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Two-Dimensional Kinetic Model for Fast Electron Transport in Compressed Core Plasmas

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Abstract

A Fokker-Planck type two-dimensional transport code for relativistic electrons in dense plasmas has been developed. Being written in cylindrical coordinates with axial symmetry, the code can be used for analyzing the core-plasma heating in spherical targets in fast ignition scheme. The feature of energy deposition of beam electrons injected into cylindrical D-T plasma is examined. The contribution of self-generated electromagnetic field to core heating profile is estimated.

Keywords:

fast ignition, relativistic electron beam, energy deposition, Fokker-Planck calculation, pinch by magnetic field, CIP-IDO scheme

1. Introduction

In the so-called fast ignition (FI) approach to inertial confinement fusion, an high intensity short-pulse laser is focused on a pre-compressed pellet to generate a beam of relativistic electrons which should heat the fuel to ignition temperature. [1] Thus, the study of electron propagation and energy deposition in dense plasma is essential in this scheme.

The propagation of fast electrons to the dense core region, as well as laser-plasma interaction, has been actively studied by means of particle simulations. On the other hand, little is reported about the energy deposition process in the dense core plasma. Since the collisions between the fast electrons and the bulk plasma particles become significant in the dense core region, a Fokker-Planck (FP) type calculation would be appropriate for analyzing the core plasma heating.

Recently we developed a FP type transport model that can calculate the time- and space-dependent energy deposition rate of fast electrons in dense plasma. [2] The collision terms includes (a) short-range ($b < \lambda_D$) binary collisions with the plasma particles and (b) long-range ($b > \lambda_D$) collective effect, *i.e.* collisions between screened particles. The effect of self-generated electric field is taken into account by assuming a current neutrality condition. So far, however, the code has been written for one-dimensional (1-D) planer coordinate system; the effect of magnetic field has not been included. For more realistic examination of the core heating process, multi-dimensional (at least two-dimensional (2-D) in coordinate space) calculations are indispensable.

In this paper, we first extend our 1-D kinetic transport model to 2-D one; the code is written in cylindrical coordinates with axial symmetry. Next, we examine the feature of energy deposition of fast electron beam injected into stationary D-T plasmas. We also estimate the contributions of the self-generated electric and magnetic fields to the energy deposition profile.

2. Analysis model and calculation scheme

A relativistic FP-type transport equation is necessary for analyzing the behavior of fast electrons. A general form of the relativistic FP collision term was derived by Braams and Karney. [3] Their collision term, however, is too complicated to be used for numerical study. To reduce it to a simple and tractable form, Nakashima and Takabe [4] assumed that $|\boldsymbol{u} - \boldsymbol{u}_j| \approx |\boldsymbol{u}|$, where $\boldsymbol{u} \equiv \boldsymbol{p}/m_e$, $\boldsymbol{u}_j \equiv \boldsymbol{p}_j/m_j$ ($j = e, i$), $\boldsymbol{p}$ (or $\boldsymbol{p}_j$) is the momentum and m_e (or m_i) is electron (or ion) rest mass.

In this study, we also adopt the same approximation as in ref. 4; the collision term is reduced to a simple (linearized) form. Furthermore, we extend it so as to include the long-range collective effect. [2]

In the present analysis, we consider the 2-D (r-z) cylindrical coordinate system (Fig. 1). Then, the relativistic FP-type transport equation is expressed as

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$$\begin{aligned}
&\frac{\partial f}{\partial t}+\frac{p}{\gamma m_e}\left[\mu\frac{\partial f}{\partial z}+\sqrt{1-\mu^2}\cos\omega\frac{\partial f}{\partial r}-\frac{\sqrt{1-\mu^2}}{r}\sin\omega\frac{\partial f}{\partial\omega}\right]\\
&\quad+\left[F_z\mu+F_r\sqrt{1-\mu^2}\cos\omega\right]\frac{\partial f}{\partial p}-F_r\frac{\sin\omega}{p\sqrt{1-\mu^2}}\frac{\partial f}{\partial\omega}\\
&\quad+\left[F_z\left(1-\mu^2\right)-F_r\sqrt{1-\mu^2}\cos\omega\right]\frac{1}{p}\frac{\partial f}{\partial\mu}\\
&=\left(\frac{Y_e n_e}{m_e}+\frac{Y_i n_i}{m_i}\right)\frac{m_e^2}{p^2}\frac{\partial}{\partial p}\left(\gamma^2 f\right)\\
&\quad+\frac{1}{2}\left(Y_e n_e+Y_i n_i\right)\frac{m_e}{p^3}\left[\frac{\partial}{\partial\mu}\left\{\gamma\left(1-\mu^2\right)\frac{\partial f}{\partial\mu}\right\}+\frac{\gamma}{1-\mu^2}\frac{\partial^2 f}{\partial\omega^2}\right]+S
\end{aligned}\tag{1}$$

