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Kinetic Simulation for Infinitely Long Cylindrical High-Beta Plasma with Field-Null Surface

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Abstract

The kinetic simulation for an infinitely long cylindrical high-beta plasma with field-null surface is carried out. The Vlasov equation is solved in order to study finite Larmor radius (FLR) effect of ions near the field-null region and the fluid equation is used for electrons. The ion distribution function and the radial profiles of macroscopic physical quantities are shown. It is found that the results from the present kinetic scheme are deviated gradually from the one fluid magnetohydrodynamic (MHD) equilibrium.

Keywords:

high-beta plasmas, field-reversed configuration, field-null, Vlasov equation, rigid rotor, FLR effect

1. Introduction

High beta plasmas exhibit kinetic behaviors due to large ion gyro-radii compared to their scale length. Existence of a field-null point makes it difficult to predict the plasma behavior because the particle motion near the point is complicated. A fast ion with the larger canonical angular momentum draws the betatron orbit and contributes to generate the diamagnetic ion current. On the other hand, a slow ion near the surface shows a stochastic behavior in its motion and drifts along the paramagnetic direction. Therefore, obtaining velocity distributions is a key issue for the high-beta plasmas. The particle simulation or the particle and fluid electron simulation (i.e., the hybrid simulation) are useful numerical tools to describe a global characteristic of the high-beta plasma. A physically valid weighting of super-particle is needed in the particle simulation modeling. Another kinetic method is to solve the Vlasov and Maxwell's equations. The time evolutions of the distribution functions are simulated by solving the Vlasov equation. The electric field is calculated from the fluid equation of motion for massless electrons, and the evolution of magnetic field is calculated from Faraday's law. The objectives of this study are to predict a high-beta plasma structure of an electromagnetically relaxed state and to demonstrate kinetic effects in the plasma with field-null surface by the plasma kinetic theory.

Preliminary results are obtained for infinitely long and cylindrically symmetric plasma whose averaged beta value is about 0.9. In this case the real space variable is the radial position alone, and the velocity space variables are the radial and azimuthal velocity components. According to Spivey's work, the only shifted-Maxwellian distributions that satisfy the Vlasov equation for systems with cylindrical symmetry are rigid rotors with constant temperature [1]. The comparison between Spivey's theory and numerical results are made here. This study will be applied in near future to the Field-Reversed Configurations (FRCs) with several unknown physical issues.

2. Simulation model

We study a high-beta plasma that is infinitely long axially and has an axisymmetry (see Fig. 1). Mathematically, this requires $\partial/\partial\theta = \partial/\partial z = 0$. Since the Larmor radius is comparable to the scale length of the high-beta plasma, it is better to use the kinetic equation for ions. On the other hand, MHD is still applicable for electrons, because effects of the field-null are localized. Moreover, computation time increases significantly if we solve the kinetic equation also for electrons; the time interval must be small in order to satisfy the Courant condition. Therefore, we choose a hybrid scheme where ions' behavior is simulated by solving the Vlasov equation and electrons' one are computed with the fluid equation.

Vlasov equation described in the cylindrical coordinate is

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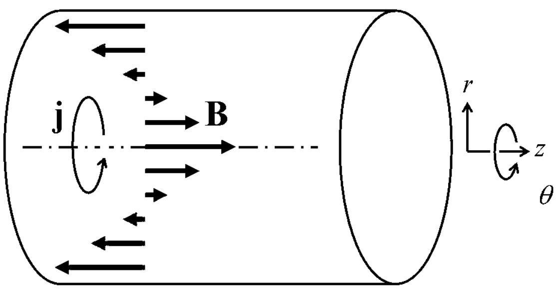

Fig. 1 Schematic view of infinitely long plasma with axisymmetry.

$$\frac{\partial f_i}{\partial t} + v_r \frac{\partial f_i}{\partial r} + \left[\frac{v_\theta^2}{r} + \frac{q_i}{m_i} (v_\theta B_z + E_r) \right] \frac{\partial f_i}{\partial v_r} + \left[\frac{v_r v_\theta}{r} + \frac{q_r}{m_i} (-v_r B_z + E_\theta) \right] \frac{\partial f_i}{\partial v_\theta} = 0 . \quad (1)$$

There is no axial component of force acting on ions in a case of $\partial/\partial\theta = \partial/\partial z = 0$, thus $\mathrm{d}v_z / \mathrm{d}t = 0$. Consequently, the ion distribution never deviates in v_z from the initial form. The ion distribution function f_i is normalized as

$$n_i = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_i\,(r, v_r, v_\theta) \mathrm{d}v_r \mathrm{d}v_\theta \,.$$

Quasineutrarity $n_i = n_e = n$ is assumed in the present study.

