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Cyclotron-Resonance Accelerations by a Generalized EM Wave

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Abstract

Particle accelerations by a one-dimensional, electromagnetic (EM), dispersive pulse propagating along an external magnetic field are investigated. As the pulse amplitude increases, cyclotron-resonance accelerations (CRA) evolve to nonlinear-trapping, and cause particle reflections. Both the amplitude and dispersion strongly affect those accelerations, revealing interesting phenomena of resonance bifurcations.

Keywords:

cyclotron-resonance, bifurcation, acceleration, electromagnetic wave, dispersion effect

1. Introduction

Accelerations of charged particles by an EM wave/pulse have been studied extensively [1]. Here, particle accelerations by a one-dimensional (1D), EM, dispersive wave packet in a magnetic field are studied. The pulse has a Gaussian profile with the following generalized linearly polarized electric field (and its self-consistent magnetic field) directed perpendicular to the z-axis along which it propagates: $E_x(z, t) = E_0 e^{-\{(z - v_g t)/l\}^2 + i(k_0 z - \omega_0 t + \theta)}$; here, E_0, z, v_g, t, l are the amplitude, the position, the central group-velocity, the time, and a measure of the pulse-length, respectively, and θ is the phase constant; furthermore, $k_0(\omega_0)$ is the wave number (angular frequency) of the carrier wave. If the acceleration is relatively small, the beam-like particles with only longitudinal initial velocities v0 experience the following perpendicular velocity shifts [1],

$$\Delta v_\perp = \frac{\sqrt{\pi}}{2}\frac{|qE_0|\Delta t}{m\gamma_0}\left[e^{-\{\omega_0 + \Omega/(\alpha\gamma_0)\}^2 \Delta t^2/2} + e^{-\{\omega_0 - \Omega/(\alpha\gamma_0)\}^2 \Delta t^2/2} + 2\cos(2\theta)\, e^{-\{\omega_0 + \Omega/(\alpha\gamma_0)\}^2 \Delta t^2/4 - \{\omega_0 - \Omega/(\alpha\gamma_0)\}^2 \Delta t^2/4}\right]^{1/2} \tag{1}$$

with $\alpha = 1 - v_p/v_0$, $\Delta t = l/(\beta v_p)$, $\beta = |(v_0 - v_g)/(v_0 - v_p)|$ and $\gamma_0 = (1 - v_0^2/c^2)^{-1/2}$.

Other notations are standard. The phase velocity is defined as $v_p = \omega_0/k_0$. Among dispersion effects only $v_g \neq v_p$, i.e., $v_p = 0.1c$ and $v_g = 0.05c$, will be considered, assuming that the time-scales for other effects are sufficiently longer than those for accelerations. The assumption should be particularly valid for stationary pulses like solitons. We will present below some numerical solutions to the equation of motion of electrons [1], and analyze them, increasing E_0.

2. Results

2.1 Linear cyclotron-resonance

We start with a linear case, which may be well described by Eq. (1). Fig. 1 depicts among others velocity shifts of electrons with initial velocity v_0 after penetrating an EM wave packet with $E_n = eE_0/mc\omega_0 = 0.001$ and $l_n = l/(c/\omega_0) = 2.0$.

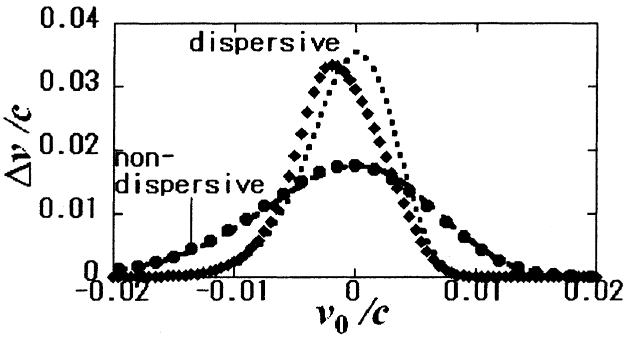

Fig. 1 Perpendicular velocity shifts of electrons after interaction with a linear pulse are plotted as a function of v_0. The dashed curve and circles respectively show analytical and numerical solutions for final perpendicular velocity-shifts due to a *nondispersive* wave packet with $v_p = v_g = 0.1c$, respectively, which are in excellent agreement with each other. The dotted curve shows analytical perpendicular velocity shifts (1), while diamonds depict numerical ones for a dispersive wave packet with $v_p = 2v_g = 0.1c$.

