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Control of Chaos by Linear and Nonlinear Feedback Methods

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Abstract

The chaotic orbits in the Rossler system are controlled into a periodic cycle by two methods; the delayed feedback controlling method which continues to control chaos by self-controlling feedback, and a control system including on-line trained neural network controller. It is found that (1) stabilization of chaotic orbits by the former method depends on the initial conditions and the gain parameters and that (2) the linear neural controller fails to control chaotic orbits in the Rossler system and choice of the threshold function (nonlinear function) is found to be essential for the second method.

Keywords:
chaos, delayed feedback controlling method, neural controller

1. Introduction

Chaotic phenomena are frequently observed in plasma discharges and in the fundamental plasma experiments [1]. Recent researches extend not only to identify but also to control chaos itself [2]. In fusion plasmas, the control of plasma turbulence is a critical issue to attain the self-ignition conditions. Therefore, techniques of controlling chaos might be useful even for fusion plasmas. To establish controlling method of plasma turbulence, we will investigate the controlling of chaos as the first step. During the initial phase of developing turbulence, the pichfork bifurcation is sometimes observed in experiments, which is described by a simple model with a few degrees of freedom [3]. Therefore, we expect these controlling methods are usufull to control chaos in the initial phase of developing turbulence.

The original controlling method of chaos called OGY method is developed by Ott, Grebogi and Yorke. It utilizes the existence of UFP (unstable fixed point) embedded within the chaotic attractor [4]. The chaotic orbit is stabilized by applying small perturbations into the system. However, this method requires the knowledge of the location of UFP so that the location should be tracked in advance by the linear prediction. Later, Pyragas proposed the delayed feedback controlling method which does not require the knowledge of the location of UFP or unstable periodic orbit (UPO) by modifying the OGY method [5]. The alternative method of controlling chaos is given by aplication of the neural network controller (NNC). On-line trained linear neural controller is proposed by Konishi and Kokame, and is applied to control the chaotic orbit in the two dimensional map system such as an Henon map [6].

In this paper, both methods are tested to the Rossler system to check these methods are applicable to not only a map system but also to coupled ordinary differential equations with a few degrees of freedom. Performance of these control methods is compared with each other. It is found that the control of chaos by the Pyragas method is strongly affected by the gain of perturbation and is sensitive to the initial conditions for the period three cycle. We also apply the NNC to the Rossler system. It is found that the original linear NNC fails to control the Rossler system and is only available for a map system. It is concluded that the nonlinear threshold function is essential to control a chaotic orbit in the Rossler system and that the NNC with the hyperbolic tangent function shows the best performance. This paper is organized as follows. In Sec. **2**, the method of delayed feedback control is applied to the Rossler system, and the sensitivity of parameters on the controlling chaos is examined. In Sec. **3**, the NNC is applied to the Rossler system. The dependence of the threshold function is examined. The chaotic orbit is controlled by using the NNC with the hyperbolic tangent function as the threshold function. In Sec. **4**, we summarize the results.

2. Chaos control by delayed feedback method

The Rossler system with a perturbation $F(t)$ is given by

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$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = -y - z \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = x + 0.2 + F(t), \quad F(t) = k[y(t-\tau) - y(t)] = k[D(t)] \ (t > 50) \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = 0.2 + z(x - 5.7), \ F(t) = 0 \ (t > 50) \end{cases} \quad (1)$$

where $y(t-\tau)$ is delayed output signal and k is the gain of perturbation. This system has a chaotic attractor. Figure 1 shows the $x-y$ phase portrait of the Rossler system without perturbation ($k = 0$), where the chaotic attractor is clearly seen.

