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Toroidal Mode Activity of Neutral Beam Injected Field-Reversed Configuration

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Abstract

The ion particle and electron fluid hybrid simulation is carried out for tangential neutral beam (NB) injected Field-Reversed Configuration (FRC). Ionization of neutral beam particles is simulated by the Monte-Carlo method. By assuming a highly elongated FRC, two dimensional structure and behavior are investigated. The time evolution of toroidal mode for the ion density and pressure is calculated. In spite of the presence of beam ions, the structure with the toroidal mode $n = 1$ and 3 is found to be dominant, contrary to the higher modes; a mechanism responsible for this, however, has not yet been identified. The toroidal electric field induced by the friction force between plasma electrons and the NB ions is little observed in the present calculation.

Keywords:

field-reversed configuration, neutral beam injection, charge exchange, hybrid simulation, hall effect

1. Introduction

The neutral beam injection (NBI) into the Field-Reversed Configuration (FRC) was firstly carried out at the FRC injection experiment (FIX) device [1]. It has been found that the NBI is effective to improve the confinement [1,2] and to heat the plasma electrons [3]. Since the FRC decays rapidly, it is indispensable to drive the diamagnetic azimuthal plasma current and to heat the core plasma; detailed investigation on NBI is, therefore, required. To diagnose the efficiency of NBI, a simulation study is useful. Moreover, by the simulation one can make a proposal of the most effective condition to an experimental side. Recent research interest is on the possibility of current drive and core plasma heating, because the poloidal flux maintenance is an important issue in the FRC [4]. Therefore, we study time evolution for the global behavior of the NB injected FRC by a self-consistent numerical simulation. A hybrid fluid electron and both plasma and beam ion particle simulation is employed to calculate a valid contribution of the ion motion to the electromagnetic fields; the ion near the field-null circle (see Fig. 1) has a comparable gyro-radius to the characteristic length of the magnetic field variation. To discuss the heating of FRC plasma, the electron temperature is evolved to hold the heat balance equation and the electron pressure gradient force is considered in the equation of motion for massless electrons. Being assumed a highly elongated FRC and supposed that the NBI is done tangentially to the plasma, as shown in Fig. 1, the temporal evolution of plasma is analyzed in two-dimensional cross-sectional plane. Hydrogen atom with a smaller Larmor radius than the other atom is injected as a neutral beam, and it is ionized by the charge exchange with a deuterium ion, and the direct ionization by both electron and ion impact; these interactions are reproduced by the Monte-Carlo method. The experimental data of cross-sections are acquired from the NIFS data base [5].

2. Simulation model

Since the FRC plasma has a high beta value, its non-uniformity of magnetic field is intrinsic; it results in the fact that ion's Larmor radius is comparable with the field-null circle radius. Thus the magnetohydrodynamic

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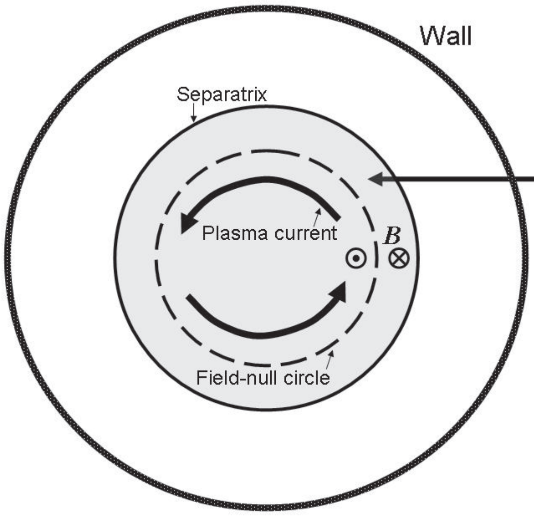

Fig. 1 Two dimensional geometry of tangential neutral beam injection. The beam trajectory is shifted from the center, and it is almost tangential to the field-null circle.

(MHD) picture is not applicable to ions, although the MHD picture for electrons still holds. Therefore, it is necessary to describe the FRC plasma by the two-fluid model.

