Charging of Dust Particles in Magnetic Field

Yukihiro TOMITA[1)], Gakushi KAWAMURA[1)], Takatoshi YAMADA[2)] and Osamu ISHIHARA[2)]
[1] *National Institute for Fusion Science, 322-6 Oroshi-cho, Toki 509-529,2 Japan*
[2] *Faculty of Engineering, Yokohama National University, Yokohama 240-8501, Japan*

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Absorption cross-section to a spherical dust particle for an electron/ion is studied in weak uniform magnetic field. In this study an orbit of a charged particle (an ion or an electron) heading to a charged dust particle at rest along the magnetic field is analyzed analytically and numerically. Because of the Lorentz force in the presence of magnetic force, the charged particle with the same sign as the dust charge approaches closer to the dust than the orbit without magnetic field, indicating larger absorption cross section. On the other hand the charged particle with the opposite sign of dust charge leaves further the dust, indicating the absorption cross section smaller than that in the absence of magnetic field. Our study reveals that the charge of the dust particle with negative floating potential more negative than that in the absence of magnetic field. For example, the charge state of the negative dust particle with a radius of 1 mm increases from 6.63×10^5 to 6.86×10^5 for the 1 eV ions and electrons in the magnetic field of 10 G. For the higher energy of plasmas with 10 eV, dust charge is found to increase from 6.63×10^6 to 6.70×10^6 due to the effects of magnetic field.

Keywords: dust, charging, magnetic field, Lorentz force, floating potential

1. Introduction

The generation, growth and transport of dust particles in fusion plasmas are one of the interesting topics as well as in the astrophysical, space, laboratory, and processing plasmas. One of the particular attentions in fusion devices is associated with absorption of radioactive tritium [1, 2]. After operation of plasma discharges, the treatment, collection and disposal of the radioactive dusts are one of key issues from the viewpoint of the safety. The dust density in fusion plasmas is as low as around 10^{-4} m^{-3} [3 - 5], where the collective effects of dust particles are negligibly small.

In order to study dynamics of the dust particle in plasmas the charge state of the dust particle is one of the essential issues. In this study the charged particle absorption cross-section by the spherical dust particle in weak magnetic field is investigated analytically and numerically. The absorption cross-section in the absence of magnetic field was expressed by the OML (Orbit Motion Limited) theory [6, 7]. The OML theory, where energy and angular momentum of a charged particle are conserved in an infinite Debye length limit, has been widely applied to charging of a dust particle in space plasmas as well as laboratory plasmas. The particle orbit heading to the charged dust particle along the magnetic field deviates from a straight one due to the electrostatic force by a dust particle. This deviation changes its absorption cross-section of the dust.

2. Equation of motion

The charged particle orbits of j-th species ($j = e, i$) in the magnetic field are investigated in the cylindrical coordinates (ρ, θ, z), where the uniform magnetic field B_0 is applied to the axial z-direction (Fig.1). The unmovable point dust particle is located at the origin (O) with the charge q_d. The charged particle with the charge q_j starts to move from the initial position ($\rho = b_{in}$, $\theta = 0$, $z = z_{in}$) with the velocity $(\upsilon_\rho = \upsilon_\theta = 0, \upsilon_z = \upsilon_{j,in})$. The equations of motion of the j-th charged particle in this system are,

ρ direction:

$$m_j[\frac{d^2\rho}{dt^2} - \rho(\frac{d\theta}{dt})^2] = q_j E_r \frac{\rho}{r} + q_j B_0 \rho \frac{d\theta}{dt}, \qquad (1)$$

and z direction:

$$m_j \frac{d^2 z}{dt^2} = q_j E_r \frac{z}{r}. \qquad (2)$$

Here m_j is the mass of the charged particle, r is the radial particle position from the origin (O) $(r^2 = \rho^2 + z^2)$ and E_r is the radial electrostatic field due to the charged dust particle,

$$E_r = \frac{q_d}{4\pi\varepsilon_0 r^2}. \qquad (3)$$

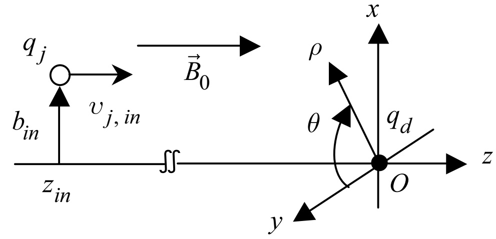

Fig.1 Coordinates and initial position and velocity of a charged particle, where .an unmovable point dust particle is set at the origin (O).

