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Study of Particle Diffusion in a Stochastic Magnetic Field: DIA and beyond

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Abstract

The particle diffusion in a stochastic magnetic field is studied by applying the functional integral method to a kinetic equation. It is found within the framework of the Direct Interaction Approximation (DIA) that the cross-field diffusion coefficients can be obtained by solving nonlinear ordinary differential equations. An extended DIA for the diffusion coefficient is also presented by incorporating the effect of particle trajectories.

Keywords:
particle diffusion, stochastic magnetic field, DIA, functional integral method, subensemble, decorrelation trajectory

1. Introduction

The particle transport due to a stochastic magnetic field has been intensively studied in the context of thermonuclear fusion research. A kinetic equation in a presence of stochastic magnetic field is the reasonable starting point for the theoretical treatment of this problem. However, solving such a stochastic kinetic equation is extremely difficult, and thus many different approaches and approximations are presented for this difficult problem.

Balescu *et al*. [1-3] have presented an interesting analytically tractable model describing the particle transport in a fluctuating magnetic field. In this model, the fluctuating components of the magnetic field are assumed to be Gaussian random process, and the mutual collisions between particles are modeled by the random variation of velocity. Even though the problem is quite simplified by using this model, the rigorous mathematical treatment is still impossible except for the limited case. In this paper, we proceed to investigate this particle-diffusion model further. In Sec. **2**, the particle-diffusion coefficient is studied within the framework of the Direct Interaction Approximation (DIA) by using the functional integral method [4-6]. We extend the previous theory [3] to the form applicable to fast particles and also extend to a finite shear configuration. In Sec. **3**, we incorporate the idea of the decorrelation trajectory method proposed by Vlad *et al*. [7] into our functional integral formulation, and present the extended DIA by taking into account the effect of particle trajectories.

2. Diffusion coefficient by DIA

In this section, we study the particle transport in a stochastic magnetic field within the framework of the DIA. Let us consider a magnetic field given by

$$\boldsymbol{B}(\boldsymbol{r}) = B_0\,[\boldsymbol{e}_z + b_x(\boldsymbol{r})\boldsymbol{e}_x + b_y(\boldsymbol{r})\boldsymbol{e}_y], \tag{1}$$

where $\{\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z\}$ are the unit vectors in the Cartesian coordinates (x, y, z), $B_0\boldsymbol{e}_z$ is the constant averaged magnetic field, and $B_0[b_x(\boldsymbol{r})\boldsymbol{e}_x + b_y(\boldsymbol{r})\boldsymbol{e}_y] \equiv B_0\boldsymbol{b}$ is the fluctuating magnetic field with the random variables $b_x(\boldsymbol{r})$ and $b_y(\boldsymbol{r})$ of Gaussian process. We start with a kinetic equation for a distribution function

$$\partial_t f + v_\parallel \partial_z f + \sqrt{\varepsilon} v_\parallel \boldsymbol{b}\cdot\nabla_\perp f - C(f) = 0, \tag{2}$$

where $\nabla_\perp = \boldsymbol{e}_x\partial/\partial_x + \boldsymbol{e}_y\partial/\partial_y$; the distribution function f is normalized as $\int f(\boldsymbol{r}, t)\mathrm{d}\boldsymbol{r} = 1$; and the subscripts $\parallel$ and $\perp$ refer to the parallel and perpendicular to the averaged magnetic field. In (2) we have introduced a formal ordering parameter ε, which physically means $\varepsilon = \mathrm{O}(\boldsymbol{b}^2 l_\parallel^2/l_\perp^2)$, where $l_\parallel$ and $l_\perp$ are the characteristic parallel and perpendicular lengths. The formal parameter ε is finally set to be unity. The effect of Coulomb collisions is modeled by using the Gaussian random variation $\boldsymbol{\eta}(t)$ of velocity as [1-3]

$$C(f) = -\sqrt{\varepsilon}\,\eta_\parallel(t)\,\partial_z f - \sqrt{\varepsilon}\,[\boldsymbol{\eta}_\perp(t) + \sqrt{\varepsilon}\,\eta_\parallel(t)\boldsymbol{b}(\boldsymbol{r})\,]\cdot\nabla_\perp f. \tag{3}$$