The delayed time τ corresponds to the period of the UPO or UFP, when $F(t) = 0$, i.e., $D^2(t) = [(y(t) - y(t-\tau)]^2 = 0$. From numerical results with different τ, we will estimate a value of τ which gives the local minimum of $D^2(t)$ for $k = 0.2$. Figure 2 shows the dependence of the time averaged perturbation $\langle D^2(t)\rangle$ on τ for $k = 0.2$. The time average is performed during the time interval t = 150 – 300. $\langle D^2(t)\rangle$ is calculated for each value of τ with different initial conditions such as $x_0 = y_0 = z_0 = 0.5, 1.0, 1.5, \ldots, 10$ and the corresponding 20 values of $\langle D^2(t)\rangle$ for each τ are shown. It is found that $\tau = 5.90$, 11.75, 17.50 correspond to the period-one, two, three, respectively. Other local minimum correspond to the UFPs.

Next, we numerically find k that gives the local minimum values of $D^2(t)$ while fixing the period, τ obtained in Fig. 2. Figure 3 shows the dependence of $\langle D^2(t)\rangle$ on k in the cases with $\tau = 5.90$ (thick line), $\tau = 11.75$ (dotted line) and $\tau = 17.50$ (dotted dashed line). It is seen that $k = 0.18$, $k = 0.12$ and $k = 0.06$ correspons to the local minimum for period one, two and three cycles, respectively. Figure 4 shows period-one cycle of the Rossler system. Two cases with ($\tau = 5.90$, $k = 0.18$) and ($\tau = 5.90$, $k = 0.8$) are shown for the initial condition of $x_0 = y_0 = z_0 = 5.5$. The thick line corresponds to the case with $k = 0.18$ and the dotted line to the case with = 0.8. It is shown that the choice of k is important to stabilize UPO.

We investigate the sensitivity of controlling chaos on initial conditions for each period τ with an optimized value of k. It is found that the control of chaos on the period-one cycle ($\tau = 5.90$) or two cycle ($\tau = 11.75$) does not depend on the initial condition. However, the control of chaos on the

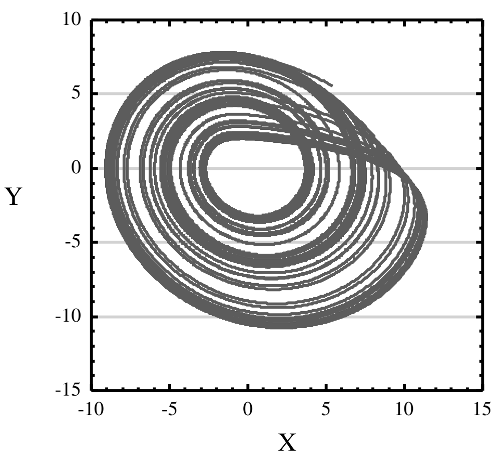

Fig. 1 Chaotic attractor of the Rossler system.

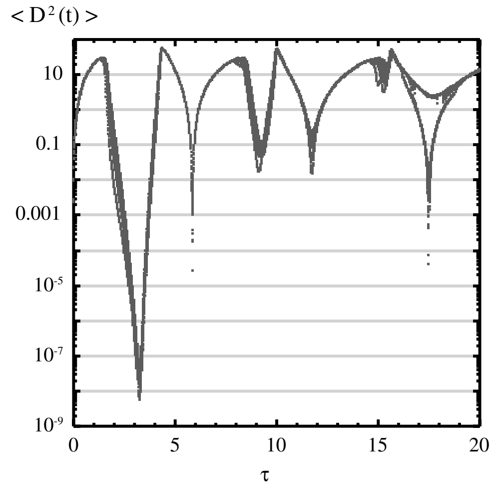

Fig. 2 Dependence of $\langle D^2(t)\rangle$ on τ.

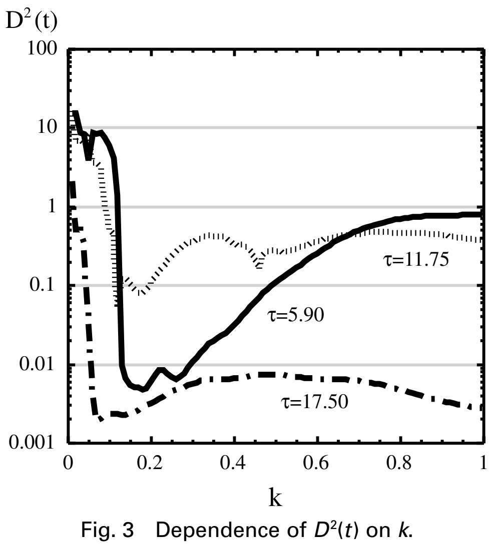

Fig. 3 Dependence of $D^2(t)$ on k.

