J. Plasma Fusion Res. SERIES, Vol. 6 (2004) 299–302

Formation of the X-Point in Helical Plasmas upon Injection of High Energy Particles

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(Received: 9 December 2003 / Accepted: 23 March 2004)

Abstract

The injected high energy charged particles could drive the localized currents in plasma, and thereby they can modify the magnetic field structure, particularly can form the X-point. The analytical description of X-point due to the localized current is presented on the base of magnetic surfaces function with analysis. The formation of large $\nabla n/n$ region in magnetic configuration with the X-point by localized current is predicted. The bifurcation of the electric field is expected by change in $\nabla n/n$. The helical magnetic configurations with X-points, which can be realized on LHD (Large Helical Devices) are proposed.

Keywords:

X-point, magnetic island, magnetic configuration, plasma bifurcation

1. Introduction

In the real experiment it is not so simple to achieve improved plasma confinement (H-mode regime). The bifurcation of the electric field and its effect on the plasma confinement strongly depend on the target plasma state. It is necessary to find the physics mechanisms, which can trigger the bifurcation of the electric field. One of the candidates to control the electric field in plasma is injection of the charged particles proposed in [1,2]. To such candidates there can be related the modification of the magnetic field and magnetic surfaces structure, particularly the formation of X-point, another type than island X-point, with the use of electron and ion injection. The difference in the X-points produced with the current in local island diverter coils and the X-point from the current in plasma is explained below.

2. Electron and ion paths and possibility to produce the current in helical plasma

2.1 Equation of motion for charged particles

The motion of high-energy charged particles is studied by using the Newton equation with Lorenz force taking into account a relativistic factor that makes possibly to take into account the Larmor radius also. This system of equations has the following form

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F} = q\left[\vec{E} + \frac{1}{c}\left(\vec{V}\times\vec{B}\right)\right] \tag{2.1}$$

Here $\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{p} = \gamma m_0\vec{V}$, $\gamma = (1 - V^2/c^2)$ is relativistic factor, $\vec{r}$ is the radius vector of the particle, $\vec{V}$ is vector velocity, $\vec{p}$ is the relativistic momentum, q and m_0 is the charge and mass of the particles, $\vec{E}$ and $\vec{B}$ represents the magnetic and the electric field respectively and c is the speed of the light. Because the particles have high energy, the collisions are not taken to account in this research.

2.2 Main magnetic field

The main magnetic field ($\vec{B} = \nabla\Phi$) is modelled by using of the magnetic field potential

$$\Phi = B_0\left[R\varphi - \frac{R}{m}\sum_n \varepsilon_{n,m}\left(r/a_h\right)^n \sin\left(n\vartheta - m\varphi\right) + \varepsilon_{1,0} r\sin(\vartheta)\right]. \tag{2.2}$$

Here B_0 is the magnetic field at the circular axis of torus, R and a_h are the major and minor radii of the helical winding system, r, ϑ, φ are coordinates connected with the circular axis of the torus, r is the radial variable, ϑ and φ are the angular variables along the minor and major circumference

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of the torus respectively; ϑ increased in the opposite direction to the main normal of the circular axis of the torus. The number of the magnetic field period along the torus is denoted by m and the helical winding polarity number denoted as l; $\varepsilon_{n,m}$ are the coefficient of the harmonics of the magnetic field. For results presented here the parameter of LHD [3] are taken: B_0 = 3T, R = 390 cm, a_h = 97.5 cm, l = 2, m = 10. The parameters $\varepsilon_{n,m}$ and $\varepsilon_{1,0}$ are taken in a way which is explained in [3].

The perturbation magnetic field is described by scalar potential

$$\Phi_p = B_0 a_h \frac{\varepsilon_{n,m,p}}{m_p}\left(r / a_h\right)^{m_p} \sin\left(m_p\vartheta - n_p\varphi + \delta_{m,n,p}\right) \quad (2.3)$$

2.3 Simulation of injected particles motion

The simulation of the injected charged particle motion has been performed with goal to find the path in the confinement region in which particles could move closely to one circular loop along circular axis of torus. As candidates for such path production the next injection points are reviewed: the centre of island (O-point), X-point of the helical field and X-point of m/n = 1/1 island.

It is found that upon injection of electrons and ${}^{+2}_{3}$He ions in the centre of island for the most of values of $V_\|/V$, where $V_\|$ is the particle velocity parallel to magnetic field line at the position of particle injection, the injected particles are “chaotically blocked” (after several oscillations the helically trapped electron periodically escape mirror filed and turn to toroidally blocked and vice versa) inside the island even if $V_\|/V$ = 0.9. But they became passing when $V_\|/V$ = 0.98 and their trajectory is the closed-loop-like around the main axis of the torus Fig. 1. The current path for ions lies inside drift island with very small square in the meridian cross-section. The value of this square is connected directly with relation $V_\|/V$ and with decreasing this value the square of drift island in meridian cross-section will increase. For injection in the vicinity of the X point of island and X-point of the helical field they become passing but the main part of trajectory follows the drift surface and does not form the closed-loop path.