The Gaussian random variables $\boldsymbol{b}(\boldsymbol{r})$, $\eta_\parallel(t)$ and $\boldsymbol{\eta}_\perp(t)$ are assumed to be no correlation among them, and the magnetic fluctuation and the collisional variation have the following

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statistical properties:

$$\langle \eta_{\|}(t)\,\eta_{\|}(t')\rangle = \chi_{\|}\nu R[\nu(t-t')], \qquad (4)$$

$$\langle \boldsymbol{\eta}_{\perp}(t)\,\boldsymbol{\eta}_{\perp}(t')\rangle = \chi_{\perp}\nu R[\nu(t-t')]\,\boldsymbol{I}, \qquad (5)$$

$$\langle \boldsymbol{b}(\boldsymbol{r})\boldsymbol{b}(\boldsymbol{r}')\rangle = \langle \nabla A(\boldsymbol{r})\times\boldsymbol{e}_z \nabla' A(\boldsymbol{r}')\times\boldsymbol{e}_z\rangle \equiv \boldsymbol{F}(\boldsymbol{r}-\boldsymbol{r}') \qquad (6)$$

with $R[\nu(t-t')] = \exp(-\nu|t-t'|)$ and

$$\langle A(\boldsymbol{r})A(\boldsymbol{0})\rangle = \beta^2\lambda_{\perp}^2 \exp\left[-\frac{z^2}{2\lambda_{\|}^2}-\frac{\boldsymbol{r}_{\perp}^2}{2\lambda_{\perp}^2}\right], \qquad (7)$$

where $\boldsymbol{r}_{\perp} = x\boldsymbol{e}_x + y\boldsymbol{e}_y$; $\boldsymbol{I} = \boldsymbol{e}_x\boldsymbol{e}_x + \boldsymbol{e}_y\boldsymbol{e}_y$; $\chi_{\|} = v_T^2/2\nu$ and $\chi_{\perp} = v_T^2\nu/2\Omega^2$ are the classical parallel and perpendicular diffusion coefficients; v_T and Ω are the thermal velocity and the Larmor frequency ; ν is the collision frequency; $\lambda_{\|}$ and $\lambda_{\perp}$ are the parallel and perpendicular correlation lengths; β represents the amplitude of fluctuating magnetic field; and $\langle\,\cdot\,\rangle$ denotes the ensemble average over the random variables $\boldsymbol{b}(\boldsymbol{r})$, $\eta_{\|}(t)$ and $\boldsymbol{\eta}_{\perp}(t)$. For later use, we define

$$\varphi(\tau) = \int_0^{\tau} \mathrm{d}\bar{\tau}\, R(\bar{\tau}), \quad \psi(\tau) = \int_0^{\tau} \mathrm{d}\bar{\tau}\,\varphi(\bar{\tau}). \qquad (8)$$

Let us introduce a generating functional with the external sources $\zeta(t)$ and $\xi(t)$

$$Z[\zeta,\xi] = \mathcal{N}\int \mathcal{D}[f']\mathcal{D}[\hat{f}]\; e^{-S/\varepsilon} \equiv e^{W/\varepsilon} \qquad (9)$$

with $S = -\log\langle e^{-L/\varepsilon}\rangle - f'(1)\zeta(1) - \hat{f}(1)\xi(1)$ and

$$L = \hat{f}(1)\Big\{\partial_t + \Big[v_{\|} + \sqrt{\varepsilon}\eta_{\|}(t)\Big]\partial_z + \sqrt{\varepsilon}\Big[(v_{\|}+\sqrt{\varepsilon}\eta_{\|}(t))\boldsymbol{b}(\boldsymbol{r}) + \boldsymbol{\eta}_{\perp}(t)\Big]\cdot\nabla_{\perp}\Big\} f'(1), \qquad (10)$$

where the index 1 denotes $1 = (\boldsymbol{r}, t)$; $\int\mathcal{D}[f']\mathcal{D}[\hat{f}]$ means the functional integral; $\mathcal{N}$ is the multiplicative constant irrelevant to the following calculation; and the integration over repeated indices is assumed in the case of no confusion. Using this generating functional, we define

$$g(1) \equiv \varepsilon\frac{\delta W}{\delta\zeta(1)}, \quad G(1-1') \equiv \varepsilon^2\frac{\delta^2 W}{\delta\xi(1')\,\delta\zeta(1)}. \qquad (11)$$

