Effects of Small Wavenumber Alfvén Waves on Particle Acceleration

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Energetic charged particles are accelerated by turbulent Alfvén waves via resonant interaction. We study effects of non-resonant Alfvén waves on energy diffusion by using test particle simulations. When only resonant Alfvén waves with small amplitude exist, simulated diffusion coefficients are similar to that by the quasi-linear theory. If the non-resonant Alfvén waves are added in the wavenumbers smaller than the resonant wavenumber, it is found that non-linear effects by the non-resonant Alfvén waves enhance the energy diffusion. The non-linear enhancement becomes stronger with increasing the energy density of the non-resonant Alfvén waves.

Keywords: cosmic rays, acceleration of particles, diffusion in energy space, MHD turbulence

1. Introduction

Non-thermal emission observed in astrophysical sources (e.g. supernova remnants, radio galaxies, clusters of galaxies) is produced by relativistic particles (cosmic rays). Acceleration at astrophysical shocks is believed to be a mechanism for the origin of the relativistic particles. On the other hand, there are sources which are difficult to be explained through the shock acceleration. Radio halo emission in a cluster of galaxies is such a source. The radio halo is diffuse synchrotron radio emission seen in the central region on Mpc scale [4]. The radio halos are observed in merging clusters. This might suggest that the merger shocks accelerate relativistic electrons responsible for the halo emission. However, X-ray observations show that the shocks in the inner regions of the clusters have low Mach numbers. According to [1], the acceleration efficiency at these shocks is expected to be low. It has been shown that acceleration of the relativistic electrons by MHD turbulence is suitable to explain the radio halo emission (e.g., [2, 10, 11]).

One of the acceleration mechanisms by MHD turbulence is acceleration by Alfvén waves. The resonant interaction of particles with the Alfvén waves results in diffusion in energy space, which can be described by a diffusion coefficient [12]. In the quasi-linear theory, the pitch angle-averaged diffusion coefficient in momentum space for particles with a Lorentz factor of γ is given by [2, 3]:

$$D_p(p) = \frac{2\pi^2 e^2 v_A^2}{c^3} \int_{k_{res}}^{k_{max}} \left[1 - \left(\frac{v_A}{c} \mp \frac{\Omega m}{pk}\right)^2\right] \times \frac{P(k)}{k} dk, \quad (1)$$

where $\Omega = \Omega_0/\gamma$ is the gyrofrequency, c is the speed of light, v_A is the Alfvén velocity, and $P(k)$ is the power spectrum of the Alfvén waves. The resonant wavenumber is given by $k_{res} = \Omega_0/(c\gamma)$, where we assume $v_A \ll c$ and the pitch angle cosine $\mu = 1$.

The quasi-linear theory relies on two assumptions. First, the wave amplitude is small. Second, the wave phases are random. When these assumptions are invalid, it has been found that nonlinear effects modify the quasi-linear results. It was shown that the nonlinear effects by non-resonant large amplitude waves enhance the particle acceleration [9, 13]. When the waves phases are strongly correlated, it was shown that the particles are accelerated efficiently [7].

The particles with a Lorentz factor γ resonate with the Alfvén waves having the wavenumbers larger than the resonant wavenumber k_{res}. In addition, Alfvén waves having wavenumbers smaller than k_{res} can scatter the particles as discussed in section 3. When the power spectrum of the turbulent Alfvén waves is assumed to be a single power law $P(k) \propto k^{-w}$ with an index $w > 1$, amplitude of the Alfvén waves becomes larger as k is smaller. In this case, the non-linear effects by the non-resonant Alfvén waves may enhance the energy diffusion. The aim of this paper is to investigate effects of Alfvén waves having wavenumbers smaller than k_{res} on the particle acceleration by using test particle simulations.

