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Radial and Poloidal Structure Coupled with Shear Viscosity under the Existence of a Large Flow Shear in Tokamaks

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Abstract

A large poloidal flow is generated in an improved confinement state in tokamaks. Poloidal shock has been predicted theoretically when a poloidal flow velocity increases. An extension of the model to include the poloidal structure is made to obtain a self-sustained two dimensional structure. Its special cases with no shear viscosity (decoupled) and large shear viscosity are solved.

Keywords:

poloidal shock, two dimensional structure, shear viscosity, $E \times B$ flow shear, tokamak

1. Introduction

The H-mode [1] offers a great advantage for improving plasma confinement in tokamaks. A large poloidal flow is generated by a large radial electric field in the tokamak edge region in H-mode [2], and a shear flow is responsible for improving confinement by suppressing anomalous transport [3]. Many studies have been carried out to understand the formation mechanism of a steep structure in the radial direction [4-6].

In the tokamak edge region in H-mode, the poloidal flow increases to $M_p \sim 1$, where M_p is the poloidal Mach number. Appearance of a poloidal shock structure in the potential and density profile has been predicted theoretically with such a fast poloidal flow [7,8]. A measurement in the CCT tokamak shows a poloidal density variation [9]. The existence of a steep poloidal structure induces a deviation of constant density contours from magnetic surfaces [10], and makes a large poloidal electric field, which gives a large $E \times B$ flow in the radial direction [11]. However, transport analyses are often carried out with flux surface averaged quantities, i.e. the poloidal structures are neglected. Neoclassical calculation with low order Fourier expansion has been carried out to include poloidal asymmetry [12], but it can take account of only mild poloidal variations. There remains a key question whether poloidal shocks are formed or not in H-mode plasmas. If shocks are established, an extension of the previous model to include the poloidal structure should be needed for a quantitative transport analysis in H-mode.

A two dimensional structure with coupled radial and poloidal variations must be considered, if a shock really exists. In this paper a self-sustained two dimensional structure model with shear viscosity (μ) is proposed. The existence of the diamagnetic flow can also weaken the shock structure [13]. In our approach, shear viscosity is taken into account to consider radial structural coupling. Interaction between quantities on different flux surfaces through shear viscosity reveals a two dimensional structure in tokamaks. Special cases with $\mu = 0$ and $\mu \gg 1$ are solved. These calculations give a typical shock structure and a structure determined by shear viscosity, respectively.

2. Two dimensional structure

2.1 Basic equations

To construct model equations for the structure with evaluation of self-sustained two dimensional effects, we take the same assumption as Ref. [7]. Poloidal variations of the density and the electrostatic potential are considered, but that of the temperature is neglected. Electrons are isothermal, ions are adiabatic, and $n_i = n_e \equiv n$ is taken, where n_i and n_e are the ion and electron density, respectively. Extending the model in Ref. [7], radial flow and shear viscosity are taken into account. By these terms, radial and poloidal structures are coupled with each other. The structures are governed by the momentum balance equation,

$$m_i n \frac{\mathrm{d}}{\mathrm{d}t} \vec{V}_i = \vec{J} \times \vec{B} - \vec{\nabla}(p_i + p_e) - \vec{\nabla} \cdot \vec{\pi}_i , \qquad (1)$$

where $\vec{V}_i$ is the flow velocity, $\vec{J}$ is the plasma current, p_i and p_e are the ion and electron pressure, $\vec{\pi}_i$ is the viscosity tensor of ions, and m_i is the ion mass. The viscosity of electrons is

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neglected because it is smaller by a factor of the order of $\sqrt{m_e/m_i}$. In tokamaks the toroidal symmetry is satisfied, so the parallel component and averaged poloidal component of the momentum balance are given to be

$$\begin{aligned}&-\frac{nIB_p}{KB^2 r}\frac{\partial \Phi}{\partial \theta}\frac{\partial}{\partial \psi}\left[\frac{1}{2}\left(\frac{KB}{n}\right)^2\right]+\frac{B_p}{r}\frac{\partial}{\partial \theta}\left[\frac{1}{2}\left(\frac{KB}{n}\right)^2\right]\\&+\frac{IB_pB_\varsigma}{B^2 r}\frac{\partial \Phi}{\partial \theta}\frac{\partial}{\partial \psi}\left[\frac{I}{B_\varsigma}\frac{\partial \Phi}{\partial \psi}\right]-\frac{KB_pB_\varsigma}{nr}\frac{\partial}{\partial \theta}\left[\frac{I}{B_\varsigma}\frac{\partial \Phi}{\partial \psi}\right]\\&=-\frac{B_p}{m_i r}\frac{\partial}{\partial \theta}\left(\frac{\langle P_e\rangle}{\langle n\rangle}\ln N+\frac{5}{2}\frac{\langle P_i\rangle}{\langle n^{5/3}\rangle}n^{2/3}\right)\\&-\frac{1}{m_i n}\left(\vec{B}\cdot\vec{\nabla}\cdot\vec{\pi}_i\right)_{bulk}-\frac{1}{m_i n}\left(\vec{B}\cdot\vec{\nabla}\cdot\vec{\pi}_i\right)_{shear}\end{aligned} \quad , (2)$$