In the limit of $\zeta = \xi = 0$, the one-point function $g(1)$ becomes the ensemble-averaged distribution function, i.e. $g(1)|_{\zeta=\xi=0} = \langle f(1)\rangle$. Performing the ensemble average in S and using the identity

$$\int \mathcal{D}[f']\mathcal{D}[\hat{f}]\frac{\delta}{\delta\hat{f}(1)}e^{-S/\varepsilon} = 0, \qquad (12)$$

we can derive the evolution equation for $g(1)$. This evolution equation is considered up to the order ε by applying the Wentzel-Kramers-Brillouin (WKB) approximation. Let us now define the cross-field diffusion tensor by

$$\boldsymbol{D}_{\perp}(t) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}\boldsymbol{r}\,\boldsymbol{r}_{\perp}\boldsymbol{r}_{\perp}g(\boldsymbol{r},t). \qquad (13)$$

Then, the anomalous diffusion tensor due to the magnetic fluctuation is expressed in terms of the response function G as

$$\begin{aligned}\boldsymbol{D}_{\perp}^{an}(t) = &\int_0^t \mathrm{d}t_2\{\nu\chi_{\|}R[\nu(t-t_2)] + v_{\|}^2\} \\ &\times\int \mathrm{d}\boldsymbol{r}\boldsymbol{F}(\boldsymbol{r})G(\boldsymbol{r}, t-t_2) \\ &+ (\nu\chi_{\|})^2\int_0^t \mathrm{d}t_2\int_0^{t_2}\mathrm{d}t_3\int_0^{t_3}\mathrm{d}t_4 \\ &\times\{R[\nu(t-t_2)]R[\nu(t_3-t_4)] \\ &+ R[\nu(t-t_3)]R[\nu(t_2-t_4)]\} \\ &\times\int\mathrm{d}\boldsymbol{r}_2\int\mathrm{d}\boldsymbol{r}_3\int\mathrm{d}\boldsymbol{r}_4\boldsymbol{F}(\boldsymbol{r}_2+\boldsymbol{r}_3+\boldsymbol{r}_4) \\ &\times G(\boldsymbol{r}_2, t-t_2)\,\partial_{z_3}G(\boldsymbol{r}_3, t_2-t_3) \\ &\times\partial_{z_4}G(\boldsymbol{r}_4, t_3-t_4) \\ &- v_{\|}\nu\chi_{\|}\int_0^t\mathrm{d}t_2\int_0^{t_2}\mathrm{d}t_3\{R[\nu(t-t_2)] \\ &+ R[\nu(t_2-t_3)]\} \\ &\times\int\mathrm{d}\boldsymbol{r}_2\int\mathrm{d}\boldsymbol{r}_3\boldsymbol{F}(\boldsymbol{r}_2+\boldsymbol{r}_3) \\ &\times G(\boldsymbol{r}_2, t-t_2)\,\partial_{z_3}G(\boldsymbol{r}_3, t_2-t_3). \end{aligned} \qquad (14)$$

Taking the functional differentiation of the evolution function for $g(1)$ with respect to $\zeta(1')$, neglecting the three point function $\delta G(1-2)/\delta\xi(1')$, and using the spatial and temporal Markovian approximation give us the equation for the response function up to the order ε

$$\begin{aligned}&[\partial_t + v_{\|}\partial_z - D_{\|}(t-t')\partial_z^2]\,G(1-1') \\ &-D_{\perp}^{cl}(t-t')\nabla_{\perp}^2 G(1-1') \\ &-\nabla_{\perp}\cdot\boldsymbol{D}_{\perp}^{an}(t-t')\cdot\nabla_{\perp}G(1-1') = \delta(1-1'),\end{aligned} \qquad (15)$$

where $D_{\|} = \chi_{\|}\varphi\,[\nu(t-t')]$ and $D_{\perp}^{cl} = \chi_{\perp}\varphi[\nu(t-t')]$ are the classical parallel and perpendicular diffusion coefficients, respectively. The approximation leading to the evolution equation for g and the equation (15) for G is referred to as the DIA in this paper.