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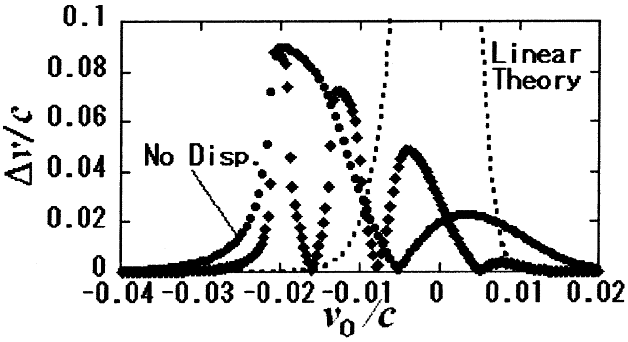

Fig. 2 Perpendicular electron velocity-shifts after interaction with a weakly nonlinear pulse are plotted as a function of v_0. The perpendicular velocity shifts due to the nondispersive wave packet shown here by squares demonstrate a bifurcation of cyclotron resonance. Other symbols are the same as in Fig. 1.

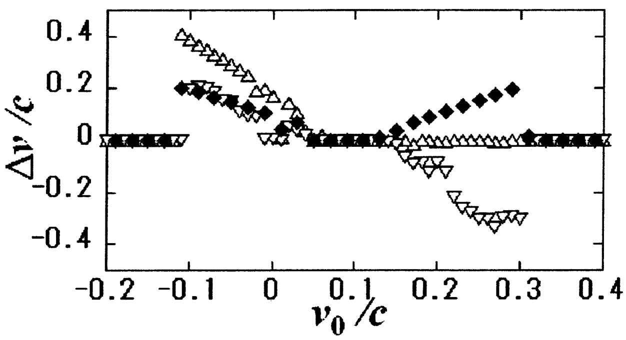

Fig. 3 Parallel and perpendicular electron velocity-shifts due to a nonlinear pulse are plotted as a function of v_0. Parallel accelerations are generally stronger than perpendicular ones

Since the magnetic field strength will be held constant at $\Omega_e = \omega_0$, the electrons with $v_0 \approx 0$ are cyclotron-resonant. Overall agreement between the theory and numerical results is rather good. The reasons for the small discrepancies in the dispersive case are small relative velocities between the particles and the pulse as well as the dispersion. The numerical results indicate the absence of θ dependence (not shown). Perpendicular acceleration is dominant for linear pulses. Hence, this is a transit-type acceleration, through which particles penetrate the entire wave packet. In this velocity range the dispersion increases accelerations and reduces the resonance width by factor of two, which may be analytically reduced from the factor $1/\beta = 2$ in Eq. (1) [1].

2.2 Weakly nonlinear cyclotron-resonance

Fig. 2 shows final velocity shifts of electrons for a weakly nonlinear wave packet with $E_n = 0.01$. Here, evidently the linear theory hardly accounts for the numerical values, which show much broader acceleration region as well as multi-peaks. The dispersion slightly reduces the resonance width, while the maximum velocity shifts remain hardly affected. However, the most outstanding dispersion effect is the further bifurcation of the resonance peaks. The strongest acceleration occurs at $v_0 = -0.02c$, while that in Fig. 1 occurs at $v_0 \approx 0$. At this stage the θ dependence is still absent (not shown). Owing to the relatively small longitudinal accelerations (not shown), the particles traverse the entire wave packet, while being accelerated.

