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Solitary waves of the Kadomstev-Petviashvili equation in warm dusty plasma with variable dust charge, two temperature ion and nonthermal electron

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Abstract: The propagation of nonlinear waves in warm dusty plasmas with variable dust charge, two temperature ion and nonthermal electron is studied. By using the reductive perturbation theory, the Kadomstev-Petviashivili (KP) equation is derived. Existence of rarefactive and compressive solitons is analyzed.

Keywords: dust, soliton, nonthermal

1. Introduction

The study of dusty plasmas represents one of the most rapidly growing branches of plasma physics. Usually the dust grains are of micrometer or sub-micrometer size and their masses are very large. Rao et al. [1] have reported the existence of dust acoustic waves (DAW) for low frequency in unmagnetized dusty plasma; theoretically. Experimental observations have confirmed the existence of linear and nonlinear feature of both dust acoustic waves (DAW) and dust ion acoustic waves (DIAW) [2]. Laboratory observations of dust acoustic waves and compare them with available theories have done with Barkan et al. [3]. Moreover study of dusty plasmas media are very attractive because of their theoretical features and also their applications which have been observed in the earth's magnetosphere, cometary tail, planetary rings and so on [4]. In most investigations reductive perturbation method has been used for deriving the Korteweg-de Vries (KdV), Zakharov-Kuznetsov (ZK) and Kadomstev-Petviashvili (KP). KP equation has been derived for dust acoustic wave in hot dusty plasma by Duan [5]. KP, modified KP and coupled KP equation for dusty plasma with two ions have been obtained by Lin and Duan [6]. The charging process of dust particles is an important effect which has been investigated in [7]. The nonlinear properties of dust acoustic waves in magnetized dusty plasmas with variable dust charges have been studied by Ghosh et al. [8]. Zhang and Xue by using the KdV equation have studied effects of the dust charge variation and nonthermal ions on dust acoustic solitary structures in magnetized dusty plasmas [9]. Role of negative ions in dusty plasma with variable dust charge have been investigated by Ghosh [10]. Also, Gill et al. [11] have derived KP equation for dusty plasma with variable dust charge and two temperature ions, but the warm dusty plasma with variable dust charge, nonthermal electron and two temperature ions were not considered. Studying the dust temperature effect and charge effect are very important for understanding their influences on dusty plasmas. In the presented paper, the warm dusty plasma with the variable dust charge, two temperature ions and nonthermal electrons has been considered. In section 2, the basic set of equations is introduced and in section 3 by using the reductive perturbation method (RPM) the KP equation has been derived. We discuss on solutions of KP equation in section 4. Conclusions and remarks are given in section 5.

2. Basic equations

We consider the propagation of dust acoustic waves in collisionless, unmagnetized warm dusty plasma consisting of electrons, two temperature ions and high negatively charged dust grains. Total charge neutrality at equilibrium requires that

$$n_{0e} + n_{0d} Z_{0d} = n_{0il} + n_{0ih} \tag{1}$$

where n_{0e}, n_{0d}, n_{0il} and n_{0ih} are the equilibrium values of electrons, dust, lower temperature ions and higher temperature ions number densities respectively. Z_{0d} is the unperturbed number of charges on the dust particles. The following set of normalized two dimensional equations of continuity, motion for the adiabatic dust and Poisson, describe dynamics of dust acoustic wave in such plasma

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$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d u_d)+\frac{\partial}{\partial y}(n_d v_d)=0$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}+v_d\frac{\partial u_d}{\partial y}+\frac{\sigma_d}{n_d}\frac{\partial P_d}{\partial x}=Z_d\frac{\partial \phi}{\partial x}$$

$$\frac{\partial v_d}{\partial t}+u_d\frac{\partial v_d}{\partial x}+v_d\frac{\partial v_d}{\partial y}+\frac{\sigma_d}{n_d}\frac{\partial P_d}{\partial y}=Z_d\frac{\partial \phi}{\partial y} \qquad (2)$$

$$\frac{\partial p_d}{\partial t}+u_d\frac{\partial p_d}{\partial x}+v_d\frac{\partial p_d}{\partial y}+3p_d(\frac{\partial u_d}{\partial y}+\frac{\partial v_d}{\partial y})=0$$

