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Numerical Studies of Magnetosonic Waves in Thermal-Equilibrium, Multi-Ion-Species Plasmas

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Abstract

Magnetosonic waves propagating perpendicular to an external magnetic field are studied with attention to the effect of multiple ion species. First, power spectra of magnetic field fluctuations and autocorrelation functions in thermal equilibrium plasmas are numerically obtained. In a multi-ion-species plasma, besides $\omega \simeq kv_A$ mode, numerous waves are present near many different ion cyclotron frequencies. The autocorrelation function of the quasi-mode consisting of these waves is not recovered to its initial value, owing to the phase mixing of these waves. Next, with particle simulations, evolution of a macroscopic perpendicular disturbance is investigated. In a multi-ion-species plasma, this disturbance is damped. The energy is transferred to from the magnetic field to the ions.

Keywords:

magnetosonic waves, multi-ion-species plasma, wave damping, energy transport

1. Introduction

The presence of multiple ion species introduces many interesting effects on magnetosonic waves [1-7]. For instance, in a two-ion-species plasma, the magnetosonic wave is split into two modes. Nonlinear pulses of these modes are damped, even when they propagate perpendicular to the magnetic field [5-7]. The damping is due to energy transfer from the pulse to heavy ions [8,9]. Periodic waves are not damped even in this case. However, the collective behavior of these waves in a multi-ion-species plasma would be different from that in a single-ion-species plasma.

Recently, a study has been made on collective behavior of ion Bernstein waves in thermal-equilibrium plasmas with multiple ion species [10]. Each perpendicular ion Bernstein wave with $\omega \simeq n\Omega_i$ is undamped in a collisionless plasma [11], where Ω_i is the ion cyclotron frequency and n is the integer. In a single-ion-species plasma, the autocorrelation function of the quasi-mode consisting of these waves shows periodic behavior with time period $2\pi/\Omega_i$. On the other hand, in a multi-ion-species plasma, the autocorrelation function is initially damped and is not recovered. This is caused by the phase mixing of numerous waves excited at the harmonics of many different ion cyclotron frequencies. This damping mechanism could be important in space plasmas where many ion species exist, with each species having many different ionic charge states.

In this paper, we study perpendicular magnetosonic waves in thermal-equilibrium, multi-ion-species plasmas where each particle species has its own Maxwellian velocity distribution. We assume that all the ion species have an equal temperature, while electrons can have a different temperature because relaxation time between electrons and ions via collisions is very long.

In Sec. **2**, we numerically calculate power spectra and autocorrelation functions of magnetic field fluctuations due to the magnetosonic waves. In a single-ion-species plasma, the autocorrelation function is not damped, because the wave with $\omega \simeq kv_A$ is dominant mode. Here, v_A is the Alfvén speed and k is the perpendicular wavenumber. On the other hand, in a multi-ion-species plasma, besides this mode, numerous waves are present near many different ion cyclotron frequencies. Owing to the phase mixing of these waves, the autocorrelation function does not return to its initial value. In Sec. **3**, evolution of a macroscopic disturbance and associated energy transport are studied by particle simulations. In a multi-ion-species plasma, the macroscopic disturbance is damped, and the energy is transferred from the magnetic field to the ions.

2. Numerical calculation

We consider magnetosonic waves propagating perpendicular to an external magnetic field in a spatially homogeneous, thermal equilibrium plasma with a temperature T. The dispersion relations of magnetosonic waves are given by

$$D_{ms} \equiv \varepsilon_{xy}^2/\varepsilon_{xx} + \varepsilon_{yy} - c^2k^2/\omega^2 = 0, \tag{1}$$

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where c is the light speed, and ε_{xx}, ε_{xy}, and ε_{yy} are defined as

$$\varepsilon_{xx} = 1 - \sum_j \sum_n \frac{\omega_{pj}^2}{\omega(\omega - n\Omega_j)} \frac{n^2}{\mu_j} \Gamma_n(\mu_j), \tag{2}$$

$$\varepsilon_{xy} = -\mathrm{i} \sum_j \sum_n \frac{\omega_{pj}^2}{\omega(\omega - n\Omega_j)} n\Gamma'_n(\mu_j), \tag{3}$$

$$\varepsilon_{yy} = \varepsilon_{xx} + \sum_j \sum_n \frac{2\omega_{pj}^2}{\omega(\omega - n\Omega_j)} \mu_j \Gamma'_n(\mu_j). \tag{4}$$

Here, the subscript j refers to electrons (e) or ion species (H, He, C, ···), Ω_j is the cyclotron frequency, and ω_{pj} is the plasma frequency. Also, $\Gamma_n(\mu_j) = \mathrm{I}_n(\mu_j)\exp(-\mu_j)$, where I_n is the modified Bessel function of the nth order, and $\mu_j = k^2\rho_j^2$ with ρ_j the gyro-radius.