author's e-mail: tomita@nifs.ac.jp

The azimuthal motion is determined from the conservation of canonical angular momentum P_θ:

$$\frac{d\theta}{dt} = \frac{1}{m_j \rho^2}(P_\theta - \frac{q_j B_0}{2}\rho^2) \,. \quad (4)$$

The initial conditions ($\rho = b_{in}$, $d\theta / dt = 0$) give the constant value of P_θ:

$$P_\theta = \frac{q_j B_0}{2} b_{in}^2 \,. \quad (5)$$

The normalized equations of motion become

$$\frac{d^2\bar{\rho}}{d\bar{t}^2} = \frac{\alpha_j}{2}\frac{\bar{\rho}}{\bar{r}^3} + \frac{\mu_j^2}{4}\frac{1-\bar{\rho}^4}{\bar{\rho}^3} \,, \quad (6)$$

$$\frac{d^2\bar{z}}{d\bar{t}^2} = \frac{\alpha_j}{2}\frac{\bar{z}}{\bar{r}^3} \,, \quad (7)$$

where the distances, velocity and time are normalized by the impact parameter b_{in}, the initial speed $\upsilon_{j,in}$ and $b_{in}/\upsilon_{j,in}$, respectively. The system in the absence of magnetic field is determined by the parameter α_j,

$$\alpha_j \equiv \frac{q_j q_d}{4\pi\varepsilon_0 b_{in}} / \frac{m_j \upsilon_{j,in}^2}{2} \,, \quad (8)$$

which is the ratio of the electrostatic potential energy at the distance of the impact parameter to the initial kinetic energy and the parameter μ_j indicates the effect of the static magnetic field,

$$\mu_j \equiv b_{in} / \frac{m_j \upsilon_{j,in}}{\left| q_j B_0 \right|} \,, \quad (9)$$

which is the ratio of the impact parameter b_{in} to the Larmor radius with respect to the initial speed $\upsilon_{j,in}$. The parameter μ_e of the electron is much larger than that of the ion for the case of ions with the sound speed c_s and the thermal speed of the electron υ_{the}:

$$\frac{\mu_e}{\mu_i} = \frac{m_i \upsilon_{i,in}}{Z_i m_e \upsilon_{e,in}} \simeq \frac{m_i c_s}{Z_i m_e \upsilon_{the}} \simeq \sqrt{\frac{m_i}{m_e}} \,, \quad (10)$$

where Z_i is the charge state of the ion. This relation indicates the effect of the magnetic field on the ion is much smaller than that of the electron (see Eq. 6).

3. Particle orbit

From the previous section the effect of magnetic field to the electron orbit is much larger than that of the ion. The dust charge in low plasma temperature is negative because of the large mobility of electrons compared to ions. On the other hand in high plasma temperature there is a possibility to charge positively due to the thermionic emission from the dust. In this study we investigate the electron absorption cross-section of negatively charged dust particle.

In order to investigate the electron orbit numerically, the start position ($z = z_{in}$) should be determined. In Fig.2 the dependence of the closest radius ($r = r_{min}$) to the dust, where the radial velocity of the charges particle is vanishing ($dr / dt = 0$), on the initial axial position (z_{in}) is shown with the parameters $\alpha_e = 1.0$ and $\mu_e = 0$ and 0.1. In the case without magnetic field ($\mu_e = 0$) the closest radius is obtained from the OML theory as $r_{min} / b_{in} = 1.618$. The orbit of the closer start to the dust deviates due to the strong Coulomb force of the dust. This figure shows that the initial position should be far from - 1000 b_{in} with the 10^{-3} accuracy. The Coulomb force is proportional to the parameter α_e, so the start point is determined as $z_{in} = -10^3 b_{in} \alpha_e$.

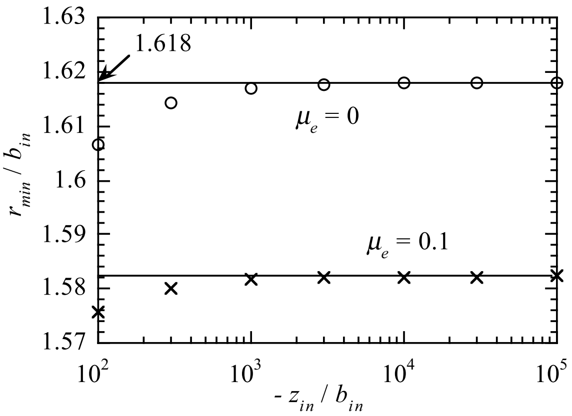

Fig.2 The closest positions as a function of initial axial position (z_{in}), where $\alpha_e = 1.0$ and $\mu_e = 0, 0.1$.