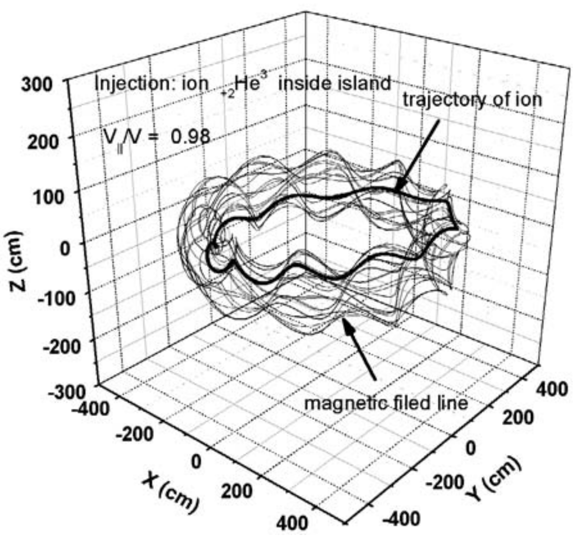

Fig. 1 3D trajectory of the ${}^{+2}_{3}$He ion with the energy W = 100 keV. This particle is injected in the centre (O-point) of island 1/1.

3. Formation of the x - point as the method to effect on plasma parameter gradient

X-point as the result of additional current in plasma can be written in the following simplified way. Simple magnetic field description below gives us the picture of the effect of controlled beam current on the magnetic configuration with the rotational magnetic transform.

Magnetic surface with the current in plasma can be described by the following expression.

$$\Psi = \int \bar{r}\iota(\bar{r}^2)d\bar{r} - \eta\ln\left[1 - 2\frac{\bar{r}}{\bar{r}_C}\cos(\vartheta - \varphi - \vartheta_C) + \left(\frac{\bar{r}}{\bar{r}_C}\right)^2\right] \quad (3.1)$$

where $\eta = \frac{2J}{c\bar{r}_C B_0}\frac{R}{a}$. Here J is the value of the produced current, ι is rotational transform, a is the small radii of the torus, r_c and ϑ_C are the radial and angular co-ordinates of the current position.

The equations for the singular points is the following

$$\sin(\vartheta - \varphi - \vartheta_C) = 0 \quad (3.2)$$

$$\iota(\bar{r}^2) + \eta\frac{1}{\bar{r}\left(1 - \dfrac{\bar{r}}{\bar{r}_C}\right)} = 0 \quad (3.3)$$

The solutions of this equation can be presented approximately with the following expressions

$$\vartheta - \varphi - \vartheta_C = \pi k, \; k = 0, \pm 1, \pm 2, ... \quad (3.4)$$

$$\bar{r}_1 = \frac{(-1)\eta}{\iota(\bar{r}^2)\big|_{\bar{r}=0}}, \; \bar{r}_2 = \bar{r}_C + \frac{\eta}{\iota(\bar{r}^2)\big|_{\bar{r}=\bar{r}_C}} \quad (3.5)$$

The X-point is formed only if $\eta < 0$.

The radial variable $\bar{r}$ as the function $\vartheta - \vartheta_C$ on the separatrix curve in the plane $\varphi = const$ may be written in a way

$$\bar{r} = \bar{r}_S + \Delta\bar{r}_S, \quad (3.6)$$

where r_S is radial co-ordinates of the separatrix position and

$$\Delta\bar{r}_S = \frac{1}{\left(\iota(0) + \dfrac{d\iota}{d\bar{r}^2}\bar{r}^2\right)\bar{r}_S}\eta\ln\frac{1 - 2\dfrac{\bar{r}}{\bar{r}_C}\cos(\vartheta - \vartheta_C) + \left(\dfrac{\bar{r}}{\bar{r}_C}\right)^2}{1 - 2\dfrac{\bar{r}_S}{\bar{r}_C}\cos(\vartheta_S - \vartheta_C) + \left(\dfrac{\bar{r}_S}{\bar{r}_C}\right)^2} \quad (3.7)$$

which can be treated as the dimension of the deviation of the curves forming the separatrix from the curve $\bar{r} = \bar{r}_S$.