2.3 Nonlinear cyclotron-resonance

Fig. 3 presents the electron velocity-shifts due to a nonlinear dispersive wave packet with $E_n = 0.1$. Diamonds depict maximum perpendicular velocity-shifts, while the upward and downward triangles show maximum and minimum parallel ones (and significant θ dependence), respectively. It is observed that strongly accelerated particles are reflected by the wave packet. This case differs from the previous ones dramatically. Its overall appearance is reminiscent of quasi-trapping (QT) [1]. However, QT by an EM pulse is expected [1] only at $E_n > (v_0 - v_p)/c > -E_n$, while Fig. 3 shows that accelerations are realized approximately at $2E_n > (v_0 - v_p)/c > -2E_n$. In addition, at $0.15 > v_0/c > 0.05$ net accelerations are suppressed. It turns out that at $0.2 > v_0/c > 0$ the cyclotron resonance condition cannot be satisfied. Then, what is the acceleration mechanism present at $0.2 > v_0/c > 0.15$ and $0.05 > v_0/c > 0$?

2.4 Phase-trapping and nonresonant cyclotron acceleration

The shift of CRA to nonlinear trapping may be explained in terms of phase trapping [2], which is an interaction mechanism applied to whistler waves and more recently to EM ion cyclotron waves; these waves are right-circularly polarized EM waves with $l = \infty$. For particles interacting with such a wave there exists a second Hamiltonian H_2 [2]:

$$H_2 = \frac{1}{2}\left(\gamma\frac{V}{c}\cos\alpha + \frac{\Omega_e}{k_0 c}\right)^2 + \gamma\frac{\omega_0}{k_0 c}\frac{B_1}{B_0}\frac{V}{c}\sin\alpha\cos\theta$$

Here, $V = |v_0 - v_p|$, γ is the Lorentz factor of the particle, B_1 is the magnetic field amplitude of the wave, α is the pitch angle of the particle measured in the wave frame, i.e., $v_\perp/(v_z - v_p) = \tan\alpha$. The lowest values of H_2 are given when $\theta = \pi$. Therefore, H_2 with $v_0 = 0$, $V = 0.1c$ and $\theta = \pi$ is plotted in Fig. 4 with various values of E_n. The electrons which start from $\cos\alpha = -1$, i.e., cyclotron resonance [2] may travel along the dashed line until they hit a corresponding curve, where they are reflected. In Fig. 4 and H_2 it is evident that as E_n is increased the resonance center shifts from $\cos\alpha = -1 (v_0 = 0)$ toward $0 (v_0 = v_p)$ and the width broadens considerably. Namely, the overall trend of electron velocity-shifts in Fig. 3 may be at least qualitatively explained in terms of the phase trapping.

3. Conclusions

As the pulse amplitude increases and/or the pulse is made more dispersive, the cyclotron resonance bifurcates,

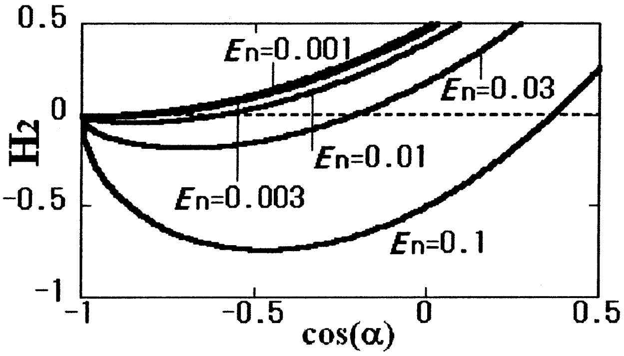

Fig. 4 The second Hamiltonian H_2 is plotted as a function of $\cos\alpha$ for E_n = 0.001, 0.003, 0.01, 0.03, and 0.1 (from top to bottom, respectively). The dashed line depicts $H_2 = 0$.

showing multi-resonance velocities, and finally transforms to phase trapping [2] characterized by large velocity shifts and broad resonance widths. Also, as the amplitude of wave packet increases, the transit-type CRA is transformed to the reflection-type phase-trapping, and the center of the resonance velocity consequently shifts to vp. It is likely that these bifurcations are caused by the finite extent of wave packet and the phase trapping.

References

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