with

$$Y_j=4\pi\left(\frac{Z_j e^2}{4\pi\varepsilon_0}\right)^2\ln\Lambda\quad(j=e,i)\,,\tag{2}$$

$$F_z=(-e)\left(E_z+\frac{p\sqrt{1-\mu^2}\cos\omega}{\gamma m_e}B_\theta\right),\tag{3}$$

$$F_r=(-e)\left(E_r-\frac{p\mu}{\gamma m_e}B_\theta\right),\tag{4}$$

where $f(r, z, p, \mu, \omega, t)$ is the distribution function of fast electrons, γ is the Lorentz factor, n_e (or n_i) is the number density of the bulk electrons (or ions), $S(r, z, p, \mu, \omega, t)$ is the source term, μ is the directional cosine of the momentum vector $\boldsymbol{p}$ relative to z axis and ω is the angle between the planes formed by $\boldsymbol{p}$ and $\hat{z}$ vectors and by the $\hat{z}$ and $\hat{\boldsymbol{r}}$ vectors; $\hat{z}$ and $\hat{\boldsymbol{r}}$ are unit vectors in the z and r directions, respectively. The Coulomb logarithm $\ln\Lambda$ is extended so as to include contributions not only from short-range binary collisions but also from long-range collective effect [2]; *i.e.*

$$\ln\Lambda=\ln\Lambda_{binary}+\ln\Lambda_{collective}\,,\tag{5}$$

$$\ln\Lambda_{binary}=\frac{1}{2}\left\{\ln\frac{1}{2\tau_{min}}+\frac{1}{8}\left(\frac{\tau}{\tau+1}\right)-\frac{(2\tau+1)}{(\tau+1)^2}\ln 2+1-\ln 2\right\},\tag{6}$$

$$\ln\Lambda_{collective}=\frac{1}{2}\ln\left(\frac{2}{3}\frac{v^2}{\lambda_D^2\omega_p^2}\right)-\frac{1}{2}\tag{7}$$

with

$$\tau_{min}=(\Delta_{DB}/2)/\lambda_D\,,\ \tau=\gamma-1\,,$$

where v is the speed of the fast electron, λ_D is the Debye length, ω_p is the plasma frequency of the bulk electrons and Δ_{DB} is the reduced de Broglie wave length.

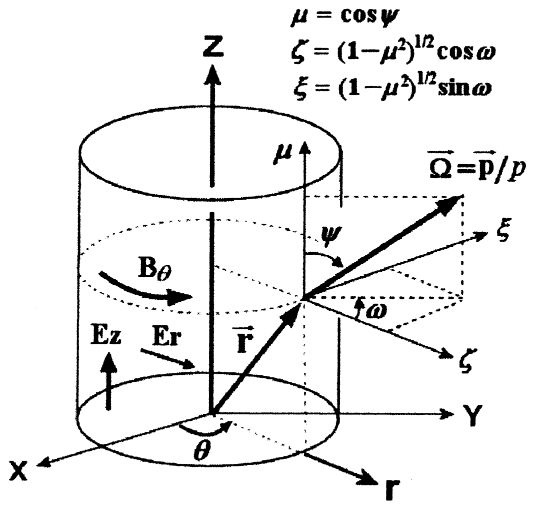

Fig. 1 2-D cylindrical coordinates system.