It is assumed that the density profile is connected with the magnetic surface with formula

$$n = n(0)\left[A_n - \left(\frac{\Psi}{\Psi_b}\right)^k\right]^g, \tag{3.8}$$

where Ψ_b is the surface which bounds the plasma, A_n, k and g are the parameters which describe the profile features. The density gradient, changed according to the formation of the X-point, effects on the radial electric field. Then the derivative of the plasma density is connected with the $\nabla\Psi$ is

$$n' = n(0)g\left[A_n - \left(\frac{\Psi}{\Psi_b}\right)^k\right]^{g-1}(-1)k\left(\frac{\Psi}{\Psi_b}\right)^{k-1}\frac{\nabla\Psi}{\Psi_b} \tag{3.9}$$

Calculations on the basis of formula (3.1) and (3.9) shows that density gradient near X-point becomes steeper than before it formation.

4. Control of the electric field with the X-point formation

We expect that the radial electric field E_r can be effected by the changes in the gradients of the density $n(r)$ and temperature $T_e(r)$ and $T_i(r)$ after formation of X-point. The ambipolar electric field can be obtained from the equation

$$\Gamma_e = \Gamma_i \tag{4.1}$$

where the expressions for the electron Γ_e and ion fluxes Γ_i can be written in the form [4,5]

$$\Gamma_e = \varepsilon_t^2\varepsilon_h^{3/2}\left(\frac{cT_e}{eB_0 r}\right)^2\frac{n}{a_p\nu_{ei}}\left[A_1\left(\lambda_n - \frac{3}{2}\lambda_{T_e} + X\right) + A_2\lambda_{T_e}\right], \tag{4.2}$$

$$\Gamma_i = \varepsilon_t^2\varepsilon_h^{3/2}\left(\frac{cT_i}{eB_0 r}\right)^2\frac{n}{a_p\nu_{ii}} \times \frac{2}{1+1.5C(1)X^2}\left[A_1\left(\lambda_n - \frac{3}{2}\lambda_{T_i} - \frac{T_e}{T_i}X\right) + A_2\lambda_{T_i}\right]. \tag{4.3}$$

Here a_p is the plasma radius and the following denotations are induced (the dot denotes the derivative with respect to radius)

$$X = \frac{ea_pE_r}{T_e},\ \lambda_n = -\frac{a_pn'}{n},\ \lambda_{T_e} = -\frac{a_pT_e'}{T_e},\ \lambda_{T_i} = -\frac{a_pT_i'}{T_i} \tag{4.4}$$

$$A_1 = \frac{1}{6}\int_b^\infty dx_j x^4 e^{-x_j},\ A_2 = \frac{1}{6}\int_b^\infty dx_j x^5 e^{-x_j} \tag{4.5}$$

The value $C(1)$ is the magnitude $C(x_i) \equiv \frac{1}{3}\left(\frac{\varepsilon_t}{\varepsilon_h}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^2\left(\frac{\upsilon_{Dhi}}{\nu_{ii}a_p}x_i^{3/2}\right)^2$ taken under $x_i = 1$, where $x_j = m_jV^2/2T_j$ and $\upsilon_{Dhi} = cT_i\varepsilon_h/eB_0r$ that can be treated as the drift velocity of ion in the helical field. The magnitude $C(1)$ can be expressed in such a manner

$$C(1) \approx \left(\frac{\varepsilon_t(r)}{\varepsilon_h(r)}\right)^{1/2}\left(\frac{T_e(r)}{T_i(r)}\right)^2\left(\frac{\varepsilon_h(r)}{r[\mathrm{cm}]}\right)^2\frac{\left(T_i(r)[\mathrm{keV}]\right)^5}{\left(n(r)[\mathrm{cm}^{-3}]/10^{13}\right)^2} \tag{4.6}$$

Here ε_t and ε_h are the coefficients of the main toroidal and helical harmonics; ε_j are the coefficients of the satellite helical harmonics. The collision frequency $\nu_{\alpha,\beta} = \frac{4\sqrt{\pi}\Lambda_c e^4 Z_\alpha^2 Z_\beta^2 n_\beta}{3\sqrt{m_\alpha}T_\alpha^{3/2}}$, where $\alpha,\beta = e,i$ and $Z_{\alpha,\beta}$ is charge numbers of species.

The ambipolar equation (4.1) can have three roots of the electric field

$$X_1 = \frac{2}{3}\sqrt{\Lambda_{n,T_e}^2 - \frac{3}{1.5C(1)}\left[1+2\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{5/2}\right]}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}q^*\right), \tag{4.7}$$

$$X_2 = \frac{2}{3}\sqrt{\Lambda_{n,T_e}^2 - \frac{3}{1.5C(1)}\left[1+2\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{5/2}\right]}\left(-q^*\right), \tag{4.8}$$

$$X_3 = \frac{2}{3}\sqrt{\Lambda_{n,T_e}^2 - \frac{3}{1.5C(1)}\left[1+2\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{5/2}\right]}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}q^*\right) \tag{4.9}$$