The fluctuation spectrum of magnetic fields due to the magnetosonic waves is written as

$$\frac{|B_{k,\omega}|^2}{8\pi} = \sum_n P(\omega)\delta(\omega - \omega_n), \tag{5}$$

with

$$P(\omega) = \frac{\pi k_{\mathrm{B}} T}{\omega \frac{\partial}{\partial \omega} D_{ms}(k,\omega)\Big|_{\omega=\omega_n}} \frac{\varepsilon_{xy}^2}{\varepsilon_{xx}} \frac{c^2 k^2}{\omega^2}, \tag{6}$$

where k_{B} is the Boltzmann constant, and ω_n is the roots of the dispersion relation $D_{ms} = 0$.

In a multi-ion-species plasma with H being major ions, there are three kinds of waves in the long wavelength region, $\mu_i \ll 1$;

$$\omega \simeq kv_A, \tag{7}$$

$$\omega \simeq \Omega_s + \frac{\omega_{ps}^2 \Omega_H}{\omega_{pH}^2 \Omega_s}(\Omega_H - \Omega_s), \tag{8}$$

and

$$\omega \simeq n\Omega_i. \tag{9}$$

Here, the subscript s in eq. (8) refers to heavy ion species (He, C, O, ···). The waves with eqs. (7) and (8) exist even in a cold plasma [4], while the mode with eq. (9) is caused by ion kinetic effects [12].

For a given wavenumber k, there are many waves with different frequencies. Autocorrelation function of the quasi-mode consisting of these waves is obtained from $P(\omega)$ though the Fourier transformation in ω as

$$C_k(\tau) = \int_{-\infty}^{\infty} |B_{k,\omega}|^2 \exp(-\mathrm{i}\omega\tau)\mathrm{d}\omega. \tag{10}$$

We pay attention to how values of $|C_k(\tau)|$ are reduced by the presence of multiple ion species. The reduction of $|C_k(\tau)|$ indicates that energy transport can be enhanced [13].

We numerically calculate specific values of $P(\omega)$ and $C_k(\tau)$ for three different plasmas and compare them. [In the calculation, we retain the terms from $n = -10$ to 10 for the ions and the $n = 0$ and 1 terms for the electrons in eqs. (2), (3) and (4).] The three plasmas that we consider are single-ion (H^+), three ion (H^+, He^{+2}, C^{+5}) and six-ion (H^+, He^{+2}, C^{+6}, O^{+6}, Si^{+9}, and Fe^{+13}) species plasmas. The cyclotron frequencies of these ions normalized to Ω_H are taken to be $\Omega_{He} = 0.5$, $\Omega_C = 0.417$, $\Omega_O = 0.375$, $\Omega_{Si} = 0.321$, and $\Omega_{Fe} = 0.232$. The densities of the ions normalized to n_H are $n_{He} = 0.1$, $n_C = n_O = 0.01$, and $n_{Si} = n_{Fe} = 0.005$. The magnetic field strength is $|\Omega_e|/\omega_{pe} = 1$. The plasma beta value is $\beta = 0.0625$.

Figure 1 shows power spectra of the mode with $k\rho_H = 0.1$ in the single-, three, and six-ion-species plasmas, where $P(\omega)$ is normalized to $\pi k_B T$. In the single-ion-species plasma, the wave with $\omega \simeq kv_A$ ($\simeq 0.42\Omega_H$) is the dominant mode. Even though there are waves with $\omega \simeq n\Omega_H$, their amplitudes are quite small. In the three-ion-species plasma, besides the $\omega \simeq kv_A$ ($\simeq 0.39\Omega_H$) mode, the waves near Ω_{He} and Ω_C are present; these frequencies are given by eq. (8) with s = He or C. The amplitudes of the waves with $\omega \simeq n\Omega_H$, $n\Omega_{He}$, and $n\Omega_C$ are much smaller. In the six-ion-species plasma, the waves near Ω_O, Ω_{Si}, and Ω_{Fe} also exist. Their amplitudes are not small, although the abundances of the heavy ions are very small.