The electromagnetic fields $\boldsymbol{B}$ and $\boldsymbol{E}$ in Eq. (1) are calculated from the fluid equation for massless electrons,

$$\boldsymbol{E} = \frac{1}{en} \nabla p_e - \boldsymbol{u}_i \times \boldsymbol{B} + \frac{1}{en} (\boldsymbol{j} \times \boldsymbol{B}), \quad (2)$$

and Maxwell's equations

$$\boldsymbol{j} = \frac{1}{\mu_0} \nabla \times \boldsymbol{B}, \; \frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}. \quad (3)$$

A solution to one-dimensional (1D) Grad-Shafranov equation is used as an initial condition. The 1D Grad-Shafranov equation for the flux function ψ is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) = -\mu_0 \frac{\mathrm{d}p}{\mathrm{d}\psi} . \quad (4)$$

The pressure p as a function of ψ is given as follows:

$$p(\psi) = \begin{cases} a_0 + a_1\psi + a_2\psi^2 + a_3\psi^3 \equiv p_{in}(\psi), \; \psi \geq 0 \\ b_0 \exp(b_1\psi) \equiv p_{out}(\psi), \; \psi < 0 \end{cases}$$

where a s and b s are coefficients which are set so as the current density and pressure at the separatrix to connect smoothly (i.e., $p_{in}\,(\psi = 0) = p_{out}\,(\psi = 0)$, $\mathrm{d}p_{in} / \mathrm{d}\psi|_{\psi=0} = \mathrm{d}p_{out}\, \mathrm{d}\psi|_{\psi=0}$), and $\mathrm{d}^2 p_{in} / \mathrm{d}\psi^2|_{\psi=0} = \mathrm{d}^2 p_{out} / \mathrm{d}\psi^2|_{\psi=0}$). Also, we give the values of pressure at the separatrix and the wall. The initial density profile is estimated from the plasma pressure

$$p = n(T_i + T_e) = nT_{tot}\,, T_i = \frac{1}{1+\alpha} T_{tot}\,, T_e = \frac{\alpha}{1+\alpha} T_{tot}$$

where total temperature T_{tot} is uniform and the parameter is constant. For the plasma in the FRC Injection Experiment (FIX) device, is about 0.5 [2]. The initial electric field is assumed to be zero, and then $u_{i\theta}$ and $u_{e\theta}$ are determined from Eqs. (2) and (3). Therefore,

$$u_{i\theta} = -\frac{1}{enB_z} \frac{\partial p_e}{\partial r} + \frac{j_\theta}{en}\,, \; u_{e\theta} = u_{i\theta} - \frac{j_\theta}{en}$$

Validity of the electron fluid equation near the wall is questionable, since the density and pressure are almost zero. In this case, the electric field and flow velocity can never be fixed by using $-\nabla p_e - en\,(\boldsymbol{E} + \boldsymbol{u}_e \times \boldsymbol{B}) = 0$. In the present study, the plasma region is defined inside the zero flux point (i.e., the separatrix), and the edge layer is located from the separatrix to the point where the density is $n(\psi = 0) / e$. The electric field and flow velocity of plasma vanish outside the edge layer. This assumption is effective to reduce the numerical noise because of the boundary condition.

The shifted-Maxwellian distribution with azimuthal flow velocity is introduced for an initial ion distribution:

$$f_i\,(r, v_r, v_\theta) = n(r) \left(\frac{m_i}{2\pi T_i} \right) \exp \left(\frac{m_i v_r^2}{2T_i} \right) \exp \left(\frac{m_i (v_\theta - u_{i\theta})^2}{2T_i} \right).$$

The Vlasov equation and fluid equations are solved numerically using the finite difference method. The advection terms in the Vlasov equation are discretized by the fourth order scheme, and a 4-step Runge-Kutta method is employed for the numerical integration in time. A filtering with the method of least squares are done for E_r and B_z.

