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Dust-Lower-Hybrid Drift Instabilities with Dust Charge Fluctuations in Inhomogeneous Dusty Magnetoplasmas

SALIMULLAH Mohammad, RIZWAN Abu M., NAMBU Mitsuhiro[1] and SHUKLA Padma K.[2]
Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh
[1]Department of Physics, Tokyo Metropolitan Institute of Technology, Tokyo 191-0065, Japan
[2]Institut für Theoretische Physik IV, Ruhr-Universität Bochum, D-44780 Bochum, Germany
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Abstract

Effects of uniform magnetic field, the plasma inhomogeneity, and the dust charge fluctuations on the drift waves have been investigated in a nonuniform dusty magnetoplasma. Charging currents of electrons and ions to a spherical dust grain in a nonuniform magnetized dusty plasma have been calculated to study the charge fluctuation damping or growth of the low-frequency drift waves. It is found that for strongly magnetized electrons and ions, the charge fluctuation damping is reduced significantly from that of an unmagnetized plasma [cf. eq. (21)]. For sufficiently hot electrons, the drift wave exihibits an instability in absence of the charge fluctuation damping [cf. eq. (25)].

Keywords:

dust-lower-hybrid drift wave, dust charge fluctuation, nonuniform magnetoplasma

1. Introduction

Dusty plasmas are encountered in space and laboratory conditions, where plasmas contain electrons, ions, and relatively massive and highly charged dust grains [1,2]. Waves and instabilities occupy the major part of basic research with emphasis on low-frequency electrostatic waves with and or without the presence of static magnetic field in dusty plasmas in recent years [3]. It has been shown that in the presence of low-frequency electrostatic waves, the dust charge fluctuations lead to an additional damping mechanism in addition to Landau damping of these modes in uniform and unmagnetized dusty plasmas [4-6].

However, most of the laboratory and astrophysical dusty plasmas are almost invariably nonuniform in density and are confined by homogeneous magnetic field either in ambient conditions, or is applied for convenience of control and confinement. Therefore, it is of much practical interest to make a rigorous investigation of the low-frequency waves and their instabilities in the presence of uniform magnetic field in an inhomogeneous dusty plasma. Recently, the effect of external uniform magnetic field on the equilibrium charging currents to spherical dust grains and the dust charge fluctuation damping of the low-frequency dust-lower-hybrid waves has been examined in a homogeneous dusty plasmas [7].

In this paper, we investigate the effects of plasma inhomogeneity and uniform magnetic field on the low-frequency regime, viz., the dust-lower-hybrid waves and their instabilities (damping or growth) in a nonuniform dusty magnetoplasma.

In Sec. **2**, we calculate the perturbed distribution function for the plasma particles in the presence of a low-frequency electrostatic wave in an inhomogeneous magnetized dusty plasma using the method of guiding center coordinates [8,9]. The expressions for the dust charge fluctuations due to the perturbed currents in the presence of the electrostatic waves in the inhomogeneous dusty magnetoplasma have been derived in Sec. **3**. Using Poisson's equation including dust charge fluctuation, we obtain the dispersion relation and the damping or growth rate of the electrostatic drift-kinetic waves at two important parameter regimes of the dusty nonuniform magnetoplasmas in Sec. **4**. The effects of magnetic field and the scalelength of inhomogeneity of the plasma are discussed in the same section. Finally, a brief discussion of the results is presented in Sec. **5**.

2. Linear perturbed distribution function

Following ref. [8,9] and employing the guiding center coordinates, we obtain the linearized perturbed distribution function for the *jth* species in terms of the Maxwellian distribution function as

$$f_j(\omega, \boldsymbol{k}) = -\frac{n_{0j}^0 q_j \Phi(\omega, \boldsymbol{k})}{T_j}\left(1 - \frac{\omega - \omega_j^*}{\omega - k_z v_z - n\omega_{cj}}\right) \times \sum_l \sum_n J_l J_n \cdot e^{\mathrm{i}(n-l)(\theta-\delta)} f_{Mj}\,, \quad (1)$$

Corresponding author's e-mail: msu@juniv.edu

where $\omega_j^* = k_y T_j/m_j\omega_{cj}L_{nj}$ is the diamagnetic drift frequency and $L_{nj} = -n_{0j}^0/n'_{0j}$ with $n'_{0j} = \partial ln n_{0j}(x)/\partial x$ as the scale length of inhomogeneity of the *jth* species.

