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A Numerical Approach to the Localization of Passive Line Integrated Neutral Particle Measurements on LHD

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Abstract

The problem of correct interpretation of passive neutral particle measurements from non-axis-symmetrical plasma has been investigated on LHD. While in active measurements the charge exchange neutral flux originates from a localized artificially created target, the naturally occurring neutral flux observed by a passive diagnostic is line-integrated. A general formulation of the measured quantity is given for an arbitrary shape of plasma parameter isolines, including the heliotron magnetic configuration. A mathematical approach is presented based on a combination of the experimental and computational diagnostic techniques to obtain radial profile data with a passive non-perturbing method.

Keywords:

neutral particle analysis, charge-exchange, locality, ion distribution, ion temperature profile, plasma heating, stellarator/heliotron, Large Helical Device (LHD)

1. Introduction

Neutral particle analysis currently represents one of the basic methods of ion parameter measurements in fusion plasma experiments. Due to the variety of particle orbit classes and complex full 3D field geometry of LHD as well as many of the modern magnetic plasma confinement devices, spatial and angular resolution is required. The physical approach to the localization of such measurements is to introduce an artificial localized target for the charge exchange process in the plasma, such as a diagnostic neutral beam or a solid pellet. This implies active measurements discussed in [1] for LHD. However, in toroidal devices with magnetic confinement one can also retrieve the spatial profiles of ion parameters from passive neutral particle diagnostic data. This paper describes a numerical approach to this problem applicable to an arbitrary shape of isolines in the diagnostic cross-section with no assumption of toroidal axial symmetry.

The measured atomic energy distribution dN/dE is a result of superposition of local ion distributions along the diagnostic sight line, taking into account the spatial distributions of the relevant plasma parameters, such as the density of charge exchange targets, and the attenuation of the atomic flux on the way out to the plasma edge. The integral relation between the plasma ion distribution function and the observed neutral particle spectrum is given in [1]. This integral has a simple form since the distance along the sight line is chosen as the integration variable. In practice the plasma parameter profiles in the integrand are expressed as functions of a more 'native' coordinate, such as minor radius for a cylindrically symmetric plasma column or a generalized radial variable ρ in the non-axis-symmetrical case of a helical plasma. The integration variable then should be changed and the formula should contain the module of the corresponding Jacobian. The detailed formulation of the problem follows in the next section. Then, possible numerical solution methods are discussed and finally, the application of these methods to the silicon detector-based neutral particle analyzer (SDNPA) [1,2] data processing is described.

2. Problem formulation

The measured quantity $\Gamma(E) = dN/dE\ dt$ is the energy resolved flux of atoms to the analyzer's collimating aperture and the sought quantity proportional to the local ion distribution function $g(E, \rho) \propto n_i(\rho) f_i(E, \rho)$ is the local birth rate of atoms at a given energy [$erg^{-1}cm^{-3}s^{-1}$]. As mentioned above, the relationship between the measured and the sought quantities has the form of an integral equation. For the following mathematical treatment let us introduce a new sought function $\tilde{g}(E, \rho(X')) = \frac{\Omega S_a}{4\pi} g(E, \rho(X'))$, where X' is the distance along the sight line and $\rho(X')$ is the local value of the effective minor radius defined as a square root of the normalized magnetic flux $(\Psi/\Psi_{LCMS})^{1/2}$. The geometrical factor includes the viewing solid angle Ω and the aperture area S_a of the neutral particle analyzer. The parameter ρ characterizes a system

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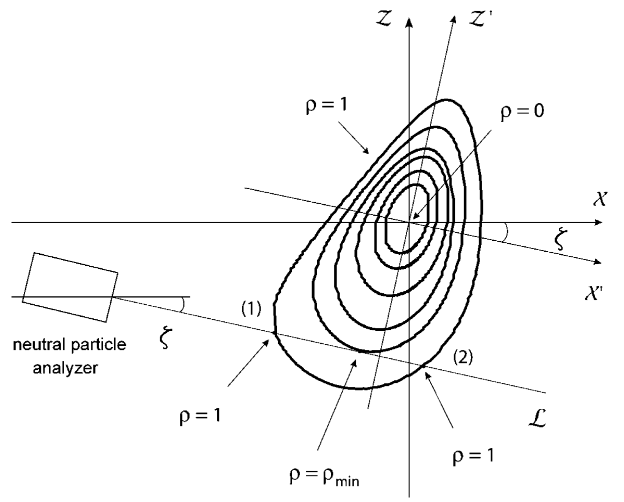

