J. Plasma Fusion Res. SERIES, Vol.6 (2004) 227–229

HINT Computation of LHD Equilibrium with Zero Rotational Transform Surface

KANNO Ryutaro, TOI Kazuo, WATANABE Kiyomasa, HAYASHI Takaya, MIURA Hideaki, NAKAJIMA Noriyoshi and OKAMOTO Masao

National Institute for Fusion Science, Toki, 509-5292, Japan

(Received: 9 December 2003 / Accepted: 20 February 2004)

Abstract

A Large Helical Device equilibrium having a zero rotational transform surface is studied by using the three dimensional MHD equilibrium code, HINT. We find existence of the equilibrium but with formation of the two or three $n = 0$ islands composing a homoclinic-type structure near the center, where n is a toroidal mode number. The LHD equilibrium maintains the structure, when the equilibrium beta increases.

Keywords:

MHD equilibrium, zero rotational transform surface, Large Helical Device

1. Introduction

In the Large Helical Device (LHD) [1], an MHD equilibrium with both deep magnetic well and high magnetic shear in the plasma core region attracts much attention from a point of view of improved MHD stability and plasma confinement. In the LHD, such an equilibrium can be realized by a large Ohkawa current [2-4] induced by counter neutral beam injection. In a plasma with a net subtractive toroidal current of about –100 kA/T, the rotational transform is expected to be below zero around the magnetic axis.

A helical equilibrium with a zero rotational transform surface was studied in Heliotron E experiment [5]. When a rotational transform at the center was below zero, strong MHD activities were observed and were guessed to be explained by an $m/n = 1/0$ resistive tearing mode, where m is a poloidal mode number and n is a toroidal mode number. The result seemed to be understood by a numerical analysis employing a low beta resistive MHD model for a straight heliotron-like configuration [6]. The numerical study of ref. [6] also showed that when the resonant surface existed near the axis, the $m/n = 1/0$ tearing instability was weak and the magnetic island width saturated. This result suggests the possibility of existence of the equilibrium having a zero rotational transform surface. From these previous studies [5,6], at first we should start to investigate whether or not an LHD equilibrium having a zero rotational transform surface can be allowed.

2. HINT computation

Numerical analysis of the equilibrium is carried out by using the HINT code [7-10]. The standard scheme of HINT is shown as follows. The first step (A-step) is a relaxation process of pressure along magnetic field lines under a condition of a fixed magnetic field $\boldsymbol{B}$. To expedite the pressure relaxation, we make an average of pressure, $\bar{p}$, along a field line, and replace a value of pressure at each grid point with the average $\bar{p}$:

$$p(u^1, u^2, u^3) \Rightarrow \bar{p}(u^1, u^2, u^3) = \frac{\int \mathrm{d}\ell \, p/|B|}{\int \mathrm{d}\ell/|B|}, \qquad (1)$$

where ℓ is a length along a field line starting from a grid point (u^1,u^2,u^3). Toroidal periods of tracing a field line are typically 20–60 in calculations of this article. In the second step (B-step), calculation of a net toroidal current density $\boldsymbol{j}_{net}$, e.g. Ohkawa current, is carried out under conditions of fixed pressure and magnetic field, if a net toroidal current exists in the equilibrium. The third step (C-step) is a relaxation process of magnetic field under a fixed pressure profile:

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p + \boldsymbol{j} \times \boldsymbol{B}, \qquad (2)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} = \nabla \times [\boldsymbol{v} \times \boldsymbol{B} - \eta(\boldsymbol{j} - \boldsymbol{j}_{net})], \qquad (3)$$

where $\boldsymbol{j} = (1/\mu_0)\, \nabla \times \boldsymbol{B}$ is the total current density. To calculate an equilibrium with a net toroidal current, we modify the Faraday equation, as shown in equation (3), which is given by the following idea. When an equilibrium with a net toroidal current has flux surfaces, Ohm's law can be modified as $\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \eta(\boldsymbol{j} - \boldsymbol{B}\langle \boldsymbol{j}\cdot\boldsymbol{B}\rangle_{net} / \langle B^2 \rangle)$, where $\langle \cdots \rangle$ means the flux surface average. Since the existence of nested flux

Corresponding author's e-mail: kanno@nifs.ac.jp

surfaces is not assumed in the HINT code, a toroidal flux is given by using contour lines of pressure. In an island and/or the ergodic region, the flux is estimated by an interpolation. Then the Faraday equation can be extended as

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} = \nabla \times \left[\boldsymbol{v} \times \boldsymbol{B} - \eta \left(\boldsymbol{j} - \boldsymbol{B} \frac{\langle \boldsymbol{j} \cdot \boldsymbol{B} \rangle_{net}}{\langle B^2 \rangle} \right) \right]. \quad (4)$$

If there exist flux surfaces in a steady state, we have $\langle \boldsymbol{E} \cdot \boldsymbol{B} \rangle = 0 = \eta(\langle \boldsymbol{j} \cdot \boldsymbol{B} \rangle - \langle \boldsymbol{j} \cdot \boldsymbol{B} \rangle_{net})$. Thus $\langle \boldsymbol{j} \cdot \boldsymbol{B} \rangle$ becomes $\langle \boldsymbol{j} \cdot \boldsymbol{B} \rangle_{net}$ in a steady state, and we obtain an equilibrium with a net toroidal current. Note that in the HINT computation, a numerical equilibrium on the way to a steady state is not meaningful in a sense of the MHD physics, and the scheme is justified when the numerical equilibrium relaxes sufficiently into a steady state.

