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Theoretical Modeling and Application of Microwave Reflectometry to Plasma Turbulence Study

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Abstract

We discuss a possibility to reconstruct turbulence fluctuation spectra from single channel microwave reflectometry data. The technique is based on the analytical solution of the full-wave equation of microwave propagation in the axially symmetric plasma. As an illustration, we apply our technique to estimate the fluctuation level and poloidal wavenumber spectrum shape in the ELM-free H mode plasma of the JT-60U tokamak.

Keywords:
plasma diagnostics, microwave reflectometry, fluctuation spectrum

1. Introduction

Microwave reflectometry has been studied theoretically and successfully applied to the measurements of plasma profile and fluctuations in a number of magnetic fusion devices [1, 2, 3, 4, 5]. The purpose of this paper is to develop a reflectometry based method of measurements of turbulence wavenumber spectra in tokamak plasmas and to apply the method to the JT-60U device. Our method does not require new hardware design and installation in addition to the conventional O-mode reflectometer, which is already in place on the JT-60U as well as on many other magnetic fusion devices.

2. Poloidal spectrum reconstruction

For the plasma frequency radial profile representable as

$$\omega_0^2(r) = \left(\frac{A}{r}\right)^2 + h \tag{1}$$

with the two constant parameters A and h, the analytical solution of the full-wave equation of microwave propagation and scattering in plasma was found in [6, 7] leading to the following expression for the simulated reflectometer signal fluctuations:

$$\begin{aligned}\delta e(t) = {} & \frac{\mathrm{i}\pi}{2}\frac{c^2}{\omega^2 - h}\int_{\sigma_s} P\,\mathrm{d}\sigma \sum_{n=0}^{\infty}\frac{\mathrm{H}_n^2(\xi)}{\mathrm{H}_n^2(\xi_a)} \\ & \times \mathrm{H}_{\nu_n}^2(\rho_a)\int_0^{\rho_a} J_{\nu_n}(\rho')\,\rho'\,\frac{1}{\pi c^2} \\ & \times \int_{-\pi}^{\pi}[\delta n\cos(n\theta)\cos(n\theta') + \sin(n\theta)\sin(n\theta')] \\ & \times E_0(\rho',\theta')\,\omega_0^2(\rho')\,\frac{\delta n(\rho',\theta',t)}{n_0(\rho')}\,\mathrm{d}\theta'\,\mathrm{d}\rho' .\end{aligned} \tag{2}$$

Here ω is the microwave frequency, fluctuations are considered to be small (Born approximation), electric field perturbations are integrated over the antenna surface σ_s, P is the antenna pattern, $\mathrm{H}_{\nu_n}^2$ and H_n^2 are the Hankel functions of the second kind, J_{ν_n} is the Bessel function, $\nu_n = \sqrt{n^2 + \frac{A^2}{C^2}}$, $\rho = \sqrt{\omega^2 - h}\,\frac{r}{C}, \xi = \omega r/c, \rho_a$ and ξ_a are the values of ρ and ξ at the edge of the plasma r_a, E_0 is the unperturbed value of microwave field, n_0 and δn are the plasma density and its fluctuation, $\delta n = 0.5$ for $n = 0$ and $\delta n = 1$ for other values of n. The unperturbed ω_0 function is assumed axially symmetric, while the density fluctuations δn depend on both spatial coordinates r, θ and time t. Variations of δn are assumed much slower than the microwave period $2\pi/\omega$. The analytical expression for the unperturbed electric field E_0 for the same assumption (1) about the shape of the plasma profile was found in [6].

In this paper we attempt to find density fluctuations δn such that the reflectometer signal simulated through the Eq. (2) closely reproduces the one obtained experimentally.