2.1 Ion particle and electron fluid hybrid simulation

The equation of motion for plasma ions as well as the beam ions is written in the form:

$$m_\alpha \frac{\mathrm{d}\boldsymbol{v}_\alpha}{\mathrm{d}t} = q_\alpha(\boldsymbol{E} + \boldsymbol{v}_\alpha \times \boldsymbol{B}) - m_\alpha \nu_{\alpha e}(\boldsymbol{v}_\alpha - \boldsymbol{u}_e), \quad \alpha = i, b. \tag{1}$$

where m_α, q_α, and $\boldsymbol{u}_e$ are the mass and charge of the α-species, and the electron flow velocity. The friction force due to the slowing down collision is taken into account in Eq. (1). The slowing down collision frequency $\nu_{\alpha\beta}$ [6] is

$$\nu_{\alpha\beta} = \sum_\beta \frac{n_\beta q_\alpha^2 q_\beta^2 \Lambda}{4\pi\epsilon_0^2 m_\alpha^2 v_\alpha^3}\left(1 + \frac{m_\alpha}{m_\beta}\right)\eta(x) \tag{2}$$

$$\eta(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t}\sqrt{t}\mathrm{d}t, \quad x \equiv \frac{m_\beta v_\alpha^2}{2T_\beta} \tag{3}$$

where Λ, v, n and T are the Coulomb logarithm, the speed, the number density and the temperature in Joule, respectively. The subscript β stands for background particles, say electrons. The typical value of Coulomb logarithm is about 13 for the NBI experiment in the FIX device [1].

In order to calculate the ion density and particle flux, the Particle-in-Cell (PIC) method [7] is used. A use of the quasi-neutrality condition gives the electron density,

$$n_e = Z_i n_i + Z_b n_b. \tag{4}$$

Here, Z_i and Z_b are the charge numbers of plasma ions and beam ion, respectively. The fluid equation of motion for massless electrons is

$$-en_e(\boldsymbol{E} + \boldsymbol{u}_e \times \boldsymbol{B}) - \nabla p_e + \boldsymbol{R}_{eb} + \boldsymbol{R}_{ei} = \boldsymbol{0}, \tag{5}$$

where the electron pressure gradient ∇p_e and friction force $\boldsymbol{R}$ by plasma ions and beam ions are considered. The friction force $\boldsymbol{R}_{e\alpha}$ at a calculation grid is calculated by summing the friction term in the ion equation of motion over the four cells neighboring the grid and by using the momentum conservation:

$$\boldsymbol{R}_{e\alpha} = -\boldsymbol{R}_{\alpha e} = \sum_n {}_{\mathrm{cell}} w_{i,j}^n m_\alpha^n \nu_{\alpha e}^n (\boldsymbol{v}_\alpha^n - \boldsymbol{u}_e). \tag{6}$$

The summation is done for the ions inside the four cells surrounding the grid (i, j), and $w_{i,j}^n$ is the weight of the n-th ion which affects the grid (i, j). The weight $w_{i,j}^n$ equals the product of the density uniformly filling a cell and the bi-linear area weight [7]. The ion density at the grid (i, j) is

$$n_{i,j} = \sum_n {}_{\mathrm{cell}} w_{i,j}^n. \tag{7}$$

The electron pressure in Eq. (5) is temporally evolved by the heat transfer between electrons and the beam and plasma ions; it is calculated by the heat balance equation

$$\frac{3}{2}n_e\left[\frac{\partial T_e}{\partial t} + (\boldsymbol{u}_e \cdot \nabla)T_e\right] + n_e T_e(\nabla\cdot\boldsymbol{u}_e) = Q_{eb} + Q_{ei}. \tag{8}$$

The electron flow velocity in Eq. (8) is obtained from the definition of current density that is calculated from the magnetic field by using the Ampère's law. With the aid of the energy conservation in collision between particles of α-species and electrons (subscript e)

$$Q_{\alpha e} + Q_{e\alpha} + (\boldsymbol{u}_\alpha - \boldsymbol{u}_e)\cdot\boldsymbol{R}_{\alpha e} = 0, \tag{9}$$

the heat generation term for electrons $Q_{e\alpha}$ in Eq. (8) is therefore obtained when the heat generation for the beam and plasma ions can be calculated by the ensemble average of particles. The rate of change in the kinetic energy of particle α due to the slowing-down collision is given as

$$\frac{\mathrm{d}E_\alpha}{\mathrm{d}t} = -m_\alpha \nu_{\alpha e}\boldsymbol{v}_\alpha \cdot (\boldsymbol{v}_\alpha - \boldsymbol{u}_e). \tag{10}$$