A relaxation process computed by the HINT code starts from the vacuum configuration with B_0 = 1.5 T and R_0 = 3.75 m, and the initial pressure profile given as $p = p_0(1 - s^4)(1 - s)$, where B_0 is the magnetic field strength at the magnetic axis, R_0 is the major radius of the axis, p_0 is pressure at the axis, and s is the normalized toroidal flux. As shown in Fig. 1, we find that an LHD equilibrium having a zero rotational transform surface is possible to exist. Here the equilibrium beta value β is 0.56 %, and the total net toroidal current I_t is –100 kA/T. We assume that a net toroidal current density modeling the Ohkawa current is given as $j \propto -(p/p_0)^2$. Profiles of rotational transforms and pressure are plotted in Fig. 2. In the field line structure of Fig. 1, we see two islands. The central island has a negative $\iota/2\pi$, and the other island with the n = 0 mode, located around the central one, has a zero rotational transform around an O-point of the central island, as shown in Fig. 2. When the equilibrium beta increases to 1.7 %, the inner region sufficiently away from the separatrix of the later island is split into two parts, and a doublet-type n = 0 island having only one X-point located at Z = 0 is formed, see Fig. 3. Note that the LHD equilibrium maintains a homoclinic-type structure [9,11] composed by the islands near the center, when β increases; see Figs. 1, 3 and 4. We can consider that this field line structure is general for both helical and tokamak plasmas, because the toroidal mode number of the islands is zero. The magnetic axis and the central rotational transform are plotted with an absolute value of the

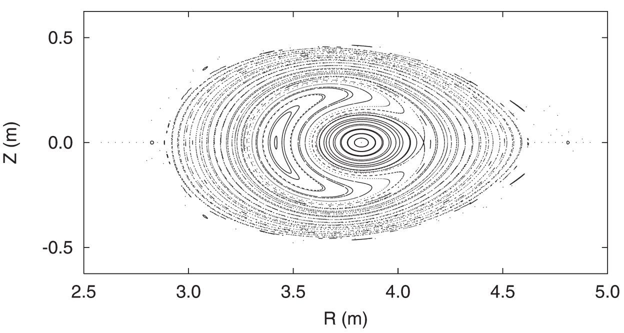

Fig. 1 Poincaré plots of field lines at the horizontally elongated poloidal cross section in the LHD equilibrium with $\beta \approx 0.56$ %. The total net toroidal current I_t is –100 kA/T.

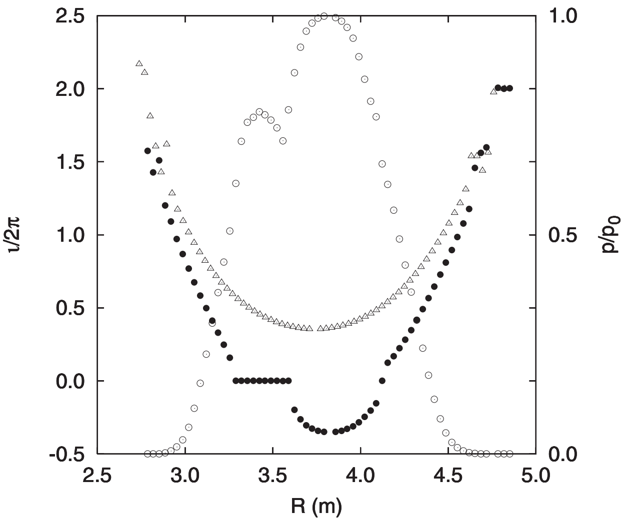

Fig. 2 Profiles of rotational transform $\iota/2\pi$ (solid circle) and pressure (open circle) along Z = 0 for $\beta \approx 0.56$ % and I_t = –100 kA/T. Open triangles represent $\iota/2\pi$ in the vacuum field.

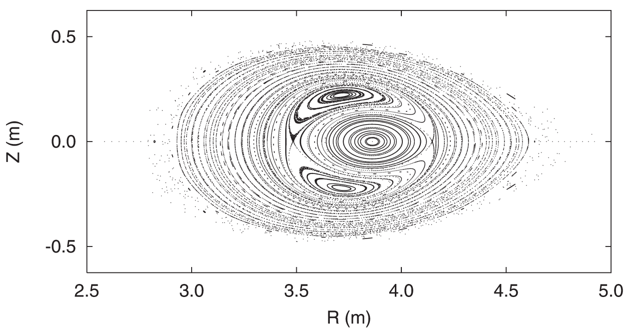

Fig. 3 Poincaré plots of field lines at the horizontally elongated poloidal cross section in the LHD equilibrium with $\beta \approx 1.7$ %. The total net toroidal current I_t is –100 kA/T.