The self-generated electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are evaluated by means of the generalized Ohm's law, Ampere-Maxwell equation, Faraday's law and the equation for total current density $\boldsymbol{J} = \boldsymbol{J}_f + \boldsymbol{J}_b$ ($\boldsymbol{J}_f$: the current density of the fast electron, $\boldsymbol{J}_b$: the current density of the bulk electron). Neglecting some terms which are not essential for the present case, we obtain

$$\boldsymbol{E}=-\eta\boldsymbol{J}_f+\frac{\eta}{\mu_0}\nabla\times\boldsymbol{B}\,,\tag{8}$$

$$\frac{\partial\boldsymbol{B}}{\partial t}=-\nabla\times\left(\frac{\eta}{\mu_0}\nabla\times\boldsymbol{B}\right)+\nabla\times\left(\eta\boldsymbol{J}_f\right),\tag{9}$$

where η is the plasma resistivity, and μ_0 is the permeability of free space. Assuming rotational symmetry, we evaluate the fields $E_z(r, z)$, $E_r(r, z)$ and $B_\theta(r, z)$.

The energy deposition rate of fast electron consists of the energy loss rates $P_B + P_C$ due to binary collisions and collective effect, and the Joule heating rate P_J through return current, *i.e.* $P_{dep} = P_B + P_C + P_J$. These rates are evaluated by

$$\begin{aligned}
&P_B+P_C=\\
&\quad-\int m_e c^2(\gamma-1)\left[\left(\frac{Y_e n_e}{m_e}+\frac{Y_i n_i}{m_i}\right)\frac{m_e^2}{p^2}\frac{\partial}{\partial p}\left(\gamma^2 f\right)\right]\mathrm{d}p^3\,,
\end{aligned}\tag{10}$$

$$P_J=\eta\left(J_{Return,r}^2+J_{Return,z}^2\right),\tag{11}$$

where $J_{Return,r}$ (or $J_{Return,z}$) is the component of return current density in the r (or z) direction. The return current density is calculated by

$$\boldsymbol{J}_{Return}=-\boldsymbol{J}_f=-e\int\frac{\boldsymbol{p}}{\gamma m_e}f\,\mathrm{d}^3\boldsymbol{p}\,.\tag{12}$$

In the multi-dimensional FP calculation, the amount of calculations becomes enormous. Therefore, we have to select or develop an adequate calculation scheme, which saves a memory and a time for calculations and fulfills a good accuracy of calculations. As for the numerical solution for the present case, the fully implicit scheme is used for t. For the space (r, z) and the momentum (p) variables, the 3-D "explicit" cubic-interpolated propagation (CIP) scheme is adopted. The angular variables μ and ω are treated by means of 2-D "implicit" interpolated differential operator (IDO) scheme. By using the CIP-IDO scheme [5], which is based on Hermite interpolations of the profile for the dependent variables over a local area, accurate calculations are possible in spite of a relatively small number of calculation-meshes, and hence the shortening of a calculation time can be expected comparing with other schemes.

In the present calculation, the reflecting boundary condition is applied on the beam axis ($r = 0$), while the free boundary condition is used for the other spatial boundaries. The upper and lower limits of the momentum p space are treated by the free boundary condition. For the angular boundaries ($\mu = \pm 1$; $\omega = 0, \pi$), the reflecting boundary condition is used. Here, we assume that the distribution function f on an arbitrary coordinate (z, r, p, μ) is symmetric with respect to the r-z plane, *i.e.* $f(z, r, p, \mu, \omega, t) = f(z, r, p, \mu, -\omega, t)$, and hence make calculations only for the half region of ω (*i.e.* $0 \le \omega \le \pi$).

3. Results and discussion

To examine the energy deposition process of fast electrons in dense plasmas, we consider a stationary D-T cylindrical plasma (ρ = 300 g/cm^3, T = 0.5 keV), and calculate the transport and slowing down of mono-energetic (1 MeV) Gaussian (the half width at half maximum is 5 µm, the peak intensity is 10^{20} W/cm^2) relativistic electron beam injected steadily into it. The source beam is directed forward ($\mu \approx 1$) and isotropic with respect to ω. In this calculation, the size of a unit space cell is 1 µm × 1 µm. The momentum variable p is divided into 15 meshes, and the angular variables μ and ω are divided into 11 meshes and 16 meshes, respectively.