The typical orbit of an electron near the negatively charged dust, which is located at the origin ($\rho = z = 0$), is shown in Fig.3, where $\alpha_e = 1.0$ and $\mu_e = 0.01$. The closest radius in the presence of in the axial magnetic field (solid line in Fig.3) becomes smaller than that in the absence of magnetic field. The orbit of a charged particle in magnetic field is characterized by three-dimensional nature rather than the two dimensional orbit. The particle has a radial velocity due to the radial electric field. This radial velocity pushes an electron in the azimuthal

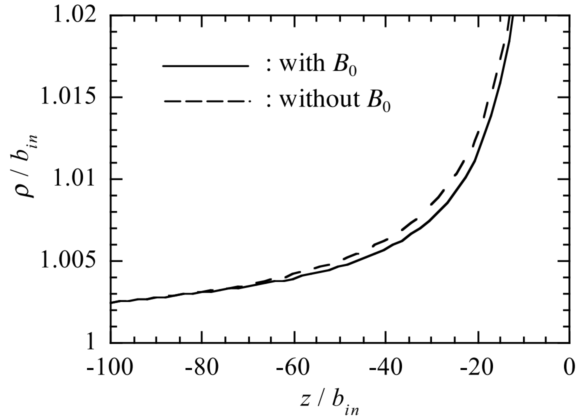

Fig.3 Typical orbit of an electron near the negatively charged dust for the case $\alpha_e = 1.0$ and $\mu_e = 0.01$. The solid and dashed lines are the orbits with and without magnetic field, respectively.

direction by the Lorentz force. This azimuthal velocity makes the radial force by the axial magnetic field. Thus the magnetic field has the second order effect on the orbit without magnetic field, see Eq. 6. As a result the radial force balance of the particle between the Lorentz force and the centrifugal force determines the radial motion of a charged particle. The radial equation of motion is expressed from Eq. 1,

$$m_j \frac{d^2\rho}{dt^2} = q_j E_r \frac{\rho}{r} + m_j \rho (\frac{d\theta}{dt})^2 + q_j B_0 \rho \frac{d\theta}{dt}, \qquad (11)$$

where the first term of the RHS is the electrostatic force by the dust particle, the second one is the centrifugal force and the third one indicates the Lorentz force. From the relation of the conservation of the canonical angular momentum (Eqs. 4 and 5), the summation of the centrifugal force and the Lorentz force is expressed as:

$$m_j \rho (\frac{d\theta}{dt})^2 + q_j B_0 \rho \frac{d\theta}{dt} = \frac{1}{m_j \rho^3}(P_\theta^2 - \frac{q_j^2 B_0^2}{4}\rho^4)$$

$$= \frac{q_j^2 B_0^2}{4 m_j \rho^3}(b_{in}^4 - \rho^4). \qquad (12)$$

For the case of the orbit of the charged particle with the same sign as the dust charge, its radius ρ is larger than the initial one (b_{in}) due to the radial electrostatic force, which means the Lorentz force is stronger than the centrifugal force all the time. This deference makes the closest radius smaller than that without magnetic field, Fig. 6. On the other hand the charged particle with the opposite sign of the dust charge leaves further the dust.

4. Electron absorption cross-section

For the weak magnetic field, i.e. small μ_e, the closest radius r_{min} is linearly proportional to the strength of magnetic field or μ_e,

$$\bar{r}_{\min}(\alpha_e, \mu_e) = \bar{r}_{\min 0}(\alpha_e) + \gamma_e(\alpha_e)\mu_e. \qquad (13)$$

Here γ_e is the constant of proportion, which depends on α_e and $\bar{r}_{\min 0}$ is the closest radius in the absence of magnetic field, which is obtained from the OML theory:

$$\bar{r}_{\min 0}(\alpha_e) = \frac{1}{2}(\alpha_e + \sqrt{\alpha_e^2 + 4}). \qquad (14)$$

In Fig.4 the closest radius is shown by the dashed line for the small μ_e, where the parameter α_e = 1.0, where the linearly approximated line is shown by the solid line. The linear approximation is valid in the range $\mu_e < 0.4$ for α_e = 1.0. In the case of the larger α_e the upper limit of μ_e for the linear approximation becomes smaller, e.g. 0.04 for α_e = 10.0.