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=Z_d n_d+n_e-n_{il}-n_{ih}$$

where u_d and v_d are velocity components of the dust particles in x and y-directions. p_d and ϕ are the pressure of the dust particles and electrostatic potential. n_d and Z_d are the dust number density and the variable charge number of dust grains. All of these variables are normalized, and related dimensionless number densities for electrons (n_e), low temperature ions (n_{il}) and high temperature ions (n_{ih}) are

$$n_e=\frac{1}{\delta_1+\delta_2-1}\left[1-\frac{4\alpha}{1+3\alpha}\beta_1 s\phi+\frac{4\alpha}{1+3\alpha}(\beta_1 s\phi)^2\right]\exp(\beta_1 s\phi)$$

$$n_{il}=\frac{\delta_1}{\delta_1+\delta_2-1}\exp(-s\phi) \qquad (3)$$

$$n_{ih}=\frac{\delta_2}{\delta_1+\delta_2-1}\exp(-\beta s\phi)$$

where

$$\beta_1=\frac{T_{il}}{T_e}\ ,\ \beta_2=\frac{T_{ih}}{T_e}\ ,\ \beta=\frac{\beta_1}{\beta_2}=\frac{T_{il}}{T_{ih}}\ ,\ s=\frac{T_{eff}}{T_{il}}=\frac{\delta_1+\delta_2-1}{\delta_1+\delta_2\beta+\beta_1}$$

$$\delta_1=\frac{n_{0il}}{n_{0e}},\delta_2=\frac{n_{0ih}}{n_{0e}},\sigma=\frac{T_d}{T_{eff}} \qquad (4)$$

Where T_d,T_e,T_{il},T_{ih} are temperature of dust, electron and low temperature and high temperature of ions α is nonthermal parameter in which determines the number of fast (nonthermal) electrons.

Z_d can be expanded respect to ϕ as follows

$$Z_d=1+\gamma_1\phi+\gamma_2\phi^2+\ldots \qquad (5)$$

3. The derivation of KP equation

According to the general method of reductive perturbation theory, we choose the independent variables as

$$\xi=\varepsilon(x-\lambda t)\quad ,\quad \tau=\varepsilon^3 t\quad ,\quad \eta=\varepsilon^2 y \qquad (6)$$

where ε is a small dimensionless expansion parameter which characterizes the strength of nonlinearity in the system and λ is the phase velocity of the wave along the x direction. We can expand physical quantities which have been appeared in (2), in term of the expansion parameter ε as

$$n_d=1+\varepsilon^2 n_{1d}+\varepsilon^4 n_{2d}+\ldots\ldots$$

$$u_d=\varepsilon^2 u_{1d}+\varepsilon^4 u_{2d}+\ldots.$$

$$v_d=\varepsilon^3 v_{1d}+\varepsilon^5 v_{2d}+\ldots.. \qquad (7)$$

$$\phi=\varepsilon^2\phi_1+\varepsilon^4\phi_2+\ldots.$$

$$P_d=1+\varepsilon^2 P_{1d}+\varepsilon^4 P_{2d}+\ldots$$

$$Z_d=1+\varepsilon^2 Z_{1d}+\varepsilon^4 Z_{2d}+\ldots$$

We Substitut (6) and (7) into equations (2).
The KP equation is derived from the above equations

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi_1}{\partial \tau}+A\phi_1\frac{\partial \phi_1}{\partial \xi}+B\frac{\partial^3\phi_1}{\partial \xi^3}\right]+C\frac{\partial^2\phi_1}{\partial \eta^2}=0 \qquad (8)$$

where

$$A=\frac{1}{2\lambda}\left\{-2+(\lambda^2-3\sigma)^2\left[(\delta_1+\delta_2\beta^2-\beta_1^2)\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)^2}-2\gamma_2\right]+3\gamma_1(\lambda^2-3\sigma)-\frac{\lambda^2+9\sigma}{\lambda^2-3\sigma}\right\}$$

$$B=\frac{1}{2\lambda}(\lambda^2-3\sigma)^2\quad ,\qquad C=\frac{\lambda}{2} \qquad (9)$$

4. Discussion

We introduce the variable

$$\chi=l\xi+m\eta-u\tau \qquad (10)$$

where χ is the transformed coordinate relative to a frame which moves with the velocity u. "l" and "m" are the directional cosines of the wave vector "k" along the ξ and η respectively, in the way that $l^2+m^2=1$.