$$\begin{aligned}&\left\langle -\frac{nIB_p}{KB^2 r}\frac{\partial \Phi}{\partial \theta}\frac{\partial}{\partial \psi}\left[\frac{1}{2}\left(\frac{KB_p}{n}\right)^2\right]+\frac{B_p}{r}\frac{\partial}{\partial \theta}\left[\frac{1}{2}\left(\frac{KB_p}{n}\right)^2\right]\right\rangle\\&=\frac{1}{m_i}\left\langle\frac{JB_pB_\varsigma}{n}\right\rangle-\frac{1}{m_i}\left\langle\frac{\vec{B}_p\cdot\vec{\nabla}\cdot\vec{\pi}_i}{n}\right\rangle_{bulk}\\&-\frac{1}{m_i}\left\langle\frac{\vec{B}_p\cdot\vec{\nabla}\cdot\vec{\pi}_i}{n}\right\rangle_{shear}\end{aligned} \quad , (3)$$

where Φ is the electrostatic potential, $K = nV_p/B_p$, $I = R^2\vec{B}\cdot\nabla\varsigma$, $(\vec{\nabla}\cdot\vec{\pi}_i)_{bulk}$ and $(\vec{\nabla}\cdot\vec{\pi}_i)_{shear}$ are the bulk viscosity, given by neoclassical process [14], and the shear viscosity, given by anomalous process [11], respectively. The radial flow is taken into account, so $\partial\Phi/\partial\theta$ terms are involved in the left side of Eqs. (2) and (3). The Boltzmann relation

$$n=\bar{n}\exp\frac{e\Delta\Phi}{T_i} \quad (4)$$

is adopted here to determine variables, where T_i is the ion temperature, $\bar{f}$ and Δf represent the spatial average and perturbed parts of quantity f, respectively. The variable that must be determined from Eqs. (2)-(4) are K, Φ and n, which have radial and poloidal variations.

2.2 Decoupled model

A decoupled limit neglecting the radial flow velocity ($V_r/V_p \ll 1$) and the shear viscosity ($\mu = 0$) is considered to show what kind of structure is formed when large E_r shear exists. This is the case solved in Ref. [7]. When the radial flow velocity can be neglected, variable K becomes a function of ψ only. The model equation (2) for the parallel component can be reduced to be

$$\begin{aligned}&\frac{2}{3}D\frac{\partial\chi}{\partial\theta}+\left(1-M_p{}^2\right)\chi+2A'\left(\chi^2-\langle\chi^2\rangle\right)\\&=\varepsilon\left[\left(M_p{}^2+2C\right)\cos\theta+D\sin\theta\right]\end{aligned} \quad , (5)$$

where $\chi = \ln(n/\bar{n})$, $M_p = KB_0/\bar{n}v_{ti}C_r$, $D = (4\sqrt{\pi}/3)[I_{ps}KB_0/(\bar{n}v_{ti}C_r{}^2)]$, $C = I^2(\partial\Phi/\partial\psi)^2/2v_{ti}{}^2B_0{}^2C_r{}^2$, $C_r{}^2 = (5/3 + T_e/T_i)/2$, $A' = M_p{}^2/2 + 5/(36C_r{}^2)$, v_{ti} is the thermal velocity of ions, and I_{ps}