2. Model

We consider a one-dimensional system with the length L. The Alfvén waves propagate parallel or anti-parallel to the background magnetic field $B_0\boldsymbol{e}_x$. The frequency ω and the wavenumber k satisfy the dispersion relation,

$$\omega = \pm k v_A. \quad (2)$$

Following [13], we define left-hand polarized waves as waves having $\omega > 0$, and right-hand polarized waves as waves having $\omega < 0$. We set four wave compo-

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nents, L^+) the left-hand polarized waves propagating in the $+x$ direction, R^+) the right-hand polarized waves propagating in the $+x$ direction, L^-) the left-hand polarized waves propagating in the $-x$ direction, and R^-) the right-hand polarized waves propagating in the $-x$ direction. A turbulent field is given by superposing circularly polarized Alfvén waves [13],

$$B_y^j = \sum_{|k|=k_{min}}^{k_{max}} b_k^j \cos(kx - \omega_k^j t + \alpha_k^j)$$
$$B_z^j = \sum_{|k|=k_{min}}^{k_{max}} b_k^j \sin(kx - \omega_k^j t + \alpha_k^j), \qquad (3)$$

where k_{min} is the minimum wavenumber and k_{max} is the maximum wavenumber. The suffix j represents the wave components, L^+, L^-, R^+, and R^-. The frequency is $\omega_k^j = +kv_A$ for L^+, R^+ components, and $\omega_k^j = -kv_A$ for L^-, R^- components. The sign of the wavenumber is positive($k = |k|$) for L^+, R^- components, and is negative($k = -|k|$) for R^+, L^- components. The wavenumber $|k|$ is given by $|k| = mk_1$, where $k_1 = 2\pi/L$. Initial wave phases α_k^j's are random in the range from 0 to 2π. The wave amplitude b_k^j is given by

$$(b_k^j)^2 = (\delta B^j)^2 \frac{m^{-w}}{\sum_{m_{min}}^{m_{max}} m^{-w}}, \qquad (4)$$

where m_{min} is the number corresponds to k_{min}, m_{max} is that corresponds to k_{max}, and $(\delta B^j)^2/8\pi$ is the wave energy density. The magnetic field and the electric field of turbulent waves are given by,

$$B_y = \sum_j [B_y^j]$$
$$B_z = \sum_j [B_z^j] \qquad (5)$$

$$E_y = \sum_j \left[\frac{\omega_k^j}{ck} B_z^j\right]$$
$$E_z = \sum_j \left[-\frac{\omega_k^j}{ck} B_y^j\right]. \qquad (6)$$

Test particles (electrons) trajectories are followed by integration of the equations of motion in the given field $\boldsymbol{B} = (B_0, B_y, B_z)$ and $\boldsymbol{E} = (0, E_y, E_z)$,

$$\frac{d\boldsymbol{u}}{dt} = \frac{q}{m_0}\left(\boldsymbol{E} + \frac{1}{c}\frac{\boldsymbol{u}}{\gamma} \times \boldsymbol{B}\right), \qquad (7)$$
$$\frac{dx}{dt} = v_x, \qquad (8)$$

where m_0 is a rest mass, q is a charge, and $\boldsymbol{u} = \gamma\boldsymbol{v}$. The normalization constants for time, velocity, and fields are Ω_0^{-1}, c, and B_0, respectively.

Initially, 1000 electrons, which have the initial Lorentz factor $\gamma_0 = 10$ and have an isotropic pitch angle distribution, are injected at random positions. The system length L is chosen so that the resonant wavenumber k_{res} corresponds to $m_{res} = 100$. In this paper, we use $v_A/c = 0.03$ and $w = 2$. Model free parameters are m_{min} and m_{max}, and the total energy density $(\delta\tilde{B})^2 = (\delta B/B_0)^2$ of the Alfvén waves in the range $m_{min} \le m \le m_{max}$.

The energy diffusion coefficient can be estimated as

$$D^{sim} = \frac{<(\gamma - \gamma_0)^2>}{2\tilde{t}}, \qquad (9)$$

where <> denotes the ensemble average and $\tilde{t} = t\Omega_0$. Since simulated diffusion coefficients fluctuate depending on the initial wave phase α_k^j, we carry out ten simulation runs with different α_k^j sets in each model of $[m_{min}, m_{max}, (\delta\tilde{B})^2]$ and average the simulated diffusion coefficients.