Fig. 1 Integration along a viewing chord crossing the system of nested isolines.

of nested contour lines corresponding to the magnetic surfaces with $\rho = 0$ referring to the magnetic axis. The structure of magnetic surfaces can be obtained from a numerical solution of Grad-Shafranov equation [3].

Suppose that the contour lines are convex closed curves without self-intersections and the neutral particle analyzer performs a scan over the angle ζ. This situation is depicted in Fig. 1. Assuming that the geometry of measurements is originally described in the laboratory coordinate system XZ, introduce a new coordinate system X' Z' so that the axis X' is parallel to the sight line L at the given angle ζ. The sight line enters the plasma at X_1' crossing the contour line $\rho = 1$, then reaches the deepest region $\rho = \rho_{min}$ for the given ζ and finally exits the plasma at X_2' crossing the contour line $\rho = 1$ again. Taking into account the atomic flux attenuation, we obtain

$$\Gamma(\zeta) = \int_{X_1'}^{X_2'} \tilde{g}[E, \rho(X')] \times e^{-\int_{X_1'}^{X'} \lambda_{mfp}^{-1}(E, \tilde{X}')\mathrm{d}\tilde{X}'} \mathrm{d}X', \qquad (1)$$

where λ_{mfp} is the mean free path of atoms with respect to ionizing collisions. The functions $\rho(X')$ and $X'(\rho)$ along the integration path determined by the angle ζ are known from the structure of the contour lines ρ = const. Let us denote $Q^+(\rho, \zeta) = \mathrm{d}X'/\mathrm{d}\rho$ on the interval between the point $\rho = \rho_{min}$ and the point (2) where $\mathrm{d}X'/\mathrm{d}\rho > 0$ and $Q^-(\rho, \zeta) = \mathrm{d}X'/\mathrm{d}\rho$ on the interval between the point (1) and the point $\rho = \rho_{min}$ where $\mathrm{d}X'/\mathrm{d}\rho < 0$. Changing the integration variable in (1) from X' to ρ yields

$$\Gamma(E, \zeta) = e^{\int_{\rho_{\min}}^{1} Q^-(\tilde{\rho}, \zeta)\lambda_{mfp}^{-1}(E, \tilde{\rho})\mathrm{d}\tilde{\rho}} \int_{\rho_{\min}}^{1} \tilde{g}(E, \rho) \times \left[Q^+(\rho, \zeta) e^{-\int_{\rho_{\min}}^{\rho} Q^+(\tilde{\rho}, \zeta)\lambda_{mfp}^{-1}(E, \tilde{\rho})\mathrm{d}\tilde{\rho}} - Q^-(\rho, \zeta) e^{-\int_{\rho_{\min}}^{\rho} Q^-(\tilde{\rho}, \zeta)\lambda_{mfp}^{-1}(E, \tilde{\rho})\mathrm{d}\tilde{\rho}} \right] \mathrm{d}\rho. \qquad (2)$$

The *a priori* knowledge about the isolines of plasma parameters allows to determine the functions $Q^+(\rho,\zeta)$ and $Q^-(\rho,\zeta)$ and to write the relation (2) in the explicit form.

3. Solution Methods

Kinetic Modeling: The problem of retrieving the ion parameter spatial profiles from $\Gamma(E, \zeta)$ can be solved by iterative comparing and joining the results of numerical modeling with the experimental data. The most complete and elaborate approach is the kinetic one. It is based on the known cross-sections of the elementary processes determining the formation of the atoms within the plasma and their passage to the periphery. The necessary electron temperature and plasma density distributions, as a rule, are known, *e.g.* from Thomson scattering and microwave diagnostic measurements. The unknown distributions of required values can be taken as free parameters. The free parameters, such as the ion temperature profile, should be variated until the numerical result matches the experimentally observed $\mathrm{d}N/\mathrm{d}E$.