Figure 2 shows time variations of autocorrelation functions normalized to their initial values $C_k(0)$. In the single-ion-species plasma, $C_k(\tau)$ oscillates with the period $2\pi/(kv_A)$ and is undamped. In the three- and six-ion-species plasmas, $C_k(\tau)$'s do not return to their initial values till the end of the calculation. As the number of ion species increases, the amplitude of the oscillation decreases more quickly, owing to the phase mixing of more waves.

If effects of collisions are entirely neglected, $C_k(\tau)$ returns to its initial value at the time of the least common multiple of all the wave periods. However, this time is extremely long in space plasmas where the number of ion species is very large (moreover, each ion species has many different ionic charge states) and numerous waves exist. On such a long time scale, the effects of collisions must be important, which reduces $|C_k(\tau)|$. Accordingly, $C_k(\tau)$ would

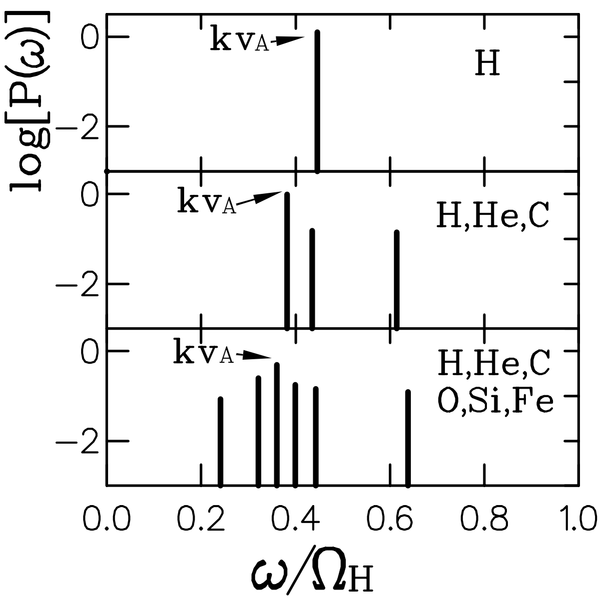

Fig. 1 Power spectra of magnetic field fluctuations with $k\rho_H = 0.1$ in three different plasmas.

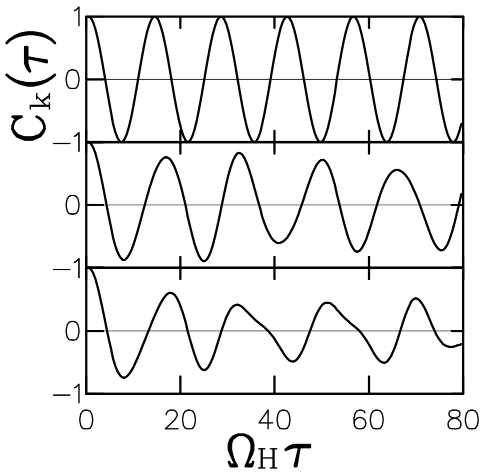

Fig. 2 Autocorrelation functions of the fluctuations with $k\rho_H$ = 0.1 in the same plasmas as shown in Fig. 1.

not be recovered and would keep smaller values than in a single-ion-species plasma.

3. Particle simulations

By means of a one-dimensional (one space and three velocity components), electromagnetic particle code with full ion and electron dynamics, we study collective behavior of magnetosonic waves in a multi-ion-species plasma. The system size is $L_x = 512\Delta_g$, where Δ_g is the grid spacing and is equal to the electron Debye length. We use periodic boundary conditions. The external magnetic field is in the z direction, and its strength is $|\Omega_e|/\omega_{pe} = 4.0$. The total numberof electrons is N_e = 262, 144. The plasma β value is $\beta = 0.03$.

We simulate single-ion (a) and four-ion (a, b, c and d) species plasmas. We choose the mass ratios as $m_a/m_e = 50$, $m_b/m_a = \sqrt{3}$, $m_c/m_a = 2$, and $m_d/m_a = \sqrt{5}$. In order to see the effect of multiple ion species with a small number of ion species, we have taken the irrational ion mass rations for b and d ions. The charges are the same, $q_a = q_b = q_c = q_d = |q_e|$. The ion densities are set to be $n_b = n_c = n_d = 0.2n_a$.