3. Dust Charge Fluctuations in Nonuniform Dusty Magnetoplasmas

We consider electrostatic perturbations (ω, $\boldsymbol{k}$) accounting for dust charge fluctuations. The charging equation for dust particles in a dusty plasma is

$$\mathrm{d}_t Q_{d1} = I_{e1} + I_{i1}, \tag{2}$$

where Q_{d1} is the perturbation of the dust charge in the presence of the perturbed electron and ion currents $I_{e1,\ i1}$ associated with the perturbed plasma particle distribution functions in the electrostatic field of the low-frequency wave. Here, ω and $\boldsymbol{k}$ are the frequency and the wavevector, respectively.

To calculate the perturbed currents of the magnetized electrons and ions, we assume $\rho_{e,i} \geq a$, where ρ_i is the Larmor radius of the species j (j equals e for electrons, i for ions and d for dust grains). We employ the guiding-center coordinates [8,9] and obtain the perturbed distribution function in the presence of the electrostatic potential $\Phi(\boldsymbol{x}, t)$, $f_{j1}(\boldsymbol{x}, \boldsymbol{v}, t)$ given by eq. (1).

In the presence of a low-frequency electrostatic field, the charging current perturbations are

$$I_{j1}(\boldsymbol{x}, t) \equiv \oint \boldsymbol{J}_j \cdot \mathrm{d}\boldsymbol{S} = 2\pi a^2 q_j \times \int (v_\perp \cos\theta + v_\perp \sin\theta + v_\parallel) f_{j1} \mathrm{d}\boldsymbol{v}, \tag{3}$$

where f_{j1} is given by eq. (1). After a straightforward calculation, we then obtain from eq. (3)

$$I_{j1}(\boldsymbol{x}, t) = -4\pi a^2 q_j n_{j0} \frac{q_j \Phi(\boldsymbol{x}, t)}{T_j} Y_j \times \exp\left(-\frac{q_j \Phi_G}{T_j}\right)\left(-\frac{T_j}{2\pi m_j}\right)^{1/2}, \tag{4}$$

where

$$Y_j = Y_j^1 + Y_j^2 + Y_j^3, \tag{5}$$

with

$$Y_j^1 = \sqrt{\frac{\pi}{2}} \sum_n \frac{n\omega_{cj}}{k_\perp v_{tj}} I_n \exp(-b_j) \times \left[1 + \frac{\omega - \omega_j^*}{\sqrt{2}\, k_\parallel v_{tj}} Z(\xi_{nj})\right], \tag{6}$$

$$Y_j^2 = -\mathrm{i}\sqrt{\frac{\pi}{2}} \frac{k_\perp v_{tj}}{\omega_{cj}} \sum_n \frac{\mathrm{d}}{\mathrm{d}b_j}[I_n \exp(-b_j)] \times \left[1 + \frac{\omega - \omega_j^*}{\sqrt{2}\, k_\parallel v_{tj}} Z(\xi_{nj})\right], \tag{7}$$

$$Y_j^3 = \sqrt{\frac{\pi}{2}} \frac{\omega - \omega_j^*}{k_\parallel v_{tj}} \sum_n I_n \exp(-b_j) \times [1 + \xi_{nj} Z(\xi_{nj})], \tag{8}$$

and $\xi_{nj} = (\omega - n\omega_{cj})/\sqrt{2}k_\parallel v_{tj}$, $b_j = k_\perp^2 v_{tj}^2/\omega_{cj}^2$, and $v_{tj} = (T_j/m_j)^{1/2}$. In obtaining eq. (4), we have assumed that the dust grain surface potential is constant. Here, $\Phi_G = Q_{d0}/a$ is the grain surface potential, and Q_{d0} is the equilibrium charge of a spherical dust grain of radius a.

4. Dispersion relations

We now consider two different parameter regimes of practical interest of the nonuniform dusty magnetoplasmas to obtain the low-frequency dust-lower-hybrid drift wave dispersion relations and their damping or growth in sub-sections **4.1** and **4.2**.