The Fourier transform of the solution to (15) is immediately obtained

$$\begin{aligned}G(\boldsymbol{k}, t-t') = \mathrm{H}(t-t')\exp\Big[&-\mathrm{i}k_{\|}v_{\|}(t-t') \\ &-k_{\|}^2\int_0^{t-t'}D_{\|}(t'')\mathrm{d}t'' - k_{\perp}^2\int_0^{t-t'}D_{\perp}^{cl}(t'')\mathrm{d}t'' \\ &-\boldsymbol{k}_{\perp}\boldsymbol{k}_{\perp}:\int_0^{t-t'}\boldsymbol{D}_{\perp}^{an}(t'')\mathrm{d}t''\Big],\end{aligned} \qquad (16)$$

where H is the Heaviside step function. Substituting (16) into (14), we find that $\boldsymbol{D}_{\perp}^{an}$ is a diagonal tensor, i.e. $\boldsymbol{D}_{\perp}^{an} = \boldsymbol{D}_{\perp}^{an}\,\boldsymbol{I}$, and the function μ defined through $D_{\perp}^{an}(t) = (\lambda_{\perp}^2/4)\mathrm{d}\mu/\mathrm{d}t$ satisfies the equation

$$\frac{\mathrm{d}^2\mu}{\mathrm{d}\tau^2} = \frac{2\bar{\chi}_\parallel \alpha^2 \mathrm{F}(\tau)}{\left[1+\bar{\chi}_\parallel \psi(\tau)\right]^{1/2}} \left\{ R(\tau) + 2\bar{v}_\parallel^2 \right.$$
$$- \frac{1}{2}\frac{\bar{\chi}_\parallel \varphi(\tau)}{1+\bar{\chi}_\parallel \psi(\tau)} \left[4\bar{v}_\parallel^2 \tau \right.$$
$$\left.\left. + \varphi(\tau)\left(1 - \frac{\bar{\chi}_\parallel \bar{v}_\parallel^2 \tau^2}{1+\bar{\chi}_\parallel \psi(\tau)}\right)\right]\right\}$$
$$\times \exp\left[-\frac{1}{2}\frac{\bar{\chi}_\parallel \bar{v}_\parallel^2 \tau^2}{1+\bar{\chi}_\parallel \psi(\tau)}\right] \tag{17}$$

with the initial conditions $\mu(0) = 0$ and $\mathrm{d}\mu(0)/\mathrm{d}\tau$, where $\tau = \nu t$, $\alpha = \beta\lambda_\parallel/\lambda_\perp$, $\bar{v}_\parallel = v_\parallel/v_T$, $\bar{\chi}_\parallel = 2\chi_\parallel/\nu\lambda_\parallel^2$, $\bar{\chi}_\perp = 2\chi_\perp/\nu\lambda_\perp^2$, and

$$\mathrm{F}(\tau) = \left[1 + \bar{\chi}_\perp \psi(\tau) + \frac{\mu}{2}\right]^{-2}. \tag{18}$$

In the $\lambda_\perp = \infty$ limit, which is equivalent to neglecting the dependence of the fluctuating magnetic field on x and y, the differential equation (17) reduces to the exact one derived without using the DIA.

In the large $v_\parallel$ limit, the differential equation (17) is approximated as

$$\frac{\mathrm{d}^2\mu}{\mathrm{d}t^2} = \frac{4\alpha^2 v_\parallel^2}{\lambda_\parallel^2}\frac{1}{(1+\mu/2)^2}\exp\left[-\frac{v_\parallel^2 t^2}{2\lambda_\parallel^2}\right]. \tag{19}$$

The explicit calculation for various parameters shows that the approximation by this equation is fairly well for $v_\parallel > v_T$ in the region of $\bar{\chi}_\parallel > 1$ and $\bar{\chi}_\perp < 1$.