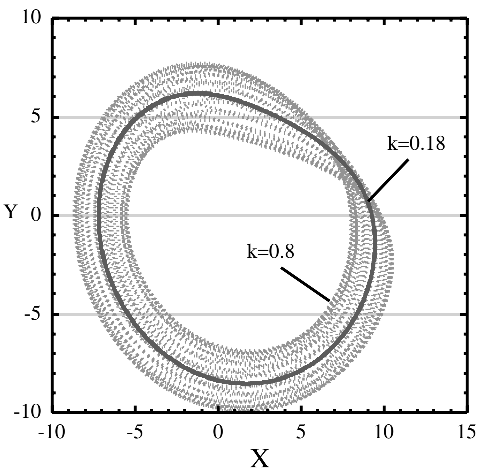

Fig. 4 Period-one cycle of the Rossler system for $\tau = 5.9$ and $k = 0.18$ and 0.8.

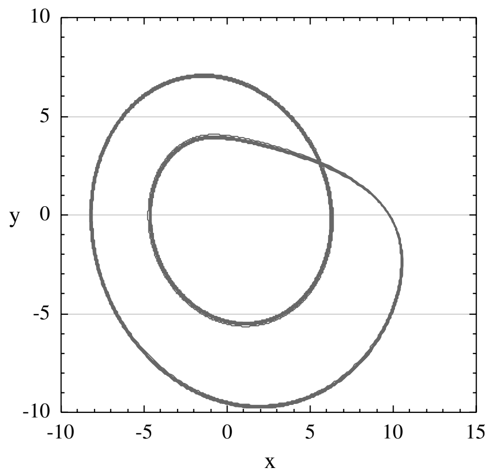

Fig. 5 the x-y phase portrait of the Rossler system controled by adding a perturbation.

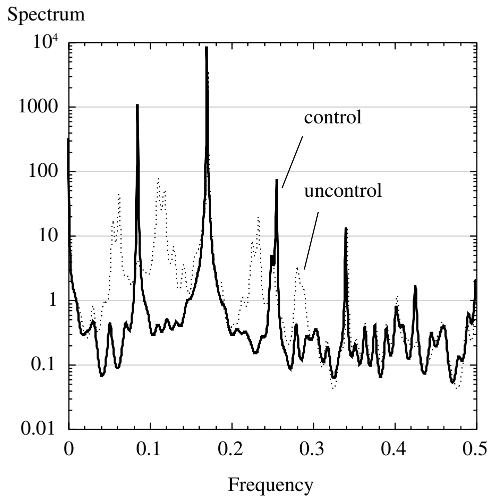

Fig. 6 Dependence of power spectrum on frequency.

period three cycle (τ = 17.5) depends on the choice of the initial condition.

Figure 5 shows the $x - y$ phase portrait of the Rossler system with τ = 11.75 for t >150. It is successfully controlled to the period-two cycle. Figure 6 shows the dependence of the power spectrum on the frequency with/without control for τ = 11.75. The peak is observed at f = 0.085 for the case with control, which implies period-two cycle ($f = 1/\tau$). It should be mentioned that the chaotic orbits of the Rossler system also converge to UFPs, which correspond to τ = 3.30 and τ = 9.30. In these cases, the value of k weakly affects the stabilization of UFP compared with stabilization of UPO.

