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Nonlinear Dynamics of Magnetic Drift Modes and Self-organization Phenomena in Turbulent Unmagnetized Plasma

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Nonlinear dynamics of the magnetic electron drift mode turbulence is outlined and spontaneous generation of large-scale magnetic structures, so called meso-scale magnetic fields, in a non-uniform unmagnetized plasma is demonstrated. The stability of such large scale structures is investigated in kinetic and hydrodynamic regime for which an instability criterion similar to the Lighthill criterion for modulation instability is found. Furthermore these large scale flows can undergo further nonlinear evolution after initial linear growth, which can lead to the formation of long-lived coherent structures.

Keywords: Magnetic electron drift mode, turbulence, Reynolds forces, large-scale flows, stability, kinetic and hydrodynamic regime, coherent structures.

1. Introduction

The excitation of magnetic fields is a current field of strong investigation in different areas. Since the end of the 1970s, experiments have shown that strong quasi-steady magnetic fields are created in laser produced plasma. These magnetic fields oscillate with a typical frequency in between the ion and the electron plasma frequency, and are fed by density and temperature gradients through the first order baroclinic vector. Moreover, phenomena occurring in such time scales may even be more important as a source of the secondary magnetic field structures and are often encountered in space physics.

It is interesting to combine the phenomena of large-scale structures spontaneously generated in a way similar to the established Reynolds stress in hydrodynamics [1] and strong magnetic fields and thus develop a nonlinear theory capable of describing the generation of such large-scale magnetic fields by small-scale turbulence and their mutual interaction. Such a self-consistent spectral two-field (magnetic field, B, and temperature, T) model has recently been developed [2]. Note that this model does not deal with flows in the original sense, since it is not flow of the particles, but rather magnetic structures that are elongated along one direction and periodic with a long wavelength along the other direction as well. Following this similarity, we call the corresponding large-scale structures "zonal magnetic fields (ZF), or magnetic streamers as it has been adopted in the literature.

With this nonlinear spectral model, we focus in the present paper on the detailed generation mechanism of large scale magnetic fields. The nonlinear evolution of such magnetic structures is also investigated.

2. Basic equations

Starting from the momentum equation together with Maxwell's equations and the energy equation, the model equations for magnetic electron drift mode turbulence can be derived and read [3]

$$\partial_t\left(B-\lambda\Delta B\right)+\beta\partial_y T = e\lambda^4\left\{B,\Delta B\right\}/m \tag{1a}$$

$$\partial_t T+\alpha\partial_y B = -e\lambda^2\left\{B,T\right\}/m \tag{1b}$$

Here α and β are inverse length scales of the density and temperature inhomogeneities and $\lambda = c/\omega_{pe}$ is the electron skin depth, the curly brackets on the right-hand side denote the Poisson brackets and are defined as $\left\{c,d\right\}\equiv\left(\nabla c\times\nabla d\right)\cdot\mathbf{z}$. Linear analysis of Eqs.(1) shows that there is purely growing solutions for $\alpha\beta<0$, so that the underlying magnetic electron drift mode turbulence is driven by gradients of temperature and density. As already mentioned, this microturbulence can spontaneously generate large scale flows. During this flow generation, thermodynamic free energy stored in gradients is converted into kinetic energy of magnetic flows by fluctuation-induced Reynolds stress and thus these gradients constitute the energy source for the magnetic structures.

Our approach uses the ansatz of multiple-scale expansion between the spatio-temporal scales of the flows and those of the microturbulence. The temperature and the magnetic field are then decomposed into a large-scale, slowly varying component (denoted with a bar) and a small-scale component, $\overline{T}+\tilde{T}$ and $\overline{B}+\tilde{B}$ respectively. Eqs.(1) has a conserved quantity corresponding to the energy integral. It is important to note that the conserved total energy contains both small-and large-scale

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components. This means that the whole wave spectrum and the interaction between different regions of the spectrum have to be included in subsequent considerations.