4.1 Strongly magnetized electrons and Ions

For the dust-lower-hybrid drift waves, we assume that the electrons and ions are strongly magnetized, and the dust component to be cold and unmagnetized :

$$\omega \ll \omega_{cj},\ \ j = e, i$$
$$k_\perp v_{tj} \ll \omega_{cj},$$
$$k_\parallel v_{tj} \ll |\omega|,\ \ |\omega - n\omega_{cj}|. \tag{9}$$

Since the drift wave frequency is typically much larger than the dust plasma and dust cyclotron frequencies ($\omega \gg \omega_{pd}$, ω_{cd}), the grains can be considered immobile.

Integrating eq. (1) with proper limits one can easity obtain the charge number density perturbation $n_j(\omega, \boldsymbol{k}) \equiv -\chi_j k^2 \Phi(\omega, \boldsymbol{k})/4\pi q_j$. Thus, we calculate the electron and ion susceptibilities with approximations, eqs. (9) as

$$\chi_e = \frac{1}{k^2\lambda_{De}^2}\left[1 + \frac{\omega - \omega_e^*}{\sqrt{2}\, k_\parallel v_{te}} Z\left(\frac{\omega - n\omega_{ce}}{\sqrt{2}\, k_\parallel v_{te}}\right)(1 - b_e)\right],$$
$$= \frac{\omega_e^*}{\omega k^2 \lambda_{De}^2} + \frac{\omega - \omega_e^*}{\omega}\left(\frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2}\right). \tag{10}$$

For $k_\parallel^2 \ll k_\perp^2$

$$\chi_e \simeq \frac{\omega_e^*}{\omega k^2 \lambda_{De}^2} + \frac{\omega - \omega_e^*}{\omega}\frac{\omega_{pe}^2}{\omega_{ce}^2}. \tag{11}$$

Similarly, for ions

$$\chi_i \simeq \frac{\omega_i^*}{\omega k^2 \lambda_{Di}^2} + \frac{\omega - \omega_i^*}{\omega}\frac{\omega_{pi}^2}{\omega_{ci}^2}. \tag{12}$$

Neglecting the inhomogeneities and considering cold magnetized electrons and ions, and unmagnetized cold dust component, one readily obtains the usual dust-lower-hybrid wave [10] propagating nearly transverse to the external uniform magnetic field from $\varepsilon = 0$

$$\omega^2 \approx \omega_{dlh}^2\left(1 + \frac{k_\parallel^2}{k_\perp^2}\frac{\omega_{pe}^2}{\omega_{pd}^2}\right), \tag{13}$$

where $\omega_{dlh}^2 = \omega_{pd}^2\,\omega_{ci}^2/\omega_{pi}^2$ and $\omega_{pi}^2 \gg \omega_{ci}^2$.

We write $n_j = n_{j0} + n_{j1}$ and $Q_d = Q_{d0} + Q_{d1}$ where the quantities subscripted by 1 denote perturbed quantities due to the presence of any low-frequency wave (ω, $\boldsymbol{k}$). Thus, from eq. (2), we obtain following Varma *et al.* [4]

$$Q_{d1} = -\mathrm{i}\beta(\omega)\Phi/\omega, \tag{14}$$

$$\beta(\omega) = \frac{a^2}{\sqrt{2\pi}}\left[\frac{\omega_{pe}}{\lambda_{De}}\exp\left(\frac{e\Phi_G}{T_e}\right)Y_e(\omega) + \frac{\omega_{pi}}{\lambda_{Di}}\left(1-\frac{e\Phi_G}{T_i}\right)Y_i(\omega)\right]. \tag{15}$$

Here, $\lambda_{Dj} = v_{tj}/\omega_{pj}$, $\omega_{pj} = (4\pi n_{j0}q_j^2/m_j)^{1/2}$, and $Y_{e,i}(\omega)$ are given by eq. (5) depending on the conditions of the wave perturbation and the plasma parameters. For the conditions of this section, eqs. (9)

$$Y_j \simeq \sqrt{\frac{\pi}{2}}\,(1-b_j)\left[\frac{k_\perp v_{tj}}{\omega_{cj}}\left\{\frac{\omega-\omega_j^*}{\omega_{cj}} + \mathrm{i}\left(1-\frac{\omega-\omega_j^*}{\omega}\left(1+\frac{k_\parallel^2 v_{tj}^2}{\omega^2}\right)\right)\right\} - \frac{\omega-\omega_j^*}{\omega}\frac{k_\parallel v_{tj}}{\omega}\right]. \tag{16}$$