The dependence of the coefficient γ_e on the parameter α_e is investigated numerically, Fig.5. The least square curve (solid line in Fig.5) indicates

$$\gamma_e = -0.114\alpha_e^{1.962}. \qquad (15)$$

This dependence is approximated in this study as:

$$\gamma_e = -0.114\alpha_e^{2.0}, \qquad (16)$$

which is shown by the dashed line in Fig.5.

By using Eq. 16, the absorption cross-section is obtained. The un-normalized closest radius is expressed from Eq. 13:

$$r_{\min}(\alpha_e^*, \mu_e^*) = r_{\min 0}(\alpha_e^*) + \gamma_e(\alpha_e^*)\mu_e^* b_{in}^2, \qquad (17)$$

where

$$r_{\min 0}(\alpha_e^*) = \frac{1}{2}(\alpha_e^* + \sqrt{\alpha_e^{*2} + 4b_{in}^2}), \qquad (18)$$

$$\gamma_e(\alpha_e^*) = -0.114\alpha_e^{*2} / b_{in}^2, \qquad (19)$$

$$\alpha_e^* \equiv b_{in}\alpha_e = \frac{-eq_d}{4\pi\varepsilon_0} / \frac{m_e v_{e,in}^2}{2}, \qquad (20)$$

$$\mu_e^* \equiv \mu_e / b_{in} = \left|eB_0\right| / m_e v_{e,in}. \qquad (21)$$

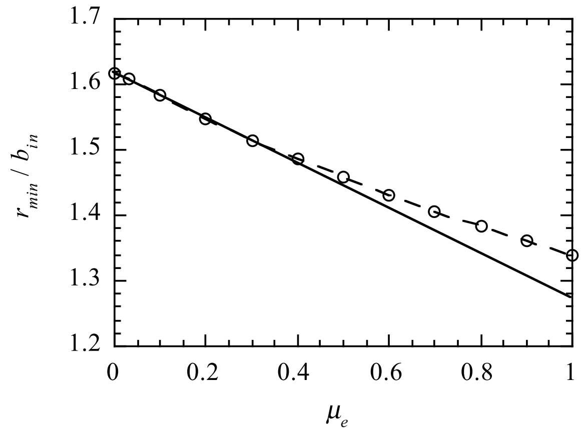

Fig.4 The closest radius as a function of parameter of magnetic field μ_e, where α_e = 1.0.

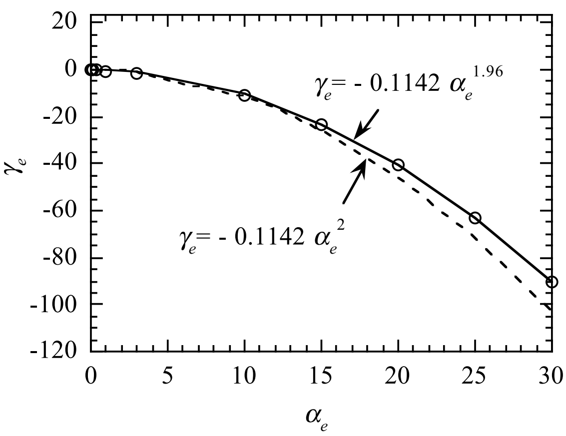

Fig.5 The dependence of the coefficient γ_e on the parameter α_e. The least square and the approximated curves are shown by the solid and the dashed lines, respectively.

This closest radius corresponds to the effective finite dust radius R_d to the absorption. From this result the absorption cross-section of an electron to the dust is obtained easily.

$$\sigma^e_{ab} = \pi b^2_{in} = \pi(R_d + 0.114\alpha_e^{*2}\mu_e^*)(R_d + 0.114\alpha_e^{*2}\mu_e^* - \alpha_e^*) \quad (22)$$

In the case without magnetic field ($\mu_e^* = 0$) the absorption cross-section becomes the conventional one from the OML theory:

$$\sigma^e_{ab0} = \pi R_d^2(1 - \alpha_e^* / R_d) = \pi R_d^2(1 + \frac{eq_d}{2\pi\varepsilon_0 R_d m_e \upsilon^2_{e,\,in}}) \quad (23)$$