3. Results

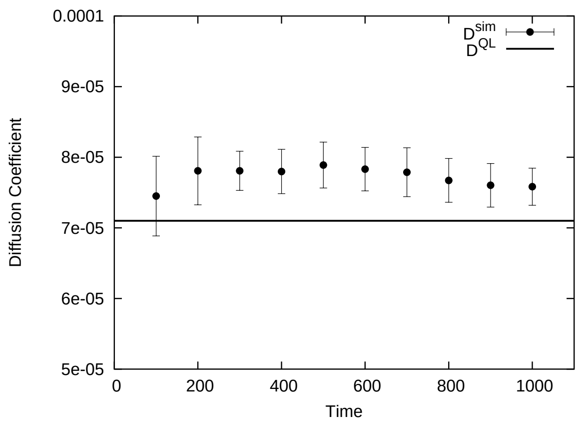

Fig. 1 The simulated diffusion coefficients D^{sim} versus $\tilde{t}$. The simulation parameters are $m_{min} = m_{res}$, $m_{max} = 400$, and $(\delta\tilde{B})^2 = 0.04$.

First, we consider the case when $m_{min} = m_{res}(= 100)$ and $m_{max} = 400$. Fig.1 shows simulated diffusion coefficients versus $\tilde{t}$ for the case of $(\delta\tilde{B})^2 = 0.04$. The filled circles with the error bars represent the averaged diffusion coefficients, and the error bars represent the standard deviation. The solid line represents the quasi-linear prediction

$$D^{QL} \sim \frac{\pi}{2}\frac{w-1}{w(w+2)}\left(\frac{v_A}{c}\right)^2 \zeta\gamma, \qquad (10)$$

where ζ is the normalized energy density of the Alfvén waves in the range $m_{res} \le m \le m_{max}$. In this model $\zeta = 0.04$, $D^{QL} \sim 7.1 \times 10^{-5}$. Fig.2 shows the simulated diffusion coefficients at $\tilde{t} = 10^3$ versus ζ. The open circles represent the quasi-linear predictions.

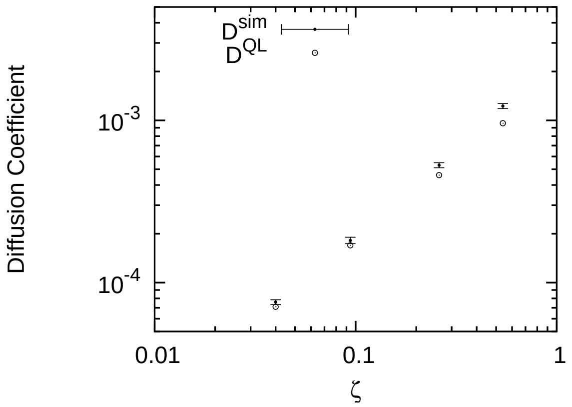

Fig. 2 The simulated diffusion coefficients at $\tilde{t} = 10^3$ versus the wave energy density ζ.

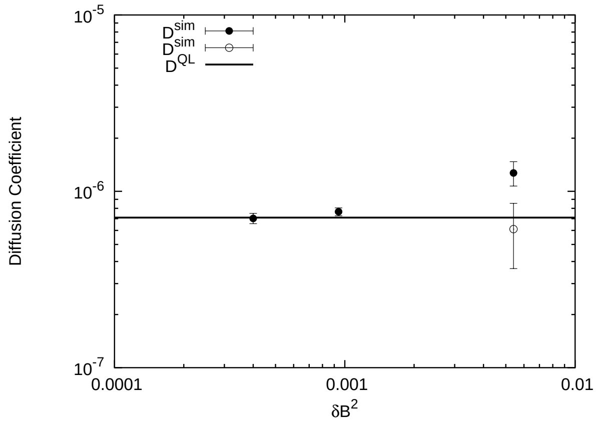

Fig. 4 The simulated diffusion coefficients at $\tilde{t} = 10^3$ versus $\delta\tilde{B}^2$. In this case $\zeta = 4 \times 10^{-4}$.