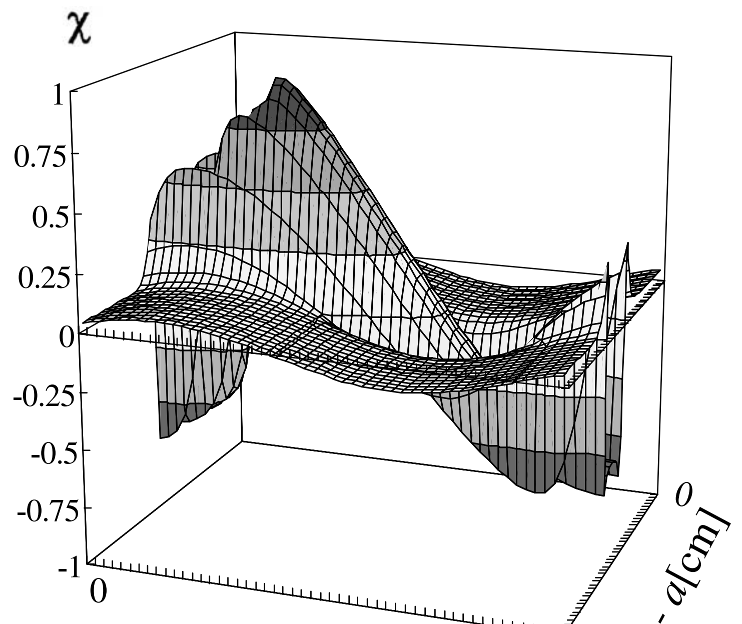

Fig. 1 χ profile when a large $E\times B$ shear flow exists, where a is the minor radius and $\theta = 0$ is defined as the low field side midplane. This is the case of a decoupled model where the radial profile of M_p is given as a parameter to solve the poloidal structure. This figure corresponds to the density perturbation profile.

is the function defined in Ref. [7]. For obtaining Eq. (5), n is replaced by the variable χ, and assumed to be $\chi \sim O(\sqrt{\varepsilon})$. Only terms up to $O(\varepsilon)$ are kept in Eq. (5). The variable χ is related to a potential perturbation $\Delta\Phi$ by Eq. (4). This equation gives a shock solution when $M_p \sim 1$. Parameter M_p corresponds to the poloidal Mach number. When the E_r profile in the radial direction is given to be the solitary structure, which is formed in electrode biasing H-mode [4], and the poloidal flow is determined by $E \times B$ flow, a two dimensional potential profile is obtained. We call the region where the solitary structure exists shear region. The χ profile is shown in Fig. 1. A large magnitude of the perturbation $\Delta\Phi \sim \bar{\Phi}$ with poloidal variation is shown. A composition of the radial solitary and the poloidal shock structure gives the total profile of Φ. Their contours correspond to the streamline of the plasma. A radial flow arises in the vicinity of the midpoint of the shear region. This is due to the existence of the shock. The shock position differs when M_p is different, so the large variation of E_r like in the solitary structure generates a flow pointing to the radial direction. This result contradicts to the assumption that V_r can be neglected, coming from decoupling in the radial direction, so radial coupling must be taken into account.

2.3 Viscosity coupling model

When shear viscosity is taken into account, the steep structures can be smoothed and there is possibility to sustain the assumption $V_r/V_p \ll 1$. In this system Eq. (2) becomes

$$-\mu \frac{r}{v_{ti}C_r}\frac{B_0}{B_p}\frac{\partial^2}{\partial r^2}\left[M_p\left(1-\chi+\frac{\chi^2}{2}\right)-E\right] + \frac{2}{3}D(1-\chi)\frac{\partial^2\chi}{\partial\theta^2} + \left(1-M_p{}^2\right)\frac{\partial\chi}{\partial\theta} + 4A'\chi\frac{\partial\chi}{\partial\theta}$$
$$= \varepsilon\left(\left\{D - \frac{\mu}{rv_{ti}C_r}\frac{B_0}{B_p}\left[2r^2\frac{\partial^2}{\partial r^2}M_p + 2r\frac{\partial}{\partial r}\left(M_p+E\right) - \left(M_p+E\right)\right]\right\}\cos\theta - M_p\left(M_p+E\right)\sin\theta\right), \quad (6)$$

where $E = I(\partial\Phi/\partial\psi)/(v_{ti}B_0C_r)$. Simplified case with M_p given by the solitary equation and strong toroidal damping $E = M_p$ is considered here. The χ profile is shown in Fig. 2. The boundary condition is $\chi = 0$ at the boundary of the shear region. This condition corresponds to no perturbation caused by a poloidal flow outside the shear region. The profile shown in Fig. 2 is for a case with large shear viscosity ($\mu = 70$ [m^2/s], which is much larger than the values estimated from experiments in the TEXTOR tokamak $\mu = 10^{-2} \sim 10^{0}$ [m^2/s] [15]) to obtain a characteristic structure formed by shear viscosity. In this case the first term of the left hand side of Eq. (6) is dominant, so an approximated solution is given as

$$\chi(r,\theta) \sim \frac{\varepsilon\, v_{ti}C_r\sqrt{D^2+4M_p{}^2}}{2\mu\, rM_p}\frac{B_p}{B_0}(r-a)(r-a+d)\sin\left(\theta+\theta_\alpha\right), \quad (7)$$