5. Charge of floating dust

In the previous section we obtain the cross-section of an electron to the negatively charged dust particle. The charge of the floating dust particle is determined from the condition of equal particle fluxes of ions and electrons to the dust particle. The ion absorption cross-section is expressed by the OML theory, where the effect of magnetic field is negligibly small:

$$\sigma^i_{ab0} = \pi R_d^2(1 + \frac{Z_i Z_d e^2}{2\pi\varepsilon_0 R_d m_i \upsilon_i^2}), \quad (24)$$

where m_i, Z_i and υ_i are the mass, charge and speed of plasma ions. The charge state of the dust is Z_d (> 0). In the case of the mono-energy charged particles, the equal particle flux gives the relation.

$$\sigma^e_{ab}(Z_d; R_d, \varepsilon_e, B_0) = \sqrt{\frac{m_e\varepsilon_i}{m_i\varepsilon_e}}\sigma^i_{ab0}(Z_d; R_d, \varepsilon_i)\,. \quad (25)$$

In the case of R_d = 1 mm, $\varepsilon_e = \varepsilon_i$ = 1, 10 eV, the dependence of the dust charge Z_d as a function of the weak magnetic field B_0 is shown in Fig.6. The charge state of the negative dust particle increases from 6.63×10^5 to 6.86×10^5 for the 1 eV ions and electrons in the magnetic field of 10 G. This increase of charge state comes from the increase of the electron absorption cross-section in the magnetic field. For the higher energy of plasmas with 10 eV, dust charge is found to increase from 6.63×10^6 to 6.70×10^6 due to the effects of magnetic field.

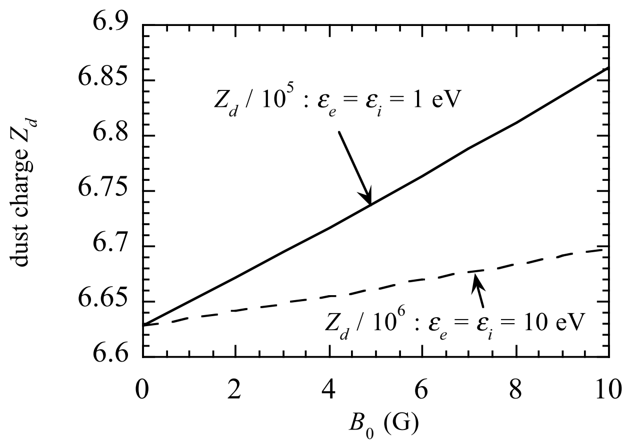

Fig.6 Charge of floating dust as a function of weak magnetic field B_0, where dust radius R_d = 1 mm and particle energy of ions and electrons = 1, 10 eV.

6. Conclusion

The absorption cross-section of the charge particle to the spherical dust particle in a weak magnetic field was investigated analytically and numerically. The effects of magnetic field to ions are found to be negligibly small compared to electrons. The closest radius of electrons becomes smaller than that in the absence of magnetic field due to the Lorentz force, indicating the absorption cross-section larger. In order to investigate the parameter dependence of the electrostatic force and the magnetic field on the electron orbits, the closest radius of an electron is approximated by the linear dependence of the strength of magnetic field and the constant of proportion is modeled by the quadratic dependence of the parameter of the electrostatic force. This model makes possible to estimate the charge state of the floating dust in a plasma. The charge state in magnetic field becomes negatively higher than the case without magnetic field. These results can be important to analyze the dynamics of the dust particle in plasmas immersed in the magnetic field. The higher order approximation of the effects of magnetic field, the absorption cross-section of electrons to the positively charged dust particle and the velocity distribution of the charged particles are left as future issues. The effects of stronger magnetic field, where the strong Larmor motions are dominant, can be studied by the statistical approach of the particle orbits.

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[1] G. Federici *et al.*, Nucl. Fusion **41**, 1967 (2001).

[2] J. Winter, Plasma Phys. Control. Fusion **46**, B583 (2004).

[3] A. Sagara, S. Masuzaki, T. Morisaki *et al.*, J. Nucl. Mater. **313-316**, 1 (2003).

[4] J.P. Sharpe, V. Rohde, A. Sagara et al., J. Nucl. Mater. **313-316**, 455 (2003).

[5] J. P. Sharpe, P. W. Humrickhouse, C. H. Skinner *et al.*, J. Nucl. Mater. **337–339**, 1000 (2005).

[6] H. Mott-Smith and I. Langmuir, Phys. Rev. **28**, 727 (1926).

[7] J .E. Allen, Phisica Scripta **45**, 497 (1992).