By integrating (10) respect to the variable χ and using the vanishing boundary condition for ϕ_1 and its derivatives up to the second-order for $|\chi|\to\infty$, we have

$$\frac{d^2\phi_1}{d\chi^2} = \frac{h}{l^4 B}\phi_1 - \frac{A}{2l^2 B}\phi_1^2 \qquad (11)$$

where

$$h = ul - m^2 c \qquad (12)$$

Equation (11) has solitonic solutions and one-soliton solution for this equation is given by

$$\phi_1 = \phi_\circ \sec h^2\left[\frac{\chi}{W}\right] \qquad (13)$$

where $h = ul - m^2 c$ and $\phi_\circ = \frac{3h}{l^2 A}$ is the amplitude while $W = 2\sqrt{\frac{l^4 B}{h}}$ is the width of the soliton.

For investigating the stability conditions of this solution, we use a method based on the energy considerations [12]. Thus we are going to find the Sagdeev potential for this situation. Equation (11) can be written as

$$\frac{d^2\phi_1}{d\chi^2} = \frac{h}{l^4 B}\phi_1 - \frac{A}{2l^2 B}\phi_1^2 = -\frac{dV(\phi_1)}{d\phi_1} \qquad (14)$$

In order to obtain the Sagdeev potential, Equation (14) is integrated to yield the nonlinear equation of motion as

$$\frac{1}{2}\left[\frac{d\phi_1}{d\chi}\right]^2 + V(\phi_1) = 0 \qquad (15)$$

where

$$V(\phi_1) = \frac{A}{6l^2 B}\phi_1^3 - \frac{h}{2l^4 B}\phi_1^2 \qquad (16)$$

It is clear that $V(\phi_1) = 0$ and $\frac{dV(\phi_1)}{d\phi_1} = 0$ at $\phi_1 = 0$. A stable solitonic solution must satisfy the following conditions [13]

$$\text{I)}\left[\frac{d^2 V}{d\phi_1^2}\right]_{\phi_1=0} < 0 \qquad (17)$$

II) There must exists a nonzero crossing point $\phi_1 = \phi_\circ$ that $V(\phi_1 = \phi_0) = 0$.

III) There must exists a ϕ_1 between $\phi_1 = 0$ and $\phi_1 = \phi_\circ$ to make $V(\phi_1) < 0$.

Thus, from (16) and (17) we have

$$\frac{d^2 V(\phi_1)}{d\phi_1}\Big|_{\phi_1=0} = -\frac{h}{l^4 B} < 0 \qquad (18)$$

The parameters, l and b are positive, Therefore h>0 or

$$ul - m^2 c > 0 \qquad (19)$$

We found that h>0, thus type of solitons depends on positive or negative values of "A".

Duan [5] has shown for warm dusty plasma with Maxwellian electron and ion and $\gamma_1 = \gamma_2 = 0$, nonlinear term is always negative. For cold dusty plasma without nonthermal electron and dust particles with constant charge [11], that is $\alpha = \sigma = 0$ and $\gamma_1 = \gamma_2 = 0$ we have

$$A = \frac{1}{2}\left[(\delta_1 + \delta_2\beta^2 - \beta_1^2)\frac{(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2\beta + \beta_1)^2} - 3\right] \qquad (20)$$

Obviously $(\delta_1 + \delta_2\beta^2 - \beta_1^2)$ is always less than $(\delta_1 + \delta_2\beta + \beta_1)$, but for term $\frac{(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2\beta + \beta_1)}$ we have

$$\frac{(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2\beta + \beta_1)} = \frac{(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2 - 1) + 1 + \beta_1 - (1-\beta)\delta_2}$$

It is clear that above term is less than 1 if $\delta_2 < \frac{1+\beta_1}{1-\beta}$ and in this case "A" is always negative and rarefactive solitons always exist. Also above mentioned term is more than 1 if $\delta_2 > \frac{1+\beta_1}{1-\beta}$ and in this case "A" can get positive or negative values and in these cases both compressive and rarefactive solitary waves can be propagated. Therefore, in (9) "A" get both negative and positive values. And thus both compressive and rarefactive solitons exist.

Now let us find the stability conditions for the above solution. From the (12) we have

$$u > \frac{m^2}{l}c, \text{ or } \quad u > (\frac{1-l^2}{l})c \qquad (21)$$

If $\frac{1-l^2}{l} > 1$ then u>c and when $\frac{1-l^2}{l} < 1$ we have u<c. Thus the soliton is stable if

$$\begin{cases} u \geq c & when \quad 0 < l \leq 0.62 \\ 0 < u < c & when \quad 0.62 < l < 1 \end{cases} \tag{34}$$

5. References

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