The starting point for the shearless configuration (1) completely agrees with that of ref. [3] by setting $v_\parallel = 0$ in (2). Therefore, for comparison with the result in ref. [3], we set $v_\parallel = 0$ in (17) and define $\hat{\mu} = \mu - \mu_0$, where $\mu_0(\tau) = 4\alpha^2(\sqrt{1+\bar{\chi}_\parallel \psi(\tau)} - 1)$ is the exact solution when $\boldsymbol{\eta}_\perp = \mathbf{0}$ and the fluctuating magnetic field $\boldsymbol{b}$ depends only on the z coordinate. The differential equation for this newly defined function $\hat{\mu}$ agrees with that obtained in ref. [3] when we neglect μ_0 in F. This neglect of μ_0 is the additional approximation to the DIA and this approximation is implicitly used in ref. [3]. However, it is found from the numerical calculation of (17) that the neglect of μ_0 in F is only valid in the quite limited region of parameters *within the framework of the* DIA.

Finally in this section, the effect of finite shear is considered in a sheared slab configuration. Restricting our consideration only around a surface $x = 0$, we write the magnetic field as

$$\boldsymbol{B}(\boldsymbol{r}) = B_0\left(\boldsymbol{e}_z + \frac{x}{L_s}\boldsymbol{e}_y\right) + B_0\boldsymbol{b}(\boldsymbol{r}), \tag{20}$$

where L_s is the shear length. In the approximation of $x/L_s \ll 1$, the diffusion tensor $\boldsymbol{D}_\perp^{an}(t, x)$ and the equation for the response function $G(1, 1') \equiv G(\boldsymbol{r}-\boldsymbol{r}', t-t'; x/L_s)$ are obtained from (14) and (15) by replacing ∂_z with $\partial_z + (x/L_s)\partial_y$.

We here note only the collisionless regime by assuming $\eta_\parallel(t) = 0$ and $\boldsymbol{\eta}_\perp(t) = \mathbf{0}$. The diffusion tensor in this regime is also diagonal for the finite shear configuration, and the diagonal components of $\boldsymbol{\mu}(t, x) = (4/\lambda_\perp^2)\int_0^t \boldsymbol{D}_\perp^{an}(t', x)\mathrm{d}t'$ are determined by the following equations:

$$\frac{\mathrm{d}^2\mu_{xx}}{\mathrm{d}t^2} = \frac{4\alpha^2 v_\parallel^2}{\lambda_\parallel^2}\frac{(1-\mathcal{F})}{(1+\mu_{xx}/2)^{1/2}(1+\mu_{yy}/2)^{3/2}} \times \exp\left[-\frac{1}{2}\left(\frac{v_\parallel^2 t^2}{\lambda_\parallel^2} + \mathcal{F}\right)\right], \tag{21}$$

$$\frac{\mathrm{d}^2\mu_{yy}}{\mathrm{d}t^2} = \frac{4\alpha^2 v_\parallel^2}{\lambda_\parallel^2}\frac{1}{(1+\mu_{xx}/2)^{3/2}(1+\mu_{yy}/2)^{1/2}} \times \exp\left[-\frac{1}{2}\left(\frac{v_\parallel^2 t^2}{\lambda_\parallel^2} + \mathcal{F}\right)\right], \tag{22}$$

where

$$\mathcal{F} = \frac{1}{1+\mu_{yy}/2}\frac{v_\parallel^2 t^2}{\lambda_\perp^2}\frac{x^2}{L_S^2}. \tag{23}$$

The equations (21) and (22) reduce to (19) when $L_s = \infty$.

3. Extended DIA

In this section, we incorporate the idea of the subensemble in ref. [7] into our functional integral formulation. Let us now consider an ensemble average over the fluctuating magnetic field and the collisional variation under the conditions $\boldsymbol{b}(\boldsymbol{r}) = \boldsymbol{b}_0$ and $\eta_\parallel(t) = \eta_{\parallel 0}$, and write this subensemble average by $\langle \cdot \rangle_0$. In analogy to (9), we introduce a generating functional

$$Z_0[\zeta, \xi] = \mathrm{N}\int \mathrm{D}[f']\mathrm{D}[\hat{f}]\, e^{-S_0/\varepsilon} \equiv e^{W_0/\varepsilon} \tag{24}$$

with $S_0 = -\log\langle e^{-\mathrm{L}/\varepsilon}\rangle_0 - f'(1)\zeta(1) - \hat{f}(1)\xi(1)$, and define the subensemble-averaged response function by

$$G_0(1-1') \equiv \varepsilon^2 \frac{\delta^2 W_0}{\delta\xi(1')\,\delta\zeta(1)}. \tag{25}$$