Figure 2 shows the spatial profile of energy deposition rate at t = 0.4 ps. Here, the calculations were made for the following three cases, *i.e.* Case (a) considering the electric and magnetic fields, Case (b) neglecting the effect of magnetic field and Case (c) neglecting the both fields. Figures 3 and 4 show the spatial profiles of self-generated electric field and magnetic field at the same time-step, respectively. The self-generated magnetic field grows about 250 Tesla at maximum (Fig. 4). Because of this strong magnetic field, the beam electrons are pinched toward the beam axis. Consequently, the energy deposition rate and the self-generated electric field are enhanced around the beam axis (see Fig. 2(a) and Fig. 3(a)). It is also shown that the self-generated electric field shortens the penetration of the beam electrons into the dense plasma. These effects would be favorable from the viewpoints of the efficient core plasma heating.

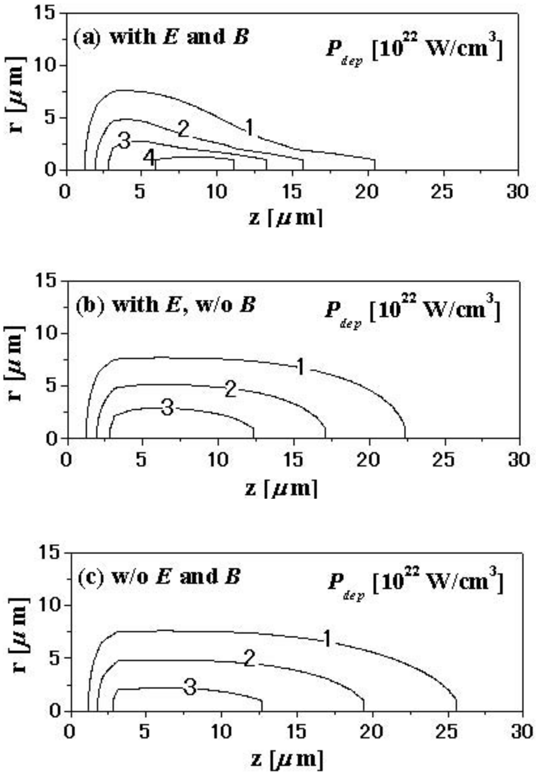

Fig. 2 Spatial profile of energy deposition rate at t = 0.4 ps. (at E_0 = 1 MeV, I = 10^{20} W/cm^3, ρ = 300 g/cm^3, T = 0.5 keV) (a) with electric and magnetic fields, (b) with electric field and without magnetic field and (c) without electric and magnetic fields.

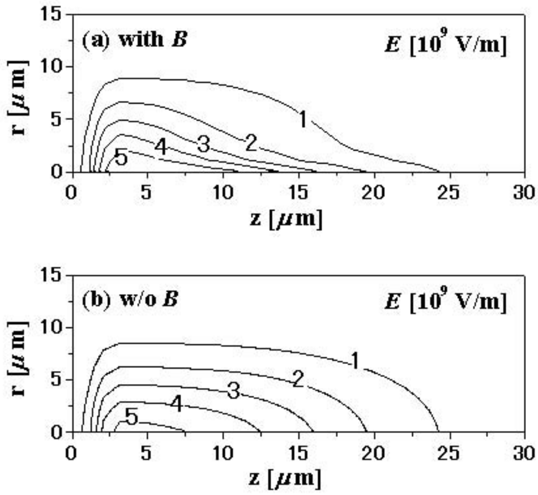

Fig. 3 Spatial profile of the self-generated electric field at t = 0.4 ps. (a) with magnetic field and (b) without magnetic field.

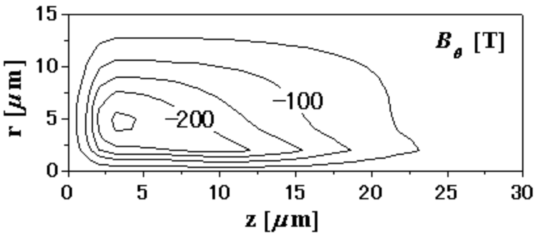

Fig. 4 Spatial profile of the self-generated magnetic field at t = 0.4 ps.

4. Concluding remarks

We have developed the 2-D FP-type transport code in cylindrical geometry and calculated the energy deposition profile of fast electron beams injected into a stationary dense D-T plasma. The code is possible to calculate precisely the time- and space-dependent heating rate (energy deposition rate) in dense plasma spheres. It has been confirmed that the self-generated electric field shortens the penetration of the beam electrons into the dense plasma, and that the self-generated magnetic field pinches the beam electrons, enhancing the energy deposition around the beam axis.

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