If we assume $k_\parallel/k_\perp \ll \omega^2/\omega_{cj}^2$ and neglect the small imaginary term, the factors, Y_j ($j = e$, i) modifying the charge fluctuation factor β of the unmagnetized plasma turn out to be

$$Y_j \approx \sqrt{\frac{\pi}{2}}\,(1-b_j)\,\frac{k_\perp v_{tj}}{\omega_{cj}}\,\frac{\omega-\omega_j^*}{\omega_{cj}}. \tag{17}$$

Let us now demonstrate how the external magnetic field and the scalelength of inhomogeneity affect the damping of a drift-kinetic wave propagating perpendicular to the magnetic field direction. For ω_{cd}, kv_{td}, $k_\parallel v_{ti}$, $k_\parallel v_{te} \ll |\omega| \ll \omega_{ci} \ll \omega_{ce}$, $|\omega| \gg \omega_{pd}$, ω_{cd} and $b_{e,i} \ll 1$, the electrons and ions are strongly magnetized, while the cold and unmagnetized dust grains are considered immobile.

The Poisson's equation in the presence of a low-frequency mode and dust charge fluctuations is

$$k^2\Phi + 4\pi(n_{e1}e - n_{i1}e - n_{d1}Q_{d0} - n_{d0}Q_{d1}) = 0, \tag{18}$$

where Q_{d1} is given by eq. (14). Inserting $n_{e1,i1} = \pm k^2\chi_{e,i}\Phi/4\pi e$, $n_{d1} = 0$, and Q_{d1} given by eq. (14), we obtain $\varepsilon(\omega, \boldsymbol{k})\Phi(\omega, \boldsymbol{k}) = 0$, where

$$\begin{aligned}\varepsilon(\omega,\boldsymbol{k}) &= 1 + \chi_e + \chi_i + \frac{\mathrm{i}\,4\pi n_{d0}\beta}{k^2\omega},\\ &= 1 + \frac{\omega_e^*}{k^2\lambda_{De}^2\omega} + \frac{\omega_i^*}{k^2\lambda_{Di}^2\omega} + \frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1+\frac{n_{e0}m_e}{n_{i0}m_i}\right) - \frac{\omega_e^*}{\omega}\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_i^*}{\omega}\frac{\omega_{pi}^2}{\omega_{ci}^2} + \mathrm{i}\,\frac{4\pi n_{d0}\beta}{k^2\omega},\\ &\simeq 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_i^*}{\omega}\frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1+\frac{T_e}{T_i}\frac{m_e}{m_i}\frac{n_{e0}}{n_{i0}}\frac{L_{ni}}{L_{ne}}\right) + \mathrm{i}\frac{4\pi n_{d0}\beta}{k^2\omega},\end{aligned} \tag{19}$$

where $k_\parallel^2 \ll k_\perp^2$ and $\omega_{pe,i}^2/\omega_{ce,i}^2 \gg 1/k^2\lambda_{De,i}^2$. Equating the real and imaginary parts from $\varepsilon = 0$, we obtain the dispersion relation of the dust-lower-hybrid drift wave as (with $\omega \equiv \omega_R + \mathrm{i}\gamma$)

$$\omega_R = \omega_i^*\left(1+\frac{T_e}{T_i}\frac{m_e}{m_i}\frac{n_{e0}}{n_{i0}}\frac{L_{ni}}{L_{ne}}\right), \tag{20}$$

and the damping rate of this drift wave due to the charge fluctuation as

$$\gamma = -\frac{4\pi n_{d0}\beta}{k^2(\omega_{pi}^2/\omega_{ci}^2)}, \tag{21}$$

for $\omega_{pi}^2 \gg \omega_{ci}^2$. Comparing this with eq.(19) of ref. [7], we note that the charge fluctuation damping of the low-frequency drift wave is less than that of the usual dust-lower-hybrid wave or dust-acoustic wave by a factor $\omega_{pi}^2/\omega_{ci}^2$ which is greater than 1. Hence, the low-frequency dust-lower-hybrid drift wave is more stable than the usual dust-acoustic and dust-lower-hybrid waves.