The plasma turbulence is represented as a superposition of the rotating modes, characterized by the poloidal and radial numbers (q, p) and yet unknown amplitudes $C_{q,p}, D_{q,p}$, which are the aim of our study:

$$\begin{aligned}\frac{\delta n(r,\theta,t)}{n_0} = {} & \sum_{q=-\infty}^{\infty} e^{2\pi\mathrm{i}q\,\frac{\theta - u_\theta t/r_c}{\Delta_\theta}} \\ & \times\left(C_{q,p}\,e^{2\pi\mathrm{i}p\,\frac{r}{\Delta_r}} + D_{q,p}\,e^{-2\pi\mathrm{i}p\,\frac{r}{\Delta_r}}\right).\end{aligned} \tag{3}$$

Here u_θ is the poloidal speed, r_c is the cut-off layer location, Δ_θ is the angle $u_\theta T/r_c$ covered by the rotating fluctuations within the time period T, Δ_r is the radial scale defined such

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as to cover all the radial wavenumbers of practical interest (normally, the same range of k_r with the poloidal wavenumbers k_θ).

Turbulent fluctuations are considered to be random functions in a sense that relative phases of different modes are random and (q,p) pairs are also random, different for different realizations of $\delta n(r,\theta,t)$. It is thus reasonable to search not for the particular realizations of $C_{q,p}$, $D_{q,p}$, which ideally reproduce the reflectometer signal on a given interval $0{:}T$, but instead, to search for the quantities $<|C_{q,p}|>$, $<|D_{q,p}|>$, averaged over many δn realizations of type (3), where the radial numbers p on each realization are randomly generated for each poloidal number q.

Substituting eq. (3) into (2), representing $\delta e(t)$ as Fourier series and taking into account the requirement of δn to be a real function ($C_{-q,-p} = C^*_{q,p}$, $D_{-q,-p} = D^*_{q,p}$), we obtain:

$$\begin{aligned} A_q &= R_{q,p} C_{q,p} + S_{p,q} D_{q,p} \\ A^*_{-q} &= R^*_{-q,-p} C_{q,p} + S^*_{-q,-p} D_{q,p}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} \begin{Bmatrix} R_{q,p} \\ S_{q,p} \end{Bmatrix} = & \frac{\mathrm{i}\pi}{2} \frac{c^2}{\omega^2 - h} \int_\sigma P \, \mathrm{d}\sigma \sum_{n=0}^{\infty} \frac{\mathrm{H}_n^2(\xi)}{\mathrm{H}_n^2(\xi_a)} \\ & \times \mathrm{H}^2_{\nu_n}(\rho_a) \int_0^{\rho_a} \mathrm{J}_{\nu_n}(\rho') \rho' \omega_0^2(\rho') \frac{1}{\pi c^2} \\ & \times \int_{-\pi}^{\pi} [\delta n \cos(n\theta) \cos(n\theta') + \sin(n\theta) \sin(n\theta')] \\ & \times E_0(\rho', \theta') e^{2\pi \mathrm{i} q \frac{\theta'}{\Delta_\theta}} \cdot \begin{Bmatrix} e^{2\pi \mathrm{i} p \frac{r'}{\Delta_r}} \\ e^{-2\pi \mathrm{i} p \frac{r'}{\Delta_r}} \end{Bmatrix} \mathrm{d}\theta' \mathrm{d}\rho'. \end{aligned} \tag{5}$$

In the eq. (4) the q index is considered to be positive, $A_q = \frac{1}{T}\int_0^T \delta e \exp(2\pi \mathrm{i} q \frac{\tau}{T}) \mathrm{d}\tau$ are the Fourier coefficients of the reflectometer signal.

For symmetric spectra $|A_{-q}| = |A_q|$ and for $|C_{q,p}| = |D_{q,p}|$ we obtain by averaging the first eq. (4):

$$C_0^2 G_\theta^2(q) = \frac{<|A_q|^2>}{<G_r^2(p)| R_{q,p}|^2> + <G_r^2(p)| S_{q,p}|^2>}. \tag{6}$$

Here we have introduced the notation $|C_{q,p}| = |D_{q,p}| = C_0 G_\theta(q) G_r(p)$, explicitly separating the poloidal G_θ and the radial G_r wavenumber spectrum profiles. The ensemble average in the denominator of the above equation is taken over many realization of (q,p) pairs, and the numerator is averaged over several realizations of the reflectometer signal. From the eq. (6) the shape of the poloidal fluctuation spectrum $G_\theta(q)$ can be determined if the radial shape $G_r(p)$ is a known function. If the fluctuations are isotropic $G_\theta(q) = G_r(p = q)$ then the spectrum shape function G_θ can be derived through eq. (6) iteratively starting with a uniform radial spectrum profile $G_r = 1$. We have noted, however, that even the first iteration provided the G_θ shape quite accurately in all the examples we have looked at. Moreover, our examples prove that even if the fluctuations are not isotropic and $G_r(p)$ is unknown, the eq. (6) with the uniform radial spectrum $G_r(p) = 1$ still defines the poloidal shape function $G_\theta(q)$ with sufficient accuracy.