3. Results and discussions

The parameters of geometry and plasma we studied are presented in Table 1. The physical quantities are all normalized as follows:

$$\bar{t} \equiv \frac{t}{1/\omega_{ci}}\,, \bar{r} \equiv \frac{r}{r_w}\,, \bar{\boldsymbol{B}} \equiv \frac{\boldsymbol{B}}{|\psi_w| / r_w^2}\,, \bar{\boldsymbol{j}} \equiv \frac{\boldsymbol{j}}{en_0 v_{thi}}\,,$$

$$\bar{\boldsymbol{E}} \equiv \frac{\boldsymbol{E}}{|\psi_w|\, \omega_{ci} / r_w}\,, \; \bar{\boldsymbol{u}}_i \equiv \frac{\boldsymbol{u}_i}{v_{thi}}\,, \; \bar{\boldsymbol{u}}_e \equiv \frac{\boldsymbol{u}_e}{v_{the}}$$

where ψ_w is the flux function on the wall surface, $\omega_{ci} \equiv e|\psi_w| / (m_i r_w^2)$, and $v_{th\alpha} \equiv \sqrt{2T_\alpha / m_\alpha}$ (α = i, e). The upper bar of normalized quantities is omitted from now on for simplicity.

The contours of ion distribution $f_i\,(r, v_r\, v_\theta)$ are presented in Figs. 2 and 3. In Fig. 2, contours of (r, $v_r = 2.5 v_{thi}$, v_θ) are drawn in $r - v_\theta$ plane. It appears that the initial distribution is changed abruptly and two peaks of distribution function are observed at $t = 3/\omega_{ci}$. The radial path length of ions with $v_r = 2.5 v_{thi}$ for the duration of $3/\omega_{ci}$ is about 0.45 m, which is comparable to the wall radius. Therefore, the calculation time is

Table 1 Geometry and plasma parameters

Parameters	Symbols	Values
External magnetic field	B_{ex}	0.05 T
Plasma temperature	$T_{tot} = T_i + T_e$	150 (100+50) eV
Reference number density	n_0	5.0×10^{19} m^3
Wall radius	r_{ci}	0.4 m

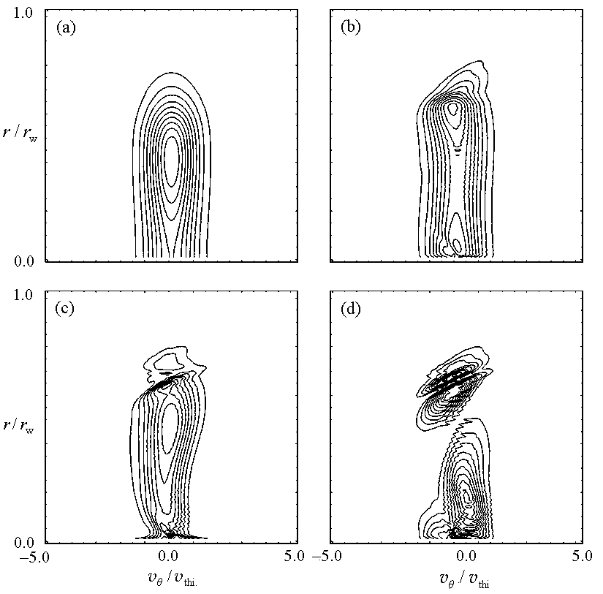

Fig. 2 Contours of the ion distribution function at (a)t = 0, (b)1ω_{ci}, (c)2/ω_{ci}, and (d)3/ω_{ci}. The figures are drawn in $v_\theta - r$ plane, and the radial velocity v_r is fixed at 2.5v_{thi}.

long enough for the plasma to transform the velocity distribution. The distortion in f_i (r, v_r v_θ) around the separatrix $r \approx 0.6r_w$ is found significant. It is found from Fig. 3, the distribution deviates from the isotropic Maxwellian. Hence ion dynamics in high-beta plasmas, in particular the plasmas with the field-null point, can cause a behavior that is never predicted by an MHD model.

Radial profiles of various physical quantities B_z, j_θ, n, E_θ, E_r, u_{ir}, $u_{i\theta}$, $u_{e\theta}$, are shown in Fig. 4. Results from the 1D Grad-Shafranov equation and the fluid equation of motion

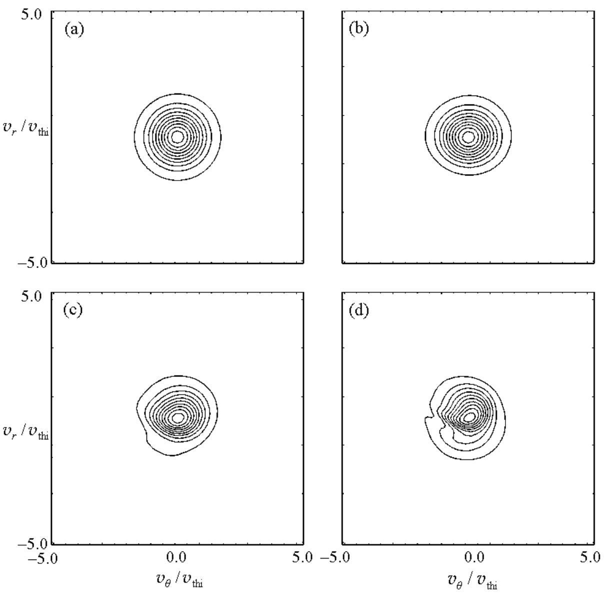

Fig. 3 Contours of the ion distribution function at (a)t = 0, (b)1ω_{ci}, (c)2/ω_{ci}, and (d)3/ω_{ci}. The figures are drawn in $v_\theta - v_r$ plane, and the radial position r is fixed at 0.4r_w.