The general scheme of this method is as follows. First, the spatial distribution of neutral atoms $n_a(\boldsymbol{r})$ should be obtained. Consider the quasistationary kinetic equation for the neutral atom distribution $n_a(\boldsymbol{r})f_a(\boldsymbol{r}, \boldsymbol{v})$ in the plasma:

$$\boldsymbol{v}\frac{\partial n_a(\boldsymbol{r})f_a(\boldsymbol{r}, \boldsymbol{v})}{\partial \boldsymbol{r}} = [s(\boldsymbol{r}, \boldsymbol{v}) + \mathrm{g}(\boldsymbol{r}, \boldsymbol{v})] - \nu n_a(\boldsymbol{r})f_a(\boldsymbol{r}, \boldsymbol{v}). \quad (3)$$

The first two terms in the round brackets on the right hand side of this equation represent the sources of atoms and the third term reflects the losses of atoms. The source function $s(\boldsymbol{r}, \boldsymbol{v})$ corresponds to the slow atoms penetrating into the plasma from the peripheral regions. The source function $g(\boldsymbol{r}, \boldsymbol{v})$ already mentioned above is the local differential birth rate of fast atoms within the plasma column. The losses of atoms due to charge exchange, ion impact ionization and electron impact ionization depend on the summary total rate of these processes $\nu = v_a/\lambda_{mfp}$.

Iterative Join: The integration of eq. (3) over the velocity space allows to obtain the neutral atom density distribution $n_a(\boldsymbol{r})$. The next step of the numerical modeling is to use this distribution to calculate the escaping neutral particle energy spectrum. Thus, at the certain chosen free parameters, one should obtain $n_a(\boldsymbol{r})$ from the kinetic equation and then plug it into formula (2) to calculate $\mathrm{d}N/\mathrm{d}E$. The actual values of the unknown free parameters are determined by iterating this procedure until the calculated atomic spectrum matches the measured one.

This method was successfully applied to tokamak data processing on the basis of the kinetic equation in cylindrical coordinates assuming the toroidal axial symmetry. In this connection the early pioneer work [4] should be mentioned. However, in case of the arbitrary isolines the calculations would be much more complicated. Alternatively, a Monte Carlo simulation method such as AURORA numerical code can be employed to calculate the neutral atom distribution. Algorithms of this kind and their applications are explained *e.g*. in [5,6].

Profile Reconstruction: Another method is possible that does not require any assumptions about the unknown ion temperature profile. Neglecting the attenuation of the atomic flux by setting $\lambda_{mfp}^{-1} = 0$ in (2) yields

$$\Gamma(E, \zeta) = \int_{\rho_{\min}}^{1} \tilde{g}(E, \rho)[Q^{+}(\rho, \zeta) - Q^{-}(\rho, \zeta)]\mathrm{d}\rho, \qquad (4)$$

which is the generalized Abel equation for arbitrary non-circular isolines. This equation belongs to the class of incorrectly posed problems, *i.e*. a small perturbation on the experimental data function $\Gamma(E, \zeta)$ can give rise to an arbitrarily large perturbation in the solution $\tilde{g}(E, \rho)$. Methods of solving incorrectly posed problems presently are abundant in literature. An algorithm of numerical solution of such integral equations by Tikhonov regularization method is described in the work [7].

4. Practical application

For the correct interpretation of line-integrated measurements it is essential to know the structure of the magnetic surfaces for the given experimental conditions. It can be calculated by such a numerical magnetohydrodynamic (MHD) equilibrium code as VMEC [3]. Thus, the local values of the effective minor radius $\rho(X')$ along the diagnostic sight line can be obtained in order to use the formalism of Sec. **2**.

From a practical viewpoint the profile reconstruction method appears to be disadvantageous since it is preferable not to neglect the attenuation. Moreover, a large data set $\Gamma(\zeta, E)$ is required for this method. It can be measured either during a steady state of a long plasma discharge by continuously scanning the plasma over the angle ζ or by changing the angle ζ in a sequence of similar plasma discharges.