To describe the interaction between small- and large-scale structures within the turbulence we will separate the whole turbulent spectrum into two parts, one describing large-scale structures with a wave vector denoted by $\mathbf{q}$ and the other describing small-scale turbulence with a wave vector denoted by $\mathbf{k}$. We therefore have the relation $|\mathbf{q}| << |\mathbf{k}|$. Note that the both $\mathbf{q}$ and $\mathbf{k}$ describe the same spectrum but different parts of it. Using these decompositions yields the qth Fourier components of (1)

$$\partial_t B_q + i\beta q_y T_q / \left(1+q^2\lambda^2\right) = -e\lambda^4 / m\left(1+q^2\lambda^2\right)\times \sum_k (\mathbf{k}\times\mathbf{q})\cdot\mathbf{z}(\mathbf{k}\cdot\mathbf{q}) B_k B_{q-k}$$

$$\partial_t T_q + i\alpha q_y B_q = -e\lambda^2 / 2m \times \sum_k (\mathbf{k}\times\mathbf{q})\cdot\mathbf{z}\left(T_k B_{q-k} - T_{q-k} B_k\right) \qquad (2)$$

It is seen that a small scale turbulence can indeed drive large-scale structures characterized by B_q and T_q via the magnetic Reynolds stress $\sum k_x k_y B_k B_{-k}$. In order to describe the nonlinear evolution of the total wave spectrum in a self-consistent way, we have to determine the "loop-back", i.e. the response of small scales to large-scale structures changes. In order to do so, it is appropriate to consider the evolution of this micro-turbulence in a medium which is slowly modulated by large scale structures. This can be conveniently done using a wave kinetic equation for the wave action density $N_k(\mathbf{r},t)$ in $\mathbf{r}-\mathbf{k}$ space. An appropriate action-like invariant takes the form $N_k = 4\alpha\left(1+k^2\lambda^2\right)|B_k|^2/\beta$. Then the corresponding wave kinetic equation can be written as

$$\partial_t N + \left(\partial\omega_k^{NL}/\partial\mathbf{k}\right)\partial_{\mathbf{r}} N - \left(\partial\omega_k^{NL}/\partial\mathbf{r}\right)\partial_{\mathbf{k}} N = 2\gamma_k N - St(N), \text{ where } N \equiv N_k(\mathbf{r},t) \qquad (3)$$

The terms on the RHS account for wave damping due to linear and nonlinear mechanisms, as well as local wave interactions. The linear frequency ω_k is modified in the presence of large scale fields. The reason is the Doppler shift induced by the large scale "flow velocity" $\mathbf{v}_f^{(q)}$, where superscript (q) means that only the large scale spectral component of magnetic field and temperature have to be considered. Therefore the nonlinear frequency $\omega_k^{NL} = \omega_k + \Delta$ has been introduced. Explicitly $\Delta = \mathbf{k}\cdot\mathbf{v}_f^{(q)}$ [2], where $\mathbf{v}_f^{(q)}$ is determined by the large scale spectral component of the magnetic field and temperature. We decompose the wave spectrum N_k into equilibrium and perturbed part, $N_k = N_0 + \tilde{N}_k$. Then, equations for N_0 and $\tilde{N}_k$ can be obtained in the spirit of quasi-linear theory. Namely, assuming that N_0 evolves in time only and with a much larger time scale than the perturbed part, we average Eq.(3) over the fast small spatial scales. This yields the equation for N_0. The equation for a perturbed part can then be found by subtracting equation for N_0 from Eq.(3) and assuming that the damping of the wave spectrum enters via the linear damping rate, γ^N. The result can be written as

$$\frac{\partial\tilde{N}_k}{\partial t} + \mathbf{v}_g\frac{\partial\tilde{N}_k}{\partial\mathbf{r}} - \frac{\partial}{\partial\mathbf{r}}\left(\mathbf{k}\cdot\mathbf{v}_f^{(q)}\right)\frac{\partial N_0}{\partial\mathbf{k}} = -\gamma^N\tilde{N}_k$$

where v_g is the group velocity of the magnetic electron drift mode. Introducing $N_k \sim \exp(-i\Omega t + i\mathbf{pr})$, where Ω and $\mathbf{p}$ is the temporal and spatial scales of perturbations, and solving resulting algebraic Eq. yields

$$\tilde{N}_k = \frac{\partial}{\partial\mathbf{r}}\left(\mathbf{k}\cdot\mathbf{v}_f^{(q)}\right)\frac{\partial N_0}{\partial\mathbf{k}} R(\Omega,\mathbf{p}) \qquad (3a)$$

Here the response function $R(\Omega,p) = i/\left(\Omega - pv_g + i\gamma^N\right)$ where γ^N is small and positive. Equations (2) and (3) constitute the basic system used to describe the nonlinear evolution of the total wave spectrum of the magnetic electron drift mode turbulence in a self-consistent way [2].

3. Excitation of large scale fields

It was stressed out above that large scale magnetic fields can be generated by small scale turbulence via magnetic Reynolds stress. It is indeed only a necessary condition for the generation and does not permit us to determine neither a sufficient condition for the excitation nor the increment. The aim of this section is to point out these conditions by investigating different regimes of large scale field generation [4, 5].

In our first approach we are looking for a general criterion for the generation of large scale fields depending on the form of the wave spectrum. We will focus our attention on the zonal magnetic fields, i.e. structures elongated perpendicular to the direction of plasma inhomogeneity. Introducing the large scale "vector" $\overline{B} \equiv \left(e\lambda^2/4m\right)\left(B_q - \sqrt{\beta\alpha^{-1}}T_q\right)\exp(i\mathbf{qr})$ and taking the difference of equations (2) yields the equation of motion for $\overline{B}$

$$\partial_t \overline{B} = e^2 \lambda^6 q^2 \int |B_k|^2 k_x k_y d^2\mathbf{k} / 4m^2 \left(1 + q^2\lambda^2\right) \tag{4}$$

To solve this equation, we first replace the magnetic field Fourier component in the integral with the wave spectrum using the definition of the wave action density N_k. Then we decompose N_k into an equilibrium and perturbed part and use (3a). We find that the problem becomes similar to that describing the instability of gas of plasmons due to coupling with ion-acoustic waves [6]. So, the evolution of the perturbed wave spectrum $\tilde{N}_k$ can be seen as a modulation of the amplitude by large scale fields. Thus ***p*** is of order of ***q*** used previously to describe the large scale fields, and the real part of Ω can be treated as the frequency of zonal field, Ω^{zf}. Assuming now $\partial/\partial t \sim -i\Omega^{zf} + \gamma^{zf}$, where γ^{zf} is the increment of zonal field generation, yields that the equation of motion for $\overline{B}$ (4) finally becomes

$$\gamma^{zf} - i\Omega^{zf} = -K_q^2 \int q_x^2 k_x \left(\partial N_0 / \partial k_x\right) M_k R(\Omega, p)\, d^2\mathbf{k}$$

here $M_k = k_y^2\lambda^2 / \left(1 + k^2\lambda^2\right)$, coefficient K_q is real. The strongest interaction between the small scale turbulence (medium) presented by its wave spectrum N_0, and the zonal fields can expected when the reaction of the medium (Ω in the response function) is in resonance with the perturbation (represented by v_g in the response function), that is, when $\Omega \simeq pv_g$ and thus $R(\Omega, p) \sim 1/\gamma^N > 0$. Because of strong interaction, we can assume $\Omega^{zf} \simeq 0$. The corresponding regime of zonal field generation is called "kinetic" because of its obvious similarities with Landau damping in the kinetic wave theory. The result of these considerations is the criterion for instability

$$\gamma^{zf} > 0 \Leftrightarrow k_x \left(\partial N_0 / \partial k_x\right) < 0 \tag{5}$$

This result is the opposite of the condition for the Langmuir turbulence [6], where the slope of the velocity distribution function must be positive for positive velocities.

If the ZF amplitude grows due to the above instability, the non-resonant response becomes important as well. We investigate the nonresonant part Ω^{zf} assuming $\Omega << \mathbf{p}\mathbf{v}_g$ and $\gamma^{zf} \simeq 0$. In this limit the response function is $R(\Omega, p) \sim -i/pv_g \sim -i/qv_g$, since $p \sim q$, and thus

$$\Omega^{zf} = -K_q^2 \int q_x^2 k_x \left(\partial N_0 / \partial k_x\right) M_k \left(qv_g\right)^{-1} d^2\mathbf{k}$$

So, as for the resonant part,

$$\Omega^{zf} > 0 \Leftrightarrow k_x \left(\partial N_0 / \partial k_x\right) < 0 \tag{6}$$

We derive rather general criterion for excitation of large scale magnetic fields by small scale turbulence, which depends on the equilibrium spectrum distribution. In order to have an exact result in the case of ZF, one has to integrate Eq.(4), or, which comes to the same after replacing the magnetic field Fourier amplitudes as before, integrate assuming $\partial/\partial t \sim -i\Omega$. It is worthwhile to note that in what follows, $\Omega \approx \Omega^{zf}$ and $q \equiv q^{zf} \approx p$ This yields

$$-i\Omega = -K_q^2 \int q_x^2 k_x \left(\partial N_0 / \partial k_x\right) M_k R(\Omega, p)\, d^2\mathbf{k}$$

For explicit integration, one has to consider a specific form of the equilibrium wave spectrum N_0. Assuming a monochromatic wave packet, $N_0^k = N_0 \delta\left(\mathbf{k} - \mathbf{k}_0\right)$, and performing the integration by parts yield the dispersion relation for ZF

$$\left(\Omega - qv_g\right)^2 = K_q^2 q^2 k_{0y}^2 N_0 \left(\partial v_g / \partial k_x\right) / \left(k_{0y}\sqrt{\alpha\beta}\right)$$

We see directly from the last equation that requirement for the instability is

$$N_0 \left(\partial v_g / \partial k_x\right) / \left(k_{0y}\sqrt{\alpha\beta}\right) < 0 \tag{7}$$

Note that the instability condition (7) is similar to the well known Lighthill criterion for the modulational instability. Explicit calculation of the derivative of the group velocity yields exact expression for the large scale zonal structure's complex frequency`

$$\Omega = qv_g - iK_q |q| |k_{0y}\lambda| N_0^{1/2} \times \sqrt{1 - 2k_{0x}^2\lambda^2 + k_{0y}^2\lambda^2} \Big/ \left(1 + k^2\lambda^2\right)^{5/2}$$

with the imaginary part being the rate of generation of large scale fields. Note that stabilization takes place for $1 - 2k_{0x}^2\lambda^2 + k_{0y}^2\lambda^2 < 0$.

4. Long term dynamics of large scale fields

We have shown in the last section that large scale magnetic fields can be generated and strengthened through instabilities and therefore also subsist for a longer time scale. That is why we turn our attention to the long term dynamics. We will concentrate on ZF.

The long term dynamics can be determined by looking for the time evolution equation of the large scale "flow velocity" $\mathbf{v}_f \equiv \mathbf{v}_f^{(q)}$. To derive this equation, we will decompose the wave spectrum into equilibrium, resonant and nonresonant first order and nonresonant second order perturbed part, $N_k = N_0 + \tilde{N}_k^r + \tilde{N}_k^{(1)} + \tilde{N}_k^{(2)}$. Now performing calculations of resonant and non-resonant parts and summing up corresponding results yields the evolution equation for "flow velocity" $\mathbf{v}_f \equiv \mathbf{v}_f^{(q)}$

$$\partial_t \partial_x v_f = D\partial_x^3 v_f + u\partial_x^2 v_f + b\partial_x^2 v_f^2 \quad (8)$$

Here $D, u,$ and b are some integral coefficients which depend on the equilibrium spectrum distribution. We now look for the stationary solutions of Eq.(8), propagating with constant velocity u_{0x} in the x direction, $v_f(x - u_{0x}t)$. Hence, after integrating Eq.(8) twice, one obtains

$$(u + u_{0x}) v_f + bv_f^2 + D(\partial v_f / \partial x) = C \quad (9)$$

where C is the integration constant. We impose the boundary conditions corresponding to a solitary wave with different asymptotic values, i.e. a "switching" or "kink" soliton, which are $v_y \to v_1, v'_y \to 0$ as $x \to -\infty$ and $v_y \to v_2, v'_y \to 0$ as $x \to \infty$. The constant C then takes the form $C = (u + u_{0x}) v_1 + bv_1^2$ and we can express v_2 through v_1, $v_2 = -v_1 - (u + u_{ox})/b$. Then the simplest solution for Eq.(9) with these assumptions is given by [7]

$$2v_f = \{v_1 + v_2 + (v_1 - v_2)\tanh[b(v_1 - v_2)x/2D]\}$$

This solution describes the transient region between two different values of the flow v_1 to v_2. We note that it is different from the stationary vortex solution found earlier [8]. The cooperative effects of the wave motion, steepening and instability give the possibility of forming stationary or moving kink solitons in between the surfaces of two different flow velocities. We would expect the effect of modifying the anomalous electron transport properties within as studied earlier. In a polar geometry, zonal fields are elongated along the θ direction. They are known to inhibit anomalous transport in the radial direction by shearing small scale turbulence [1]. However, the coherent structures found above travelling along the radial direction with the velocity u_{0x}, take trapped particles with them and hence can even attribute to radial electron transport.

The case of magnetic streamers is similar to the above case with one difference, which is the additional term in the streamer flow velocity evolution equation.

5. Conclusions

The properties of large scale fields, both zonal magnetic fields and magnetic streamers, have been investigated using a self-consistent model for magnetic electron drift wave turbulence. The small scale turbulence evolves in a medium of slowly changing variables, large scale fields, and is modulated by them. The kinetic and hydrodynamic instability of these modulations have been studied and in the hydrodynamic case, a criterion similar to the Lighthill criterion for the case of modulation instability has been found for zonal fields. Once it was shown that large scale fields can be unstable and be strengthened, their nonlinear long term evolution could be studied. We were able to stress out that both zonal magnetic fields and magnetic streamers admit the formation of stationary coherent structures in the transition layer between surfaces of different flow velocities, modifying the turbulent electron transport properties.

6. References

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