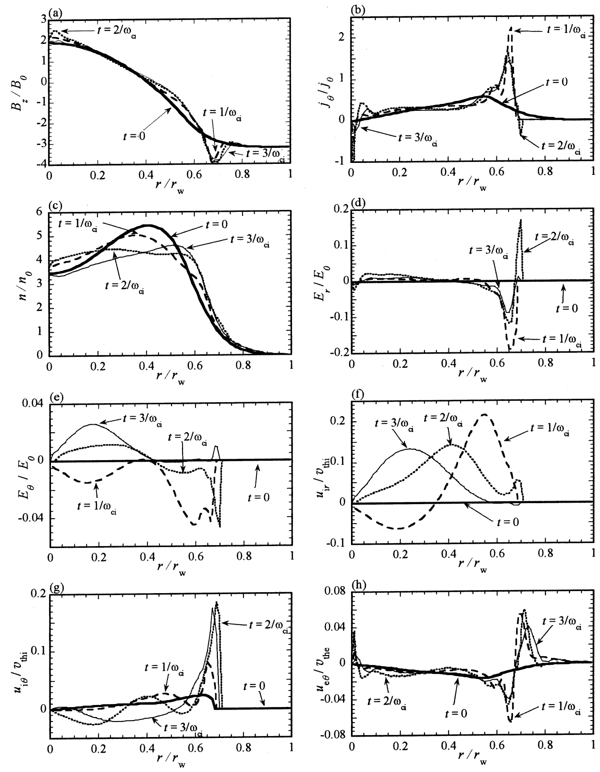

Fig. 4 Radial profiles of (a)B_z, (b)j_θ, (c)n, (d)E_r, (e)E_θ, (f)u_{ir}, (g) $u_{i\theta}$, and (h)$u_{e\theta}$ at t = 0 (solid line), 1/ω_{ci} (dashed line), 2/ω_{ci} (dotted line), and 3/ω_{ci} (thin solid line). Here, $B_0 \equiv |\psi_w|/r_w^2$, $j_0 \equiv en_0 v_{thi}$, $E_0 \equiv |\psi_w|\omega_{ci}/r_w$.

are for t = 0. Figure 4(f) shows that ions start to move inward inside the field-null surface and outward outside the surface, which flattens the density profile (cf. Fig. 4(c)). Consequently, the density gradient is increased at the separatrix compared with the initial profile. The azimuthal electric field E_θ is generated due to this radial ion flow, which is identical to the $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity (see Figs 4(e) and (f)). It appears that the generated radial electric field E_r by the effect of the Hall term is localized near the separatrix; the field rotates the plasma ions. According to Spivey's work, the only shifted-Maxwellian distributions that satisfy the Vlasov equation for systems with cylindrical symmetry are rigid rotors $u_{i\theta} = r\omega_{i\theta}$ with constant temperature [1]. The present calculation shows that the velocity distribution is gradually deviated from the initial shifted-Maxwellian, and thus the ion's rotation velocity profile is different from the rigid rotors. Moreover, inclusion of the plasma wall interaction in a calculation model can slow down the rotation velocity of plasma ions.

4. Conclusions

The kinetic simulation for an infinitely long cylindrical high-beta plasma with field-null surface has been carried out. The Vlasov equation is solved in order to study finite Larmor radius effect of ions near the field-null region and the fluid equation is used for electrons. The ion distribution function and the radial profiles of macroscopic physical quantities have been shown. It has been found that the velocity distribution is deviated from the initial shifted-Maxwellian distribution. Therefore, the kinetic simulation is necessary to predict the high-beta plasma behavior. The electric field has been found to generated, which results from the ion drift motion and the effect of the Hall term.

The gross stability of FRC will be studied with a use of the present hybrid code. It is not easy, however, to obtain the velocity distribution functions that are described by the variables in the six-dimensional phase space. If the finite difference method is applied, the number of grid meshes becomes enormous. The effective technique to reduce the number of grid meshes in the velocity space significantly will be needed.

References

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