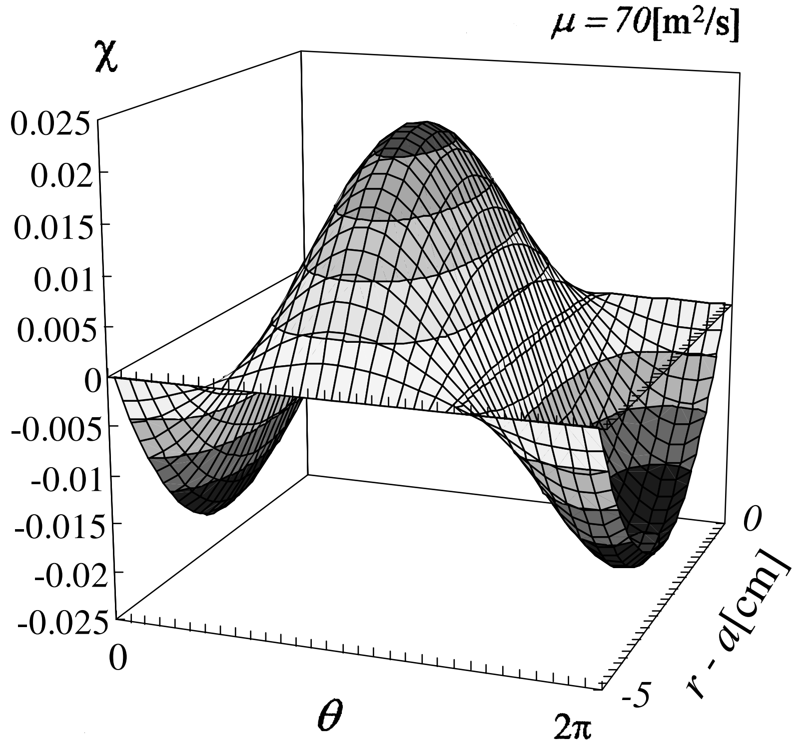

Fig. 2 χ profile for a large viscosity case with $\mu = 70$ [m^2/s]. The maximum amplitude is much smaller than that in Fig. 1.

where tan $\theta_\alpha = -D/2M_p^2$. Smoothed structure is obtained, and a poloidal variation is sufficient small ($\Delta n/\bar{n} < 4\%$) to satisfy the assumption $V_r/V_p \ll 1$. The maximum value of χ is proportional to $1/\mu$. The peak position depends on M_p, and the peak in Fig. 2 appears at that of M_p near the boundary. This structure results from the strong radial coupling with the shear viscosity and the strong boundary constraint $\chi = 0$.

3. Conclusion

The two dimensional potential structure, which plays an important role in a tokamak improved confinement state, is studied in this paper. An extension of the model to include the poloidal structure is carried out. A poloidal shock structure has been predicted when the poloidal Mach number $M_p \sim 1$. We propose two dimensional coupling model equations. A decoupled limit, which corresponds to the model in Ref. [7], shows a significant modification of the potential profile by the appearance of the poloidal shock, and generation of radial flow. The decoupled limit model is not self-consistent, and shear viscosity is taken into account to smooth the structure. A large shear viscosity case is analyzed, and a well-suppressed perturbation profile is obtained. The structure is affected by those near the boundary of the shear region because of the strong radial coupling with the shear viscosity and the strong boundary constraint. Only the limiting cases of poloidal shock and shear viscosity dominant, respectively, are considered here, and an intermediate case that the poloidal shock and shear viscosity effect are comparable is left for a future work. The intermediate case is what is expected to be realized in tokamak experiments. Figures 1 and 2 indicate that both the magnitude and the peak position of the density perturbation depend strongly on the magnitude of μ. Therefore, a measurement of the poloidal density profile can be used to estimate μ. Even the large shear viscosity case shows a little but finite poloidal variation, so a detailed measurement of the poloidal density profile gives important information on the structural formation mechanism.

Previous transport analyses based on flux surface averaged quantities have shown good qualitative agreement with experiments, and a clear shock structure has not been observed yet in experiments. An effect of the poloidal structure might be negligible as in the large shear viscosity case. On the other hand in a transient phase in L-H transition the shock is possible to affect to shorten the transition time [11].

The poloidal shock structure arises from toroidicity. Another cause of formation of a poloidal structure is a divertor configuration. A potential hill near the divertor X-point is observed in DIII-D [16]. This effect is also important for poloidal structural formation mechanism and must be taken into account.

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