Then, repeating the procedure leading to (14) and (15), we can derive the subensemble expression for the anomalous diffusion tensor

$$\boldsymbol{D}_\perp^{an}(t) = \int \mathrm{d}\boldsymbol{b}_0 \int \mathrm{d}\eta_{\parallel 0} P_1(\boldsymbol{b}_0)P_2(\eta_{\parallel 0})\boldsymbol{D}_0(t), \tag{26}$$

where $P_1(\boldsymbol{b}_0)$ and $P_2(\eta_{\parallel 0})$ are the Gaussian probabilities of $\boldsymbol{b}_0$ and $\eta_{\parallel 0}$, and $\boldsymbol{D}_0(t)$ is approximately obtained by replacing the response function $G(i-j)$ with $G_0(i-j)$ in the expression (14). The equation for $G_0(1-1')$ is

$$\left\{\frac{\partial}{\partial t'} + \boldsymbol{C}_\parallel(t-t')\frac{\partial}{\partial z'} + \boldsymbol{C}_\perp(t-t', \boldsymbol{r}-\boldsymbol{r}')\cdot\nabla'_\perp + D_\parallel(t-t')\frac{\partial^2}{\partial z'^2} + D_\perp^{cl}(t-t')\nabla'^2_\perp\right\}G_0(1-1')$$
$$+ \nabla'_\perp\cdot\boldsymbol{D}_0(t-t')\cdot\nabla'_\perp G_0(1-1') = -\delta(1-1'), \tag{27}$$

where

$$C_{\|}(t-t') = v_{\|} + \nu\chi_{\|}R[\nu(t-t')]\ \frac{\eta_{\|0}}{\langle \eta_{\|}^{2}(t)\rangle}, \qquad (28)$$

$$\boldsymbol{C}_{\perp}(t-t', \boldsymbol{r}-\boldsymbol{r}') = \frac{2C_{\|}(t-t')}{\langle \boldsymbol{b}^{2}(\boldsymbol{r})\rangle}\ \boldsymbol{b}_{0}\cdot \boldsymbol{F}(\boldsymbol{r}-\boldsymbol{r}'). \qquad (29)$$

The equations $dZ(\tau;t-t')/d\tau = C_{\|}(\tau)$ and $d\boldsymbol{X}_{\perp}(\tau;\boldsymbol{r}-\boldsymbol{r}', t-t')d\tau = \boldsymbol{C}_{\perp}(\tau, \boldsymbol{X}_{\perp}, Z)$ with $\boldsymbol{X}_{\perp}(t-t';\boldsymbol{r}-\boldsymbol{r}', t-t') = \boldsymbol{r}_{\perp} - \boldsymbol{r}'_{\perp}$ and $Z(t-t';t-t') = z-z'$ determine the characteristic trajectory to (27) in the approximation of neglecting the diffusion terms. This characteristic trajectory is similar to the decorrelation trajectory proposed in ref. [7]. The expression (26) for the diffusion tensor and the equation (27) for G_0 reduce to (14) and (15) obtained by the DIA when we neglect $\eta_{\|0}$ and $\boldsymbol{b}_0$.

4. Conclusions

The particle diffusion in the fluctuating magnetic field is investigated by applying the functional integral method to the kinetic equation (2). It is found within the framework of the DIA that the diffusion coefficient can be obtained by solving the nonlinear ordinary differential equation (17) for the shearless configuration (1). For the sheared configuration (20), the diffusion coefficient in the collisionless regime is shown to be obtained by solving the coupled equations (21) and (22). In Sec. **3**, we have formulated the extended DIA by incorporating the effect of particle trajectories. In this formulation, the transport coefficient can be calculated from the expression (26) by solving the equation (27).

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