4.2 Hot electrons

Here, we study the dust-lower-hybrid drift instability in the presence of the charge fluctuation damping in a nonuniform magnetized dusty plasma where the electrons are thermal, ions are strongly magnetized, and the dust grains are immobile (cold and unmagnetized) as $\omega \gg \omega_{pd}$, ω_{cd}. Consequently, the following conditions are valid for this case.

$$\begin{aligned}&\omega_{cd} \ll \omega \ll \omega_{ci} \ll \omega_{ce},\\ &k_\parallel v_{te} \gg |\omega|,\ |\omega-\omega_e^*|,\ |\omega - n\omega_{ce}|,\\ &k_\parallel v_{ti} \ll |\omega|,\ |\omega - n\omega_{ci}|,\\ &b_{e,i} \ll 1.\end{aligned} \tag{22}$$

Thus,

$$\begin{aligned}\chi_e &= \frac{1}{k^2\lambda_{De}^2}\left[1+\mathrm{i}\sqrt{\pi}\left(\frac{\omega-\omega_e^*}{\sqrt{2}\,k_\parallel v_{te}}\right)\right],\\ \chi_i &= \frac{1}{k^2\lambda_{Di}^2}\frac{\omega_i^*}{\omega} + \frac{\omega-\omega_i^*}{\omega}\times\left(\frac{k_\perp}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega^2}\right),\\ \chi_d &= 0.\end{aligned} \tag{23}$$

Using conditions, eqs. (22) and following the procedures of the previous Sec. **4.1**, we obtain the wave dispersion relation of the dust-lower-hybrid drift wave and the damping rate in the presence of the charge fluctuation damping ($\omega \equiv \omega_R + \mathrm{i}\gamma$)

$$\omega_R = -\frac{n_{i0}}{n_{e0}}\frac{T_e}{T_i}\frac{\omega_i^*}{1+k_\perp^2\rho_s^2}, \tag{24}$$

$$\gamma = \left[\sqrt{\frac{\pi}{2}}\frac{\omega_R}{k_\parallel v_{te}}\frac{k_y c k_B T_e}{n_{e0}e^2B_0}\frac{\partial}{\partial x}(q_{d0}n_{d0}) - 4\pi n_{d0}\beta\lambda_{De}^2\right], \tag{25}$$

where $k_\perp^2 \rho_s^2 \ll 1$. For this case, eqs. (22), the modification factors of $\beta(\omega)$ given by eq. (15), on simplification reduce to

$$Y_e \simeq \sqrt{\frac{\pi}{2}}(1-b_e)\left(\frac{\omega-\omega_e^*}{k_\parallel v_{te}} + \mathrm{i}\frac{k_\perp v_{te}}{\omega_{ce}}\right), \qquad (26)$$

$$Y_i \simeq \sqrt{\frac{\pi}{2}}\frac{k_\perp v_{ti}}{\omega_{ci}}\frac{\omega-\omega_i^*}{\omega_{ci}}(1-b_i). \qquad (27)$$

Thus, $Y_e \approx \sqrt{\frac{\pi}{2}}\frac{\omega_R}{k_\parallel v_{te}}$ and $Y_i \approx \sqrt{\frac{\pi}{2}}\frac{k_\perp v_{ti}}{\omega_{ci}}\frac{\omega_R}{\omega_{ci}}$ are less than unity reducing the charge fluctuation damping in the magnetized plasma [cf. eq. (15)].

We note from eq. (25) that when the charge fluctuation damping is negligible, the drift wave grows as an instability for $\partial(q_{d0}n_{d0})/\partial x > 0$. This instability is due to the motion of electrons in the presence of the drift wave.

5. Discussion

We have investigated the effects of an external uniform magnetic field and the plasma inhomogeneity on the low-frequency drift waves in the presence of the dust-charge fluctuations in a nonuniform dusty magnetoplasma. In the presence of the low-frequency electrostatic waves, the dust charging current perturbations have been calculated explicitly to examine the dust charge fluctuation effects. For calculating the dust charging currents, we have employed the method of the guiding center coordinates to solve the Vlasov equation and obtain the perturbed distribution function in the presence of the low-frequency drift waves. We assume the spherical dust grains and obtain the dust charge fluctuations. Effects of homogeneous magnetic field and the nonuniform dust density have been shown in the dispersion relation and the damping or growth of the drift waves. It is found that for strongly magnetiged electrons and ions, the charge fluctuation damping is reduced significantly from that of an unmagnetized plasma[cf. eq. (21)]. For sufficiently hot electrons, the drift wave exihibits an instability in absence of the charge fluctuation damping [cf. eq. (25)].

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