For an isotropic turbulence or for a given radial spectrum shape $G_r(p)$ we can determine $C_0 G_\theta(q)$ from the eq. (6) and thus finally obtain the average spectrum of density fluctuations $|C_{q,p}|$. It follows from the eq. (3) and the requirements of density δn to be a real function that the RMS amplitude of density fluctuations $\gamma = \sigma(\delta n/n_0)$ is equal to

$$\gamma = 2\sqrt{<G_r^2(p)> \sum_{q=1}^{Q} C_0^2 G_\theta^2(q)}, \tag{7}$$

where Q is the amount of modes considered in the expression (3). If the radial spectrum shape function G_r is unknown, then of course, the precise value of γ cannot be defined. A rough estimation can be done by merely assuming a certain radial spectrum type, for example, Gaussian spectrum or isotropic spectrum. In a few test examples the γ errors caused by wrong G_r assumptions were in the range of 40−60 %.

3. H-mode turbulence in JT-60U

The O-mode reflectometer, installed on the JT-60U tokamak, performs routine monitoring of plasma fluctuations using the fixed frequency microwave channels [3]. The signal amplitude and phase are registered using a quadrature detector. In this section we apply our technique to reconstruct the shape of the fluctuation spectrum from the quadrature components of the reflectometer signal during the H-mode phase of the plasma discharges #37433 and #37429 on the JT-60U tokamak. In the discharge #37433 the transition to ELM-free H mode happened at 6.329 s, ELM activity started at 6.355 s. The spatial distribution of the ambient plasma was calculated from the magnetic equilibrium condition and approximated by a cylindrical layer with $A = 1.475 \cdot 10^{14}$ s^{-1} cm and $h = -7.9426 \cdot 10^{23}$ s^{-2}. We first search for the stationary 2,010 points records within the reflectometer signal corresponding to the 35.9 GHz reflectometer channel, apply the Fourier transform to each 201 points interval and average the Fourier transform over 10 intervals to obtain the function $<|A_q|^2>$ in eq. (6). Substituting it together with the simulated functions $R_{q,p}$, $S_{q,p}$ to the formulas (6) and (7), we obtain the fluctuation spectrum shown in Fig. 1. Soon after the transition the poloidal spectrum has a monotonic shape, at later stages a maximum grows at $k_\theta = 0.6$ cm^{-1}. The poloidal velocity of fluctuations u_θ, necessary for the spectrum reconstruction, was scanned from 5 to 20 km/s. The value of 17.5 and 20 km/s led to a fluctuation spectra such that the simulated reflectometer signal (eq. (2)) closely reproduced the experimental one. It is worth noting that charge exchange recombination (CXR) spectroscopy indicates, although with possibly large errors, that the poloidal speed of carbon ions was ∼20 km/s. With the assumption of isotropic symmetric spectrum, the amplitude of fluctuations is estimated as $\gamma = 0.21\%$ at 6.336 s, $\gamma = 0.56\%$ at 6.343 s and $\gamma = 0.75\%$ at 6.347 s, increasing before the ELM activity starts. The fluctuation amplitude of the H-mode turbulence was already analyzed in our previous work [7], and the above γ values are in agreement with the previous results. Since the radial shape of the fluctuations spectrum is not known, the above