Firstly, we observed that autocorrelation functions of fluctuations propagatig perpendicular to the external magnetic field are not recovered in the four-ion-species plasma. Next, as a initial condition, we set the magnetic field to have a finite amplitude disturbance with a monochromatic cosine profile, $\delta B_z(x)/B_0 = 0.02\cos(k_0x)$, where B_0 is the external magnetic field and $k_0\rho_a = 0.01(k_0v_A = 0.68\Omega_a)$. We then study evolution of its disturbance and associated energy transport. Initially, all the ion species have equal temperature; the electron-to-ion temperature ratio is chosen to be $T_i/T_e = 0.1$.

Figure 3 shows time variations of total magnetic-field energy E_B and ion kinetic energy $K - K_0$, where K is the total energy of all the ions and K_0 is the initial one. The energies are normalized to $m_ev_{Te}^2$. The thin and thick lines denote energies in the single- and four-ion-species plasmas, respectively. In the single-ion-species plasma, the magnetic field energy and ion kinetic energy oscillate with period

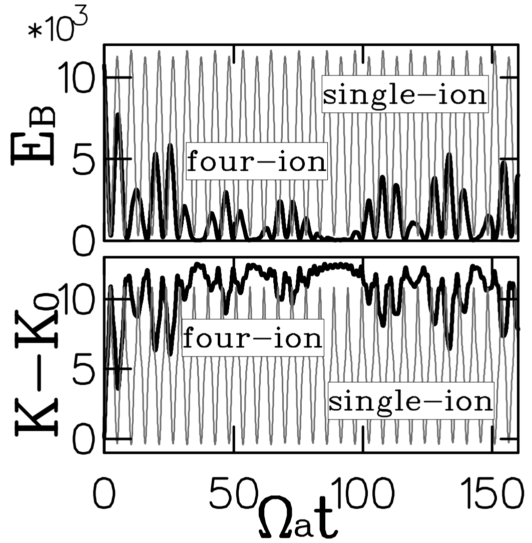

Fig. 3 Time variations of total magnetic-field energies and ion kinetic energies. The thin and thick lines represent single- and four-ion-species plasmas, respectively.

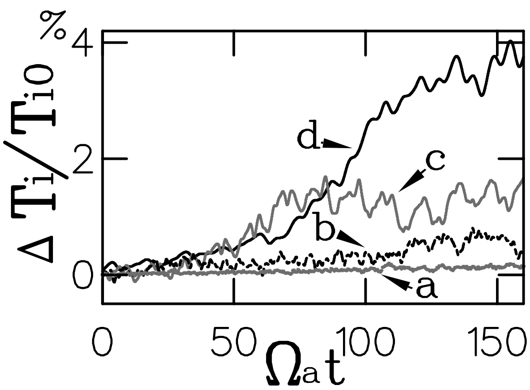

Fig. 4 Time variations of ion energies in the four-ion-species plasma.

$\pi/(k_0v_A)$. On the other hand, in the four-ion-species plasma, the magnetic field energy is reduced and does not return to its initial value. The ion kinetic energy is rapidly increased and then keeps large values.

Figure 4 shows time variations of ion energies, $\Delta T_i = T_i - T_{i0}$, where T_i is defined as

$$T_i = \int \mathrm{d}\boldsymbol{x} \int \mathrm{d}\boldsymbol{v} m_i f_i(\boldsymbol{x},\boldsymbol{v})(\boldsymbol{v} - <\boldsymbol{v}_i(\boldsymbol{x})>)^2, \tag{11}$$

with $<\boldsymbol{v}_i(\boldsymbol{x})>$ the fluid velocity at position $\boldsymbol{x}$, and T_{i0} is the initial value of T_i. The energy of the major light ions T_a does not almost change, while those of heavier ions are increased. The heaviest ions have the greatest energy increase.

4. Summary

We have studied collective behavior of perpendicular magnetosonic waves in multi-ion-species plasmas. We have numerically shown that the autocorrelation functions in a thermal-equilibrium plasma are not recovered, because, in addition to the $\omega \simeq kv_A$ mode, many waves exist near many different cyclotron frequencies. Furthermore, we have shown with particle simulations, that the macroscopic disturbance is also damped in a multi-ion-species plasma, and that the energy is transferred from the magnetic field to the ions. Associated with this evolution, the energies of heavy ions are increased. Although it seems plausible that this energy transfer is related to the damping of the autocorrelation functions, we have not yet understood the transfer mechanism. To do this, we will further investigate with particle simulations, for example, how the energies depend on the initial conditions or on the ion species.

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