When ζ is less than ~ 0.1, the simulated diffusion coefficients are similar to the quasi-linear predictions. As ζ increases, D^{sim} rises faster than the quasi-linear one, as previously reported by [7, 9, 13]. We obtain $D^{sim}/D^{QL} \sim 1.3$, when $\zeta \sim 0.54$. In this case, non-linear enhancement is weak because of the absence of non-resonant Alfvén waves having $m < m_{res}$.

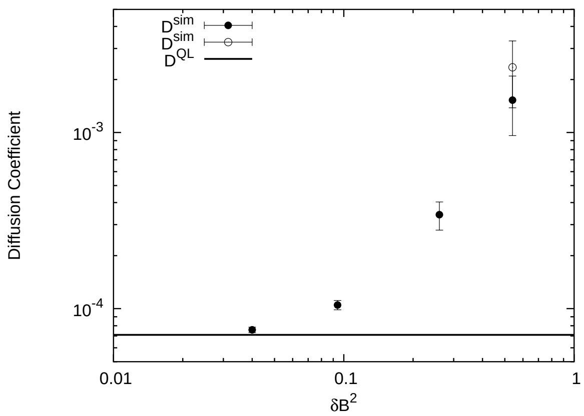

Fig. 3 The simulated diffusion coefficients at $\tilde{t} = 10^3$ versus $\delta\tilde{B}^2$. In this case, the energy density of the waves in the range $m_{res} \leq m \leq m_{max}$ is $\zeta = 0.04$.

We consider the case when $m_{min} < m_{res}$ to see how non-linear effects by the non-resonant Alfvén waves modify the energy diffusion. Fig.3 shows the simulated diffusion coefficients at $\tilde{t} = 10^3$ versus the total energy density $\delta\tilde{B}^2$ of the Alfvén waves in the wavenumber range $m_{min} \leq m \leq m_{max}$. In this case the energy density of the waves in the range $m_{res} \leq m \leq m_{max}$ is assumed to be $\zeta = 0.04$, which is sufficient for the quasi-linear treatment. The total wave energy density $\delta\tilde{B}^2$ becomes larger as m_{min} is smaller because the power spectrum is assumed to be a single power law. In the models with $m_{min} = 50$, 20, and 10, $\delta\tilde{B}^2$ is ~ 0.09, 0.26, and 0.54, respectively. The filled circles with the error bars represent the averaged diffusion coefficients, and the error bars represent the standard deviation. Since ζ is fixed, the quasi-linear diffusion coefficient represented by the solid line is constant $D^{QL} \sim 7.1 \times 10^{-5}$. The simulated diffusion coefficient becomes larger with increasing $\delta\tilde{B}^2$. Because the non-resonant waves have larger amplitude in this case, the non-linear enhancement is strong. When $\delta\tilde{B}^2 = 0.54$, we obtain $D^{sim} \sim 1.5 \times 10^{-3}$, which is an order of magnitude larger than D^{QL}.

Fig.4 shows D^{sim} at $\tilde{t} = 10^3$ versus $\delta\tilde{B}^2$ in the case when ζ is assumed to be 0.0004. In this case $D^{QL} \sim 7.1 \times 10^{-7}$. In the models with $m_{min} = 50$ and 10, $\delta\tilde{B}^2$ is 0.0009 and 0.0054, respectively. In this case the non-linear enhancement is weak and we obtain $D^{sim}/D^{QL} \sim 2$ when $\delta\tilde{B}^2 = 0.0054$.

To see the energy diffusion only by the non-resonant waves, we consider the case when the wavenumber range is $m_{min} \leq m < m_{res}$. The open circle in Fig.3 represents D^{sim} of the model with ($m_{min} = 10$, $m_{max} = 90$, $\delta\tilde{B}^2 = 0.54$). The observed D^{sim} is similar to that of the previous model with ($m_{min} = 10$, $m_{max} = 400$, $\delta\tilde{B}^2 = 0.54$). Fig.4 shows that the observed D^{sim} of the model with ($m_{min} = 10$, $m_{max} = 90$, $\delta\tilde{B}^2 = 0.0054$) is similar to D^{QL}. Even in this small amplitude case, the energy diffusion at the rate $\sim D^{QL}$ can be caused by the non-resonant Alfvén waves.