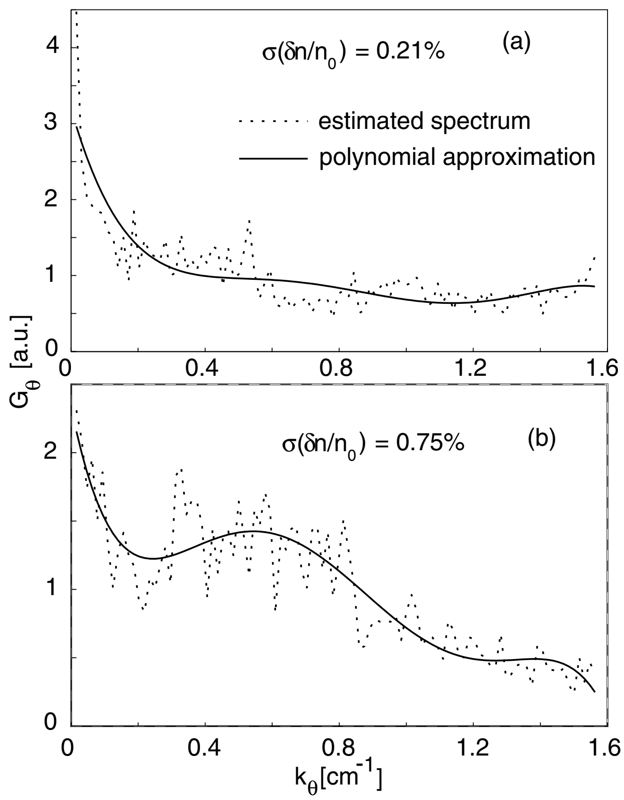

Fig. 1 Reconstructed wavenumber spectra of the JT-60U edge plasma turbulence during the H-mode phase of #37433 discharge, t = 6.336 s (a) and 6.347 s (b). The amplitude of fluctuations is estimated with the assumption of $u_\theta = 20$ km/s.

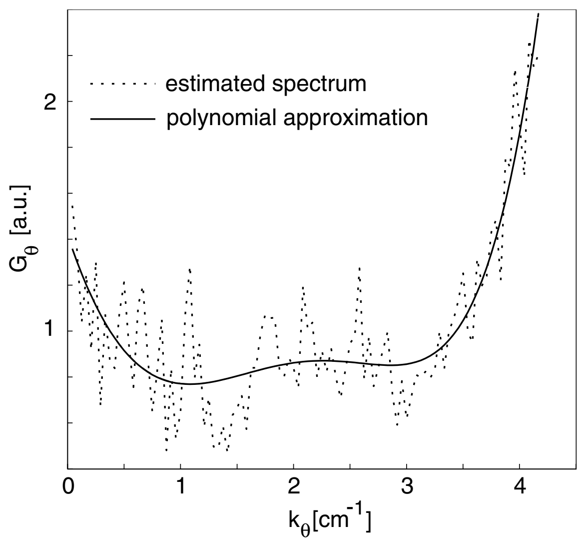

Fig. 2 Poloidal spectrum reconstructed with the assumption of $u_\theta = 7.5$ km/s.

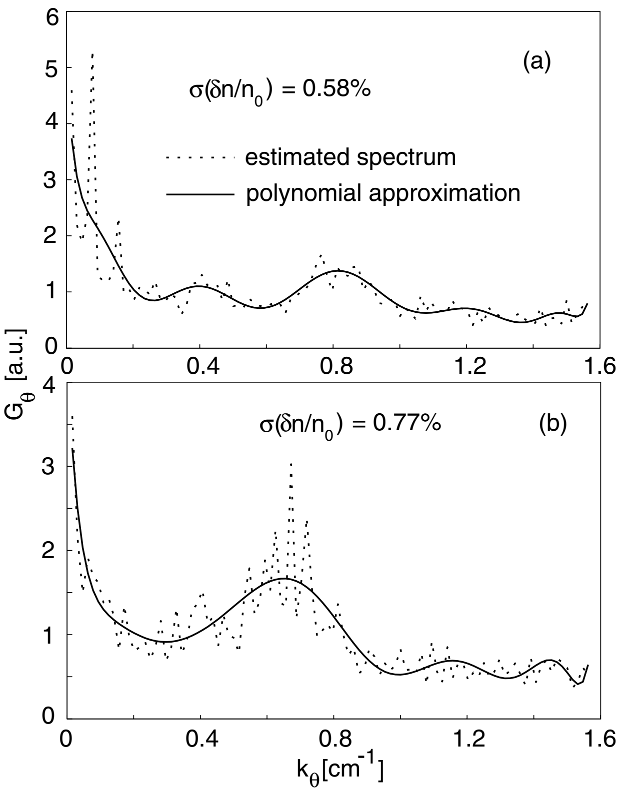

Fig. 3 Reconstructed wavenumber spectra of the JT-60U edge plasma turbulence, #37429 discharge, t = 6.272 s (a) and t=6.302 s (b).