3. Chaos control by neural controller

In this section, we use neural network controller to control the chaotic system, i.e., the Rossler system. Figure 7 shows the block diagram of NNC. Here the NNC consists of two layers such as input and output layers [5]. The following chaotic system is considered:

$$\boldsymbol{X}(t+1) = \boldsymbol{F}[\boldsymbol{X}(t)] + \boldsymbol{U}(t), \tag{2}$$

where $\boldsymbol{X}(t)$ is the state vector, $\boldsymbol{F}$ denotes chaotic system and $\boldsymbol{U}(t)$ is the control signal. When the orbits $\boldsymbol{X}(t)$ and $\boldsymbol{X}(t-\tau)$ of the chaotic system without control satisfy

$$\| \boldsymbol{X}(t) - \boldsymbol{X}(t-\tau) \| < \varepsilon \tag{3}$$

the watcher passes a control signal from the NNC to the chaotic system, and then the weights of connection from the input neuron to output neuron in the NNC are updated by back propagation method. Initial values of the weights are determined within the range of –0.01 to 0.01 by a random number. ε is assumed to be a small positive value. The NNC is given by

$$\begin{aligned} U_i(t_k) &= f(O_i(t_k) + \theta_i(k)) \\ O_i(t_k) &= \sum\nolimits_{j=1}^{N} W_{ij}(k) X_j(t_k), \end{aligned} \tag{4}$$

where j is the number of input neuron, i, the number of output neuron and k is the number of times in which the NNC is trained (i.e., the number of times the watcher operates). W_{ij} is the weight of output neuron, and $O_i(t_k)$ is the signal which goes into neuron of the output layer through the weight from ith neuron of the input layer, and $\theta_i(k)$ is the bias of the ith neuron of the output layer. The weights and the biases are updated by using

$$\begin{aligned} W_{ij}(k+1) &= W_{ij}(k) + \Delta W_{ij}(k), \\ \theta_i(k+1) &= \theta_i(k) + \Delta\theta_i(k), \end{aligned} \tag{5}$$

where

$$\begin{aligned} \Delta W_{ij}(k) &= -\eta \frac{\partial E(k)}{\partial U_i(n_k)} \frac{\partial U_i(n_k)}{\partial W_{ij}(k)}, \\ \Delta\theta_i(k) &= -\eta \frac{\partial E(k)}{\partial U_i(n_k)} \frac{\partial U_i(n_k)}{\partial \theta_i(k)}. \end{aligned} \tag{6}$$

η is the learning rate and the error is defined by

$$E(k) = k_C E_C(k) + k_U E_U(k), \tag{7}$$

with

$$E_C = \frac{1}{2} [\boldsymbol{X}(t_k+1) - \boldsymbol{X}(t_k)]^T [\boldsymbol{X}(t_k+1) - \boldsymbol{X}(t_k)], \tag{8}$$

and

$$E_U = \frac{1}{2} \boldsymbol{U}(t_k)^T \boldsymbol{U}(t_k), \tag{9}$$

We apply the NNC to the Rossler system:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = -y - z + U_1(t) \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = x + 0.2 + U_2(t) \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = 0.2 + z(x-5.7) + U_3(t). \end{cases} \tag{10}$$

In this case, the NNC consists of three input neurons and three output neurons. The three cases with different threshold functions are investigated: (1) linear function $f(x) = x$ and other parameters are $\varepsilon = 1.0$, $\eta = 0.08$, $k_C = 3.0$ and $k_U = 1.0$, (2) the sigmoid function $f(x) = \frac{1}{1+e^{-x}}$, (3) the hyperbolic tangent function $f(x) = \frac{1-e^{-x}}{1+e^{-x}}$ for period-one, two and three. It is found that the case (1) fails for all cases; the case (2) fails in period-two and three cases for $k_C = 3.0$, $k_U = 1.0$. However, the case (2) successfully controls period-three for $k_U = 4.2$. The case (3) works for all cases without changing parameters k_C and k_U. These results are shown in Figs. 8 and 9 for period-one and two cases. The dotted line corresponds to the case with the linear function and the thick line to the case with the hyperbolic tangent function. It is concluded the hyperbolic tangent function is the best candidate as the threshold function of NNC for controlling the Rossler system. The numbers of updating weights in the NNC is a good indicator whether the NNC successfully controls the chaos, or not.

