, 27]. Previous work has looked at the effect of charge variance on mode spectra for small dust clusters [27] and compared the influence of correlated versus uncorrelated charges in similar circumstances [28]. In this context, the correlation of charges can be described as the relationship between the charge variance of each particle in an oscillating cluster. For correlated charge fluctuation, the same charge variance occurs simultaneously for all particles; conversely for uncorrelated charge fluctuation, each particle has a uniquely assigned charge variance selected from a common distribution. Mode spectra were presented and analysed to assess the results in each case, whereby it became apparent that a quanti-



Fig. 2 Dependence of the half-width of the normal modes on the charge variance in the case of uncorrelated, (a), and correlated, (b) fluctuations.

tative means of assessment would be of use for future investigation. Particularly in the case of a high number of dust particles (corresponding to twice the number of modes), the mode spectra become increasingly complex such that changes are harder to discern and describe.

Consequently, the existing code was extended to allow calculations of the properties of each mode. These contribute to indicating the degree of shift and spread of the modes under different parameters, and are useful starting points to describe the complicated changes to the mode spectra with high numbers of dust particles. The code was also modified to allow multiple simulations with the same parameters, using the mean, median and standard deviation of the properties described above to judge the level of convergence of the simulation results. The modified code was then used to plot the changing properties of the modes, each point averaged over 200 loops for stability, versus charge variance for both correlated and uncorrelated charges in N=3 clusters, showing how these included properties can give a quantitative insight into mode spectra trends.



Fig. 3 Dependence of the mean frequencies of the normal modes for different charge distributions in the case of uncorrelated, (a), and correlated, (b) fluctuations.

Figure 1 shows the mean frequencies of the normal modes 1 to 6 as functions of the charge variance. Accordingly, Figure 2 shows the half-widths of the normal modes 1 to 6 as functions of the charge variance. It can be clearly seen that in the case of correlated fluctuations, an oscillatory behavior of the median frequency and the half-width takes place. The mostly influenced are the 4th and 5th modes. The absence of the 2nd and 3rd mode in all plots is explicable in that, as solely translational modes, both remain unaffected by applied charge variance and do not split. In the case of correlated charge fluctuation, the 4th and 5th modes undergo identical splitting and hence the behaviour of the 4th mode is hidden under that of the 5th.

3. Charge Distributions

Previously charge fluctuations in the simulation have been selected from a normal distribution, being randomly assigned a value with each time step such that they follow a gaussian curve. However, although it is common to prescribe the charge fluctuations to be Gaussian, there is no experimental evidence for this



Fig. 4 Dependence of the half-width of the normal modes for different charge distributions in the case of uncorrelated, (a), and correlated, (b) fluctuations.

assumption. Thus it is necessary to investigate the effect of different charge distribution functions on the dust cluster properties. For this purpose, the existing code [28] was further developed to allow selection of different probability distributions using MATLAB inbuilt statistical functions. These are pre-programmed versions of known statistical functions, allowing selection of various distributions such as those used. The inverse cumulative distribution functions, as included in the MATLAB Statistical Toolbox, were used to choose values of charge variance, while the parameters for each distribution were selected within the context of the program.

Code was first written to allow input of a chosen potential distribution function over a specified interval with defined parameters, to assess the suitability of a particular curve. Potential distributions were selected with an emphasis on their distinctness from the normal distribution, such as a left-side skew or narrower curve; these include the Laplace distribution, extreme value distribution and uniform distribution. Preliminary testing of a selection of these distributions for N=3 clusters was performed at 2%, 10% and 50% charge variance, averaged over a small number of loops, plotting the mode properties described above versus distribution to gain an immediate insight into the influence of charge distribution function on mode spectra properties.

Figure 3 shows the mean frequencies of the normal modes from 1 to 6 for different charge distributions. The numbers on the x-axis correspond to: 1, the normal distribution with the mean 0 and $\sigma = 1$; 2. the normal distribution with the mean 0 and $\sigma = 0.2$; 3, the Laplace distribution with the mean 0 and $\sigma = 1$; 4, the Laplace distribution with the mean 0 and $\sigma = 0.2$; 5, the extreme value distribution with the mean 0 and $\sigma = 1$; 6, the extreme value distribution with the mean 0 and $\sigma = 0.4$; 7, the extreme value distribution with the mean 0.7 and $\sigma = 0.3$; 8, the uniform distribution; and 9, the normal mean with the mean 0 and $\sigma = 1$ (random number generator). Accordingly, Figure 4 shows the half-widths of the normal modes from 1 to 6 for different charge distributions.