values of γ can be only treated as rough estimations of the fluctuation amplitude, with the accuracy mentioned in the previous section. It is likely that the fluctuations in the edge plasma area are not isotropic, since the longest possible radial wavelength can't exceed the density pedestal width in the H mode plasma (roughly 6 cm for the considered discharge) while the poloidal spectrum is free of that limitation. The shape of the poloidal spectrum, however, is defined with higher precision than the amplitude, because it is not strongly affected by the shape of radial fluctuation spectrum.

If the assumed u_θ is different from 20 km/s, the fluctuation spectrum changes as well. Those u_θ values less than 12.5 km/s result in unreasonable density spectra with sharp increase at large k_θ (Fig. 2), therefore we expect u_θ to satisfy the relation $u_\theta > 12.5$ km/s. In the discharge #37429 the ELM-free H mode plasma existed between 6.27 s and 6.31 s. Similar to the shot #37433, the reconstructed poloidal fluctuation spectrum gradually changed from a relatively flat to a more bulgy shape (Fig. 3), and the estimated fluctuation amplitude slightly increased before the ELMs. The value of 20 km/s is assumed for the rotation velocity of the fluctuations, assumptions less than 10 km/s resulted in the reconstructed spectrum shapes similar to Fig. 2, sharply increasing at large k_θ.

4. Discussion and summary

The analytical technique discussed in this paper is intended to enrich the conventional reflectometry method with the measurements of fluctuation poloidal spectrum and amplitude. There are plasma conditions, however, when our method is not applicable, such as the case of fast non-stationary events like ELMs and transitions. Multiple scattering by strong plasma fluctuations reduce the accuracy of the method. Another situation when our technique fails to correctly extract the fluctuation spectrum is the case when the fluctuations are not random, but instead, exhibit persistent patterns. Our method relies on an assumption of random density fluctuations when taking statistical averages, and that is why its accuracy reduces if the plasma fluctuations are not random. For example, it is possible in our model to introduce the fluctuations in a form of phase-locked structures such that the microwaves scattered from some possibly strong coherent modes would always cancel each other near the receiver, while other weaker modes wouldn't, thus drawing a misleading conclusion about the relative strength of those modes. In our treatment we didn't consider such phase-locked structures in plasma fluctuations. An interference of the random modes averaged over many realizations does not erase the information about the average amplitude of the fluctuating modes such that stronger modes always produce stronger scattering than the weaker modes of the same scale.

In summary, we have discussed a new extension of the conventional reflectometry, which allows us to reconstruct the average poloidal spectrum and estimate the amplitude of plasma turbulence. As an illustration, we have applied the spectrum reconstruction technique to the reflectometry of the JT-60U tokamak plasma.

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References

[1] E. Mazzucato and R. Nazikian, Plasma Phys. Control. Fusion **33**, 261 (1991).
[2] M.E. Manso *et al*., Plasma Phys. Control. Fusion **43**, A73 (2001).
[3] N. Oyama and K. Shinohara, Rev. Sci. Instrum. **73**, 1169 (2002).
[4] R. Nazikian, G.J. Kramer and E. Valeo, Phys. Plasmas **8**, 1840 (2001).
[5] H. Hojo, Y. Kurosawa and A. Mase, Rev. Sci. Instrum. **70**, 983 (1999).
[6] L.G. Bruskin, A. Mase, N. Oyama and Y. Miura, Plasma Phys. Control. Fusion **44**, 2305 (2002).
[7] L.G. Bruskin *et al*., Rev. Sci. Instrum. **74**, 1473 (2003).