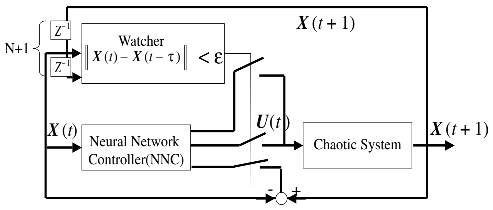

Fig. 7 Block diagram of the control system.

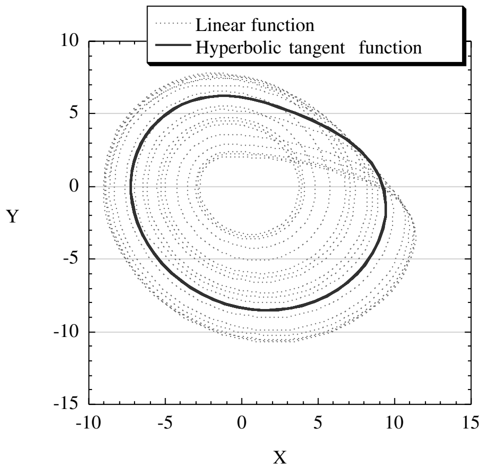

Fig. 8 Period-one cycle of the Rossler system with NNC.

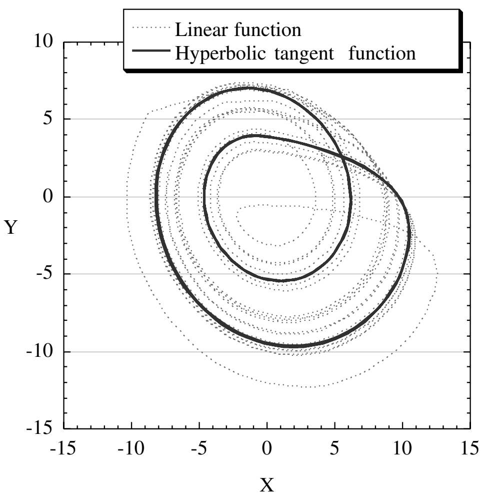

Fig. 9 Period-two cycle of the Rossler system with NNC.

Finally, we extend the NNC with three layers and examine the effect of numbers of neurons in hidden layer on controlling chaos. However, no advantage is found compared with the NNC with two layers. Therefore we conclude that the NNC with two layers and hyperbolic tangent function as the threshold function shows the best peroformance to control the Rossler system.

4. Summary and discussion

The Pyragas method and the on-line trained NNC are tested for the Rossler system. It is found that the UPO of the Rossler system is stabilized by the delayed feedback controlling method, however, the controllability depends on the choice of the initial condition and the gain of perturbation. The NNC with linear threshold function works well for the control of the chaos in map systems [6], however, it fails in the application to the Rossler system as is shown in this paper. The NNC is improved by changing the threshold function such as the sigmoid function and hyperbolic tangent function. We find that the NNC with hyperbolic tangent function shows the best performace to control the Rossler system. We also examine the NNC with three layers, however, no advantage is found in comparison with the NNC with two layers for the Rossler system. Once we find the suitable value of k, then the delayed feedback controlling method is faster than the NNC with hyperbolic tangent function, although it is not always possible for the general case.

Fully developed plasma turbulence has a large degree of freedom, therefore, simple neural network system will not work. As a future work, we will apply 3 layers neural network controller to plasma turbulence.

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References

[1] P.Y. Cheung and A.Y. Wong, Phys. Rev. Lett. **59**, 551 (1987).
[2] T. Fukuyama, K. Taniguchi and Y. Kawai, Phys. Plasmas **9**, 1570 (2002).
[3] P. Cvitanovic, *Universality in Chaos* (Adam Hilger Ltd, Bristol, 1984).
[4] E. Ott, C. Grebogi and J.A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).
[5] K. Pyragas, Phys. Lett. A **170**, 421 (1992).
[6] K. Konishi and H. Kokame, Physica D **100**, 423 (1997).