The integration of the kinetic equation for neutrals is not practical because of its complexity for helical configurations. AURORA code has been used instead.

The function $\rho(X')$ for SDNPA sight line geometry has been calculated from VMEC numerical data. For the practical computational treatment of (2) an analytic fit has been found in the form

$$\rho = A_j(\zeta)|B_j(\zeta) - X'|^{r_j(\zeta)} + C_j(\zeta). \qquad (5)$$

Here j = 1,2 corresponds to the intervals where $\mathrm{d}X'/\mathrm{d}\rho$ has different signs. Fig. 2 shows the examples of VMEC data points and solid curves calculated using (5) for three different values of the scanning angle ζ for detector #3 of SDNPA. A quite good fitting (5) can be achieved as shown in Fig. 2, so that the average $|\rho_{VMEC} - \rho_{fit}|$ along the sight line is about 0.01 or less.

Formula (2) was used to match the experimental dN/dE measured from NBI#1 and NBI#3 tangentially heated hydrogen plasma. Data from tangential sight line looking against neutral beam injector (NBI) was used. The model estimation was iteratively joined to the bulk thermal part of the measured distribution. In the calculation the experimental cross-sections were used from [8]. The attenuation was assumed to be mainly due to the secondary charge exchange and ion impact ionization. Density data from FIR interferometer was used. Maxwellian ion distribution was assumed for the bulk plasma. The agreement between the calculation and the experiment was reached for the trial T_i profile in the form $T_i(\rho) = T_i(0)(1 - \rho^2)^2$ with $T_i(0)$ 2.75 keV as it is shown in Fig 3. The lower energies are out of the measurable range. The central T_i value agrees with the crystal spectrometer data. The high-energy part of the spectrum corresponds to suprathermal pitch-angle scattered particles from NBI.

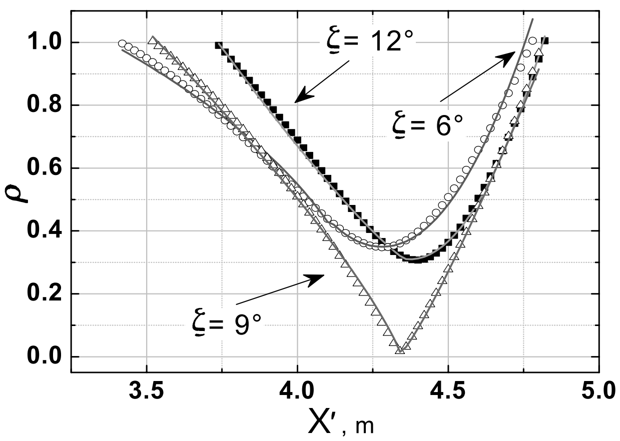

Fig. 2 VMEC data points and analytical fit curves $\rho(X')$ at R_{ax} = 3.6 m, β = 0.22% for SDNPA detector #3.

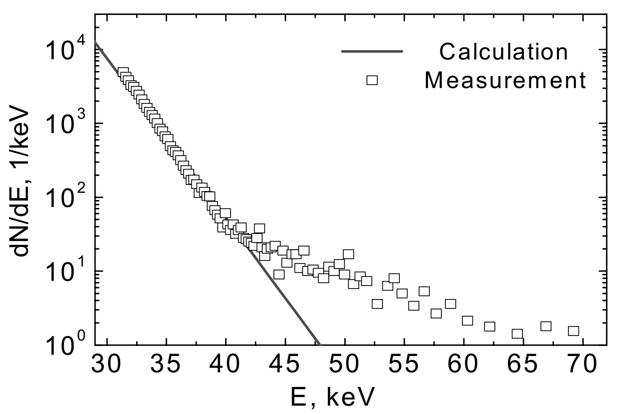

Fig. 3 Calculated (solid) and experimental (dots) atomic energy distribution from n_e = 4 × 10^{13} cm^{-3} NBI plasma.

5. Summary

The formulation of the neutral particle flux from a non-axisymmetrical plasma has been discussed for an arbitrary magnetic configuration. T_i profile retrieving from NPA data on LHD has been demonstrated.

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