The top panel of Fig.5 shows the distribution function $N(\gamma)$ at $\tilde{t} = 10^3$ of the model with ($m_{min} = m_{res}$, $m_{max} = 400$, $\delta\tilde{B}^2 = 0.04$). The solid line represents the solution of the quasi-linear diffusion equation with $D^{QL} \sim 7.1 \times 10^{-5}$. In this case the simulated distribution function fits the theoretical curve. The lower panel shows $N(\gamma)$ at $\tilde{t} = 10^3$ of the model with ($m_{min} = 10$, $m_{max} = 400$, $\delta\tilde{B}^2 = 0.54$). In this case the simulated distribution function evolves faster than the quasi-linear prediction.

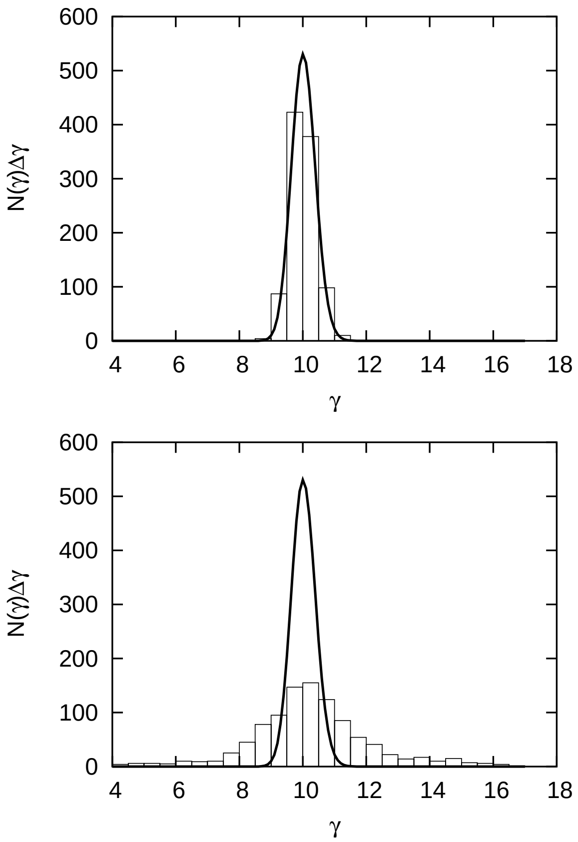

Fig. 5 Comparison of the simulated distribution functions at $\tilde{t} = 10^3$ with that in the quasi-linear theory.

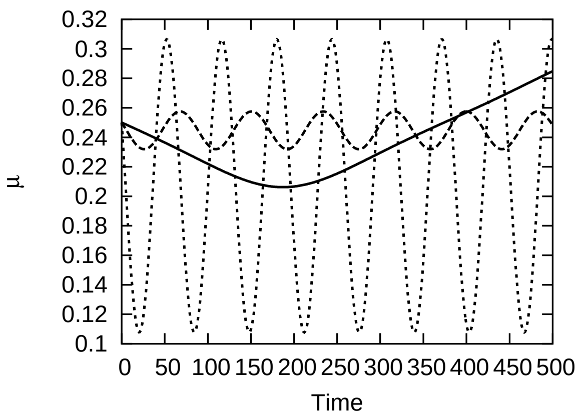

Fig. 6 The time profile of the pitch angle cosine.

Here we compare pitch angle changes by the resonant wave with that by the non-resonant wave, because the energy diffusion is caused by a number of pitch angle scatterings. We consider the case when the test particle is injected at the pitch angle cosine $\mu = 0.25$, and the Alfvén wave is the monochromatic wave having the wavenumber $k = mk_1$ and the amplitude b with $v_A = 0$. Fig.6 shows examples of the time profile of the pitch angle cosine μ. The solid line represents the model where the wave has the wavenumber $m = 400$, which is the resonant wavenumber for the particle of $\mu = 0.25$, and the wave amplitude $b = 0.003$. In this resonant wave model, the pitch angle changes slowly around $\mu = 0.25$. The dashed line and the dotted line represent the non-resonant models with $(m = 100, b = 0.01)$ and $(m = 10, b = 0.1)$, respectively. As the wave amplitude is larger, the pitch angle changes faster and larger. The fast and large pitch angle changes by the non-resonant waves enhance the pitch angle scatterings, and result in the enhancement of the energy diffusion.