Here

$$q^* \approx \frac{1}{3} + \frac{3}{C(1)\Lambda_{n,T_e}^2}\left[1 - 2\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{7/2}\frac{\Lambda_{n,T_i}}{\Lambda_{n,T_e}}\right], \tag{4.10}$$

$$\Lambda_{n,T_e} = \lambda_n - \left(\frac{3}{2} - \frac{A_2}{A_1}\right)\lambda_{T_e},\ \Lambda_{n,T_i} = \lambda_n - \left(\frac{3}{2} - \frac{A_2}{A_1}\right)\lambda_{T_i}. \tag{4.11}$$

The transition from one root to the case of three roots under the density gradient increase is seen on Fig. 2. If the density gradient and/or temperature gradient are increased by changing of flux surfaces through the formation of X-point

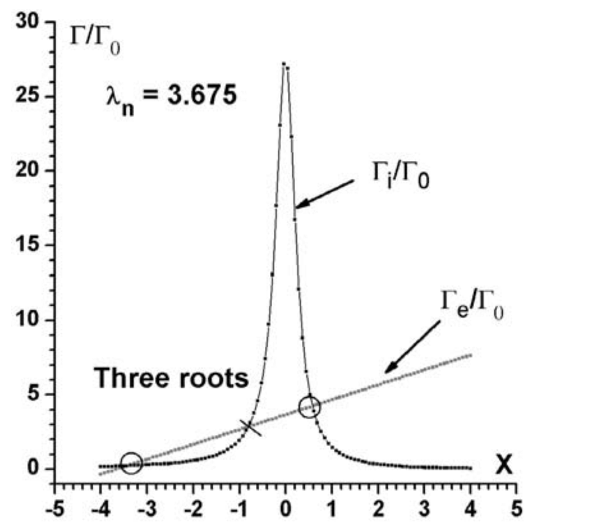

Fig. 2 Neo-classical fluxes of ions and electrons under λ = 3.675.

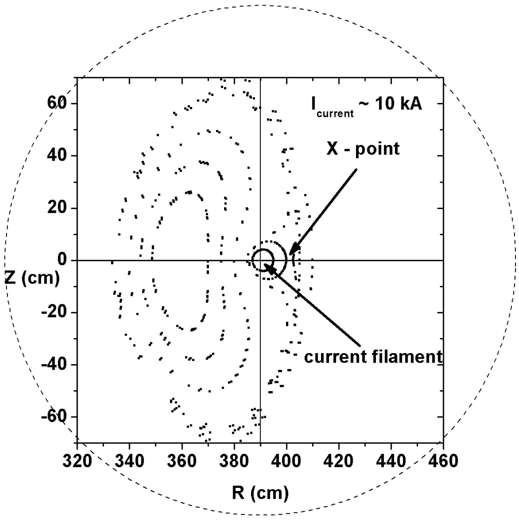

Fig. 3 Inner Shift Configuration with X-point created by "filament" current.

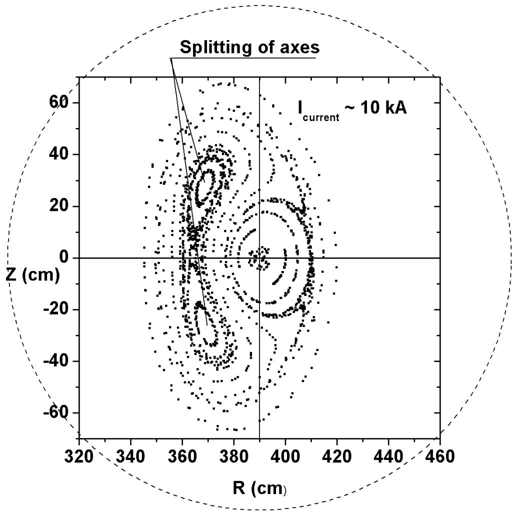

Fig. 4 Vertically Elongated Configuration with X-point created by current distributed on *r*.

as (3.9) there is a possibility for the bifurcation of the radial electric field as shown in Fig. 2. The bifurcation of the electric field is expected by change in $\nabla n/n$ in accordance with the model [6].

5. The magnetic configurations with X-point

The configurations with X-point can be created with the current strongly concentrated and this current can be modelled with the filament. Such configuration is shown on Fig. 3. The real for the experimental conditions current distribution on radii is expected. In that case the configuration with X-point is changed as shown on Fig. 4.

6. Conclusions

The X-point formation is expected due to injection of high energy particles in m/n = 1/1 island O-point. In contrast to X-point from local divertor coils the X-point from local current in plasma has stronger influence on plasma parameter and could lead to it bifurcation. The analytical treatment for roots of transition for electric field is done in neoclassical approximation. The expressions for the radial electric fields are obtained from the solution of neoclassical non-ambipolar flux balance equation.

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