4. Conclusion

To conclude, the presented preliminary results on the influence of the different charge fluctuation distributions on the cluster oscillation spectra show that such mode parameters as the median frequency and the half-width strongly depend on the character of the charge fluctuations. It was also confirmed that the oscillation spectra of dust clusters in the cases of correlated/uncorrelated fluctuations are different. The nature of this behavior still needs to be more understood. Here, we have found the new signature for the case of correlated fluctuations: the oscillatory behaviour of the modes' parameters. This fact is useful to obtain important information on the plasma correlation lengths on the basis of the analysis of the cluster mode spectra. Thus analysis of the oscillation spectra of dust clusters can be used for diagnostics of early stages of the developing dust-plasma instabilities leading to plasma fluctuations. Correlations of plasma fluctuations are related to spatial inhomogeneities developing in the gas discharge plasma. Absorption of plasma particles by dust creates a sink for the plasma making the dust-plasma sheath a highly dissipative structure. Introduction of dust also leads to appearing of new characteristic space scales related to the processes of plasma-dust interactions [1, 2]. Therefore analysis of the oscillation spectra could be used to estimate spatial non-uniformities in the near-electrode region.

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[1] S. V. Vladimirov, K. Ostrikov, and A. A. Samarian, *Physics and applications of complex plasmas* (Imperial College, London, 2005).

- [2] H. M. Thomas and G. E. Morfill, Nature (London) 379, 806 (1996).
- [3] H. Thomas, G. E. Morfill, and V. N. Tsytovich, Plasma Phys. Rep. 29, 895 (2003).
- [4] W.T. Juan, J.W. Hsu, Z.H. Huang, Y.J. Lai, and Lin I, Chin. J. Phys. 37, 184 (1999).
- [5] F. Cheung, A. Samarian, and B. James, New J. Phys. 5, 75 (2003).
- [6] D. H. E. Dubin, T. M. O'Neil, Rev. Mod. Phys. 71, 87 (1999).
- [7] P. E. Tosheck, in New Trends in Atomic Physics Proceedings of Les Houches Summer School, Amsterdam (Holland) 1, 383 (1982).
- [8] D. J. Wineland and W. M. Itano, Phys. Today 40, 30 (1987).
- [9] M. A. Reed and W. P. Kirk, Eds., Proceedings of the First International Symposium on Nanostructure Physics and Fabrication, College Station, Texas (Academic, Boston, 1989).
- [10] R. C. Ashoori, Nature (London) 379, 413 (1996).
- [11] Yu. E. Lozovik and L. M. Pomirchy, Phys. Status Solidi B 161, K11 (1990).
- [12] Yu. E. Lozovik and V. A. Mandelshtam, Phys. Lett. A 145, 269 (1990).
- [13] Yu. E. Lozovik and V. A. Mandelshtam, Phys. Lett. A 165, 469 (1992).
- [14] G. E. Astrakharchik, A. I. Belousov, and Yu. E. Lozovik, Phys. Lett. A 258, 123 (1999).
- [15] G. E. Astrakharchik, A. I. Belousov, and Yu. E. Lozovik, JETP 89, 696 (1999).
- [16] V. M. Bedanov and F. M. Peeters, Phys. Rev. B 49, 2667 (1994).
- [17] V. A. Schweigert and F. M. Peeters, Phys. Rev. B 51, 7700 (1995).
- [18] I. V. Schweigert, V. A. Schweigert, and F. M. Peeters, Phys. Rev. B 54, 10827 (1996).
- [19] L. Candido, J.-P. Rino, N. Studart, and F. M. Peeters, J. Phys.: Cond. Matt. 10, 11627 (1998).
- [20] M. Kong, B. Partoens, A. Matulis, and F. M. Peeters, Phys. Rev. E 69, 036412 (2004).
- [21] S. G. Amiranashvili, N. G. Gousein-zade, and V. N. Tsytovich, Phys. Rev. E 64, 016407 (2001).
- [22] W. T. Juan, Z. H. Huang, J. W. Hsu, Y. J. Lai, and L. I, Phys. Rev. E 58, R6947 (1998).
- [23] M. Klindworth, A. Melzer, A. Piel, and V. A. Schweigert, Phys. Rev. B 61, 8404 (2000).
- [24] A. Melzer, M. Klindworth, and A. Piel, Phys. Rev. Lett. 87, 115002 (2001).
- [25] A. Melzer, Phys. Rev. E 67, 016411 (2003).
- [26] R. Kompaneets, S. V. Vladimirov, A. V. Ivlev, V. Tsytovich, and G. Morfill, Phys. Plasmas 13, 072104 (2006).
- [27] S. Barkby, S.V. Vladimirov and A.A. Samarian, Phys. Lett. A 372, 1501 (2008).
- [28] D.J. Kedziora, S.V. Vladimirov and A.A. Samarian, Phy. Rev. E., in press (2008).