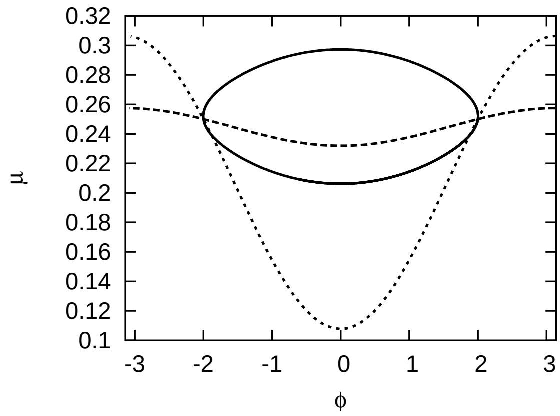

Fig. 7 The particle trajectories in $\mu - \phi$ phase space.

The non-linear enhancement can be caused by several effects. To see this, in Fig.7, we plot the particle trajectories (from the same simulation as Fig.6) in the $\mu - \phi$ space, where $\phi = \phi_w - \phi_p$ is the phase difference between the particle gyrophase ϕ_p and the wave phase ϕ_w (i.e., the angle between $\boldsymbol{v}_\perp$ and $\boldsymbol{B}_w = (B_y, B_z)$). In the resonant model represented by the solid line, the particle is trapped around the linear resonant pitch angle cosine $\mu = 0.25$ as discussed in [8]. As seen in this figure, the particle resonates with the finite amplitude wave having the resonance width in the μ space (e.g., [6]). This is the non-linear effect called the resonance broadening. We would like to study how only this effect modifies the energy diffusion as a future work. In the non-resonant models represented by the dashed line and the dotted line, the particles are not trapped, but their trajectories are modified. This orbit modification causes the fast pitch angle scattering and results in the energy diffusion enhancement. The enhanced diffusion observed in this study may be mainly caused by this effect. If the wave amplitude were large enough ($> B_0$), the non-resonant trapping discussed in [8] would modify the energy diffusion. However, this effect may not occur in the present case, because the wave amplitude is small.

4. Summary and Discussion

We have studied how the Alfvén waves having the wavenumber less than k_{res} enhance the diffusion in energy space by using the test particle simulations. We considered the case when the amplitude of the resonant waves ($k \geq k_{res}$) is sufficiently small. When only the resonant waves exist, the simulated diffusion coefficients are similar to that by the quasi-linear theory. If the wave power spectrum, which was assumed to be a single power law, extends to lower wavenumbers, it is found that the non-linear effects by the non-resonant Alfvén waves of $k < k_{res}$ enhance the energy diffusion and the simulated diffusion coefficients exceed the quasi-linear prediction. The non-linear enhancement becomes stronger with increasing the energy density of the non-resonant Alfvén waves.

Finally, we discuss particle acceleration in the clusters of galaxies with the synchrotron radio halo. The relativistic electrons with $\gamma \sim 10^4$, which emit at $\nu \sim 1$ GHz if we assume $B_0 = 1\mu$G, resonate with the Alfvén waves having the wavenumber $k \sim 6 \times 10^{-14}$ cm^{-1}. Fluid turbulence can radiate Alfvén waves through the Lighthill process [2, 5]. According to [2], the wavenumber range of the turbulent Alfvén waves is $10^{-15}\mathrm{cm}^{-1} \leq k < 10^{-10}\mathrm{cm}^{-1}$, and the energy density is $\sim 10^{-4}(B_0^2/8\pi)$. Because the wave amplitude is small, the non-linear enhancement of the energy diffusion is small. If large amplitude Alfvén waves were injected by energetic events in the clusters of galaxies (e.g., substructure merging), the diffusion in energy space would be enhanced.

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