The heat generation term for the α-species particle $Q_{\alpha e}$ is therefore

$$Q_{\alpha e} = \sum_n {}_{\mathrm{cell}} w_{i,j}^n \frac{\mathrm{d}E_\alpha^n}{\mathrm{d}t}. \tag{11}$$

Equations (5) and (8) and also the Faraday's law are transformed into the finite difference equations. Numerical integration in time for the heat balance equation and the Faraday's law is carried out using the predictor-corrector algorithm. On the other hand, we employ the adaptive step size control method to the equation of motion for the plasma and beam ions because of the high collision frequency by low temperature electrons in the open-field region.

When both the ions and the beam ions reach the wall surface, they are assumed to be absorbed in the wall material. No electric field at the simulation boundary is set to reduce the numerical noise. With the use of a digital filtering [7], the electric field profile can be smoothed. A free boundary condition is chosen for the other physical quantities.

2.2 Neutral beam injection

NB particles are ionized by the following three processes:

1) the charge exchange,
2) the direct ionization due to an impact of a plasma particle (i.e., an ion or an electron),
3) the indirect ionization through the excitation processes.

The dominant ionization process is the charge exchange. In order to consider the excitation process, a two-step calculation is necessary, which complicates the calculation. Moreover, since the cross-section of excitation process is much smaller than others, therefore we neglect this; the charge exchange and direct ionization process are taken into account in the present calculation. The interactions between plasma particles and beam particles are calculated by a use of the Monte Carlo method.

3. Results and discussion

We simulate the case that the NB particles are injected into the FRC plasma confined by the external field B_{ex} of 0.05 T and in the cylindrical tube with a radius r_w of 0.4 m; the parameters are the same as the FIX device. The plasma ion and electron temperatures (T_i and T_e) at the field-null are 100 and 50 eV, respectively. The equilibrium of FRC is obtained by solving the Grad-Shafranov equation. Assuming the uniform density $n_0 = 5.0 \times 10^{19}$ m^{-3}, we obtain the temperature profile from the equilibrium pressure profile. The NBI parameters are as follows; the beam current is 20 A and the beam energy is 800 eV, which is much lower than usual NB energy (≈15 keV) and is unrealistic. For the FIX case, however, the external field is too weak to confine the tangential beam injected fast ions; almost all of the beam ions suffer from the orbit loss on the wall surface. A stronger magnetic field is therefore required to confine the beam ion of 15 keV. An optimum external field will be calculated in a near future. In the present study, we suppose that the low energy beam source is available. As shown in Fig. 1, the NB particles are injected almost tangentially to the field-null circle. The impact parameter (i.e., the shift length of the beam path from the geometric center) is 0.18 m in our case. If the NB particle is ionized without the charge exchange process, a free electron is generated. There are few generated electrons, and they hardly affect the plasma behavior. The electrons are neglected in the present study.

After the ionization process, the beam ion encircles the geometric axis as its energy loses. Figure 2 shows the flow velocity profile of beam ions, where the radial betatron oscillation can be seen. Due to this oscillation, the beam ions sometimes move outside the separatrix. Plasma electrons existing on the beam ion's path are dragged due to slowing-down collisions, which cause the friction force $\boldsymbol{R}$ in Eq. (5). The resultant electric force balancing the friction force $\boldsymbol{R}$ is in the toroidal direction. The calculated electric field profile along with the electron density profile is presented in Fig. 3 to confirm electric field generation by the beam ions. The radial electric field is generated near the separatrix due to the Hall effect. From Eq. (5), the electric field induced by the Hall effect is

$$\boldsymbol{E} = -\boldsymbol{u}_i \times \boldsymbol{B} + \frac{\boldsymbol{j} \times \boldsymbol{B}}{en_e} + \frac{1}{en_e}(-\nabla p_e + \boldsymbol{R}_{eb} + \boldsymbol{R}_{ei}) \approx \frac{\boldsymbol{j} \times \boldsymbol{B}}{en_e} - \frac{\nabla p_e}{en_e} \approx \frac{\nabla p_i}{en_e}, \quad (12)$$

when $\boldsymbol{u}_i \approx \boldsymbol{R}_{eb} \approx \boldsymbol{R}_{ei} \approx 0$. Therefore, the radial elec-

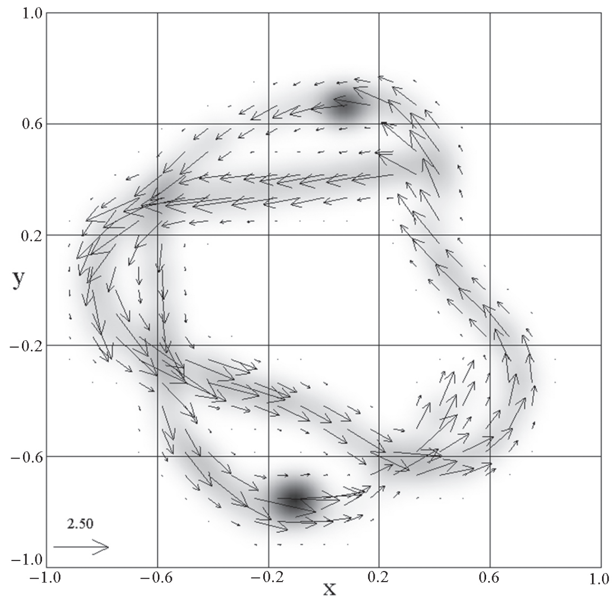

Fig. 2 Flow velocity profile of neutral beam injected fast hydrogen ions. Here, x and y are normalized by the wall radius. The beam flow velocity is normalized by the plasma ion thermal velocity $\sqrt{2T_i/m_i}$. The reference arrow is drawn in the left bottom corner, and its length corresponds to 2.5 $\sqrt{2T_i/m_i}$.

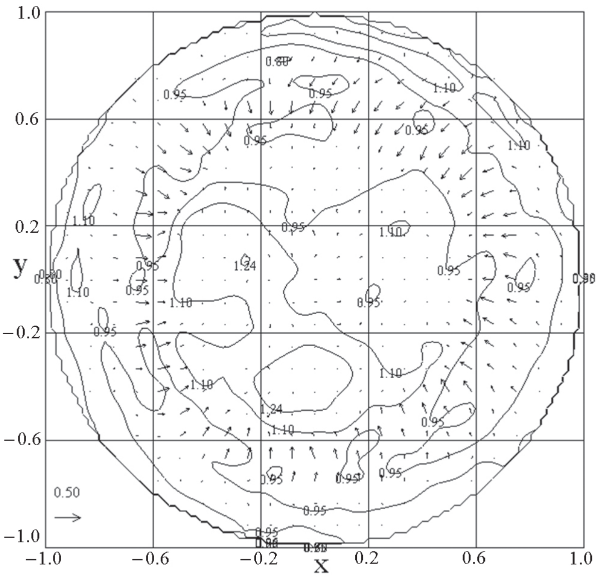

Fig. 3 Contour of electron density and electric field profile at $t = 10/\omega_{ci0}$ where $\omega_{ci0} \equiv qB_{ex}/m_i$. Electron density is normalized by 5.0×10^{19} m^{-3}. The values on the contour line represent value of the normalized density. The reference arrow is drawn in the left bottom corner, and its length corresponds to 0.5 $B_{ex}\sqrt{2T_i/m_i}$.

tric field originates in the ion pressure gradient. Contrary to the radial component, no significant toroidal electric field generated due to the electron drag by the beam ion is found along the beam ion's path. Therefore, the global motion induced by the electric field due to the friction force is little observed in our present case. Figure 4 shows the contour of electron pressure and profile of electron flow velocity. Since there is no ion flow at the initial state, the electron flow velocity equals $-\boldsymbol{j}/en_e$. In Fig. 4 (a), only the toroidal velocity component can be seen. When the ions start to move along the preferential direction, the electron flow velocity becomes $\boldsymbol{u}_i - \boldsymbol{j}/en_e$. Because of the ion shift motion, the radial component of the electron flow velocity is found in Fig. 4 (b). The ion shift motion corresponds to the toroidal mode number $n = 1$.

To study a complicated toroidal structure, the integrated mode amplitude

$$\left\| \frac{1}{\pi} \int_0^1 \int_0^{2\pi} f(\bar{r}, \theta) e^{-in\theta} \mathrm{d}\theta \bar{r} \mathrm{d}\bar{r} \right\| \qquad (13)$$

is calculated. The time evolutions of the amplitude of several toroidal modes from $n = 1$ to $n = 8$ for the ion density is presented in Fig. 5 (a) for without the NBI case and (b) for with the NBI case. The $n = 1$ and 3 mode activities are found to be dominant in both cases. It is found that the higher modes, however, never grow in our present calculation. Exceeding the $n = 1$ mode, the $n = 3$ toroidal mode for the beam injected plasma

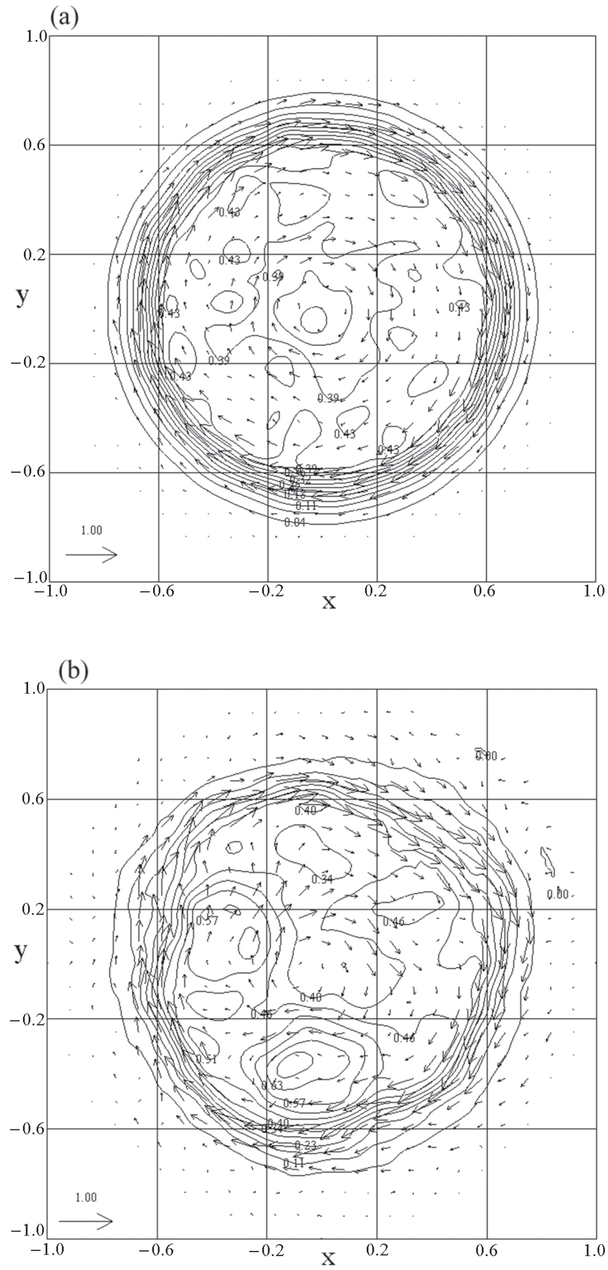

Fig. 4 Contours of electron pressure and electron flow velocity profile at (a) $t = 1/\omega_{ci0}$ and (b) $t = 10/\omega_{ci0}$ where $\omega_{ci0} \equiv qB_{ex}/m_i$. The reference arrow is drawn in the left bottom corner, and its length corresponds to the ion thermal speed $\sqrt{2T_i/m_i}$.

is found to grow rapidly. And then it saturates until $t = 10/\omega_{ci0}$. On the other hand, the $n = 1$ mode is most active without the beam ions. Although the toroidal electric field resulting from the beam ions is negligibly small, the mode activity of the beam injected plasma is different from the plasma without the NBI. The electron heat generation by the beam ions is probably responsible for the difference. The spatial profile of electron heat generation rate by the beam ions is presented in Fig. 6. The electrons are heated and then the electron pressure is elevated near the field-null where the beam ions move. Then a high radial electron pressure gradi-

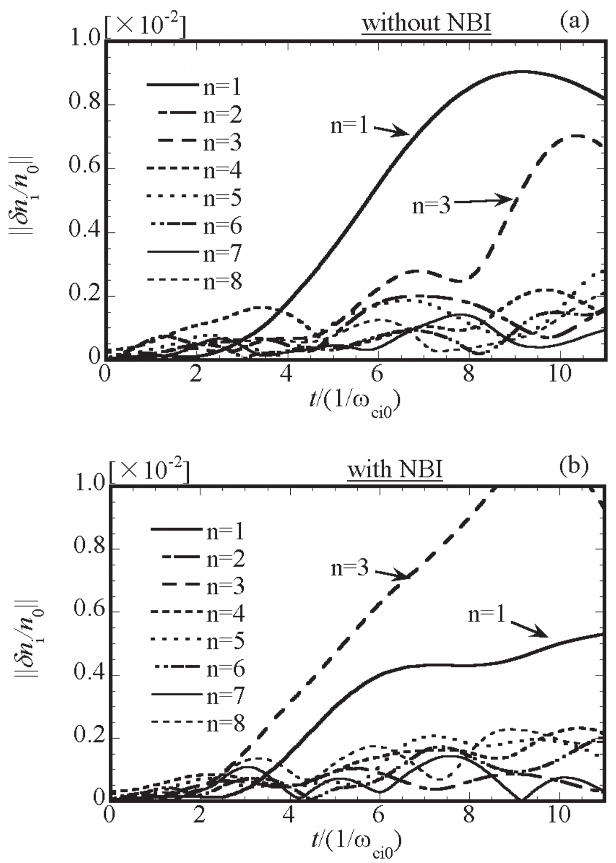

Fig. 5 Time evolution of the amplitude for several toroidal modes (from $n = 1$ to $n = 8$) of the ion density (a) for without NBI and (b) with NBI.

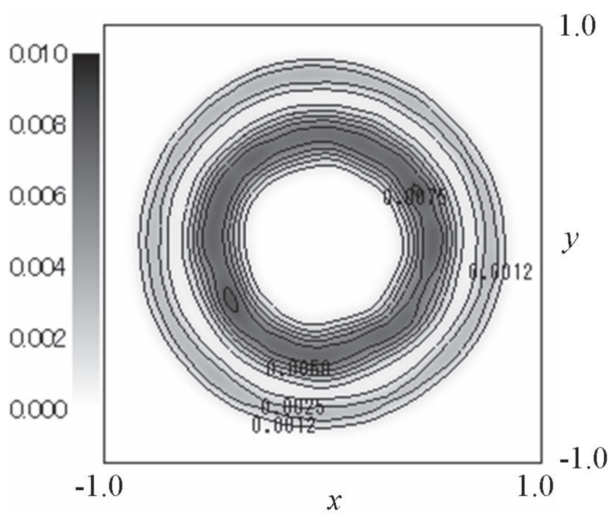

Fig. 6 The spatial profile of electron heat generation rate by the beam ions (i.e., Q_{eb} in Eq. (8)) at $t = 5/\omega_{ci0}$. Here, Q_{eb} is normalized by $n_0 T_{i0} eB_{ex}/m_i$, where $n_0 = 5.0 \times 10^{19}$ m^{-3} and $T_{i0} = 100$ eV.

ent is formed, and it can affect the radial electric field.

4. Summary

The ion particle and electron fluid hybrid simulation model is developed to study the global motion of the neutral beam injected Field-Reversed Configuration. Ionization of neutral beam is simulated by the Monte-Carlo method. Due to the Hall effect, the radial electric field is formed near the separatrix. According to the toroidal mode analysis, the ion density and pressure with the $n = 1$ and 3 toroidal mode numbers are dominant, contrary to the higher modes. In the present paper, a promising mechanism responsible for this, however, has not yet been identified; a further investigation is needed in a future study. No significant toroidal electric field due to the friction force between the beam ions and the electrons is found. In order to increase the efficiency of NBI (the beam energy of above 15 keV), a parametric study on the optimum size of wall radius and external magnetic field for confining the target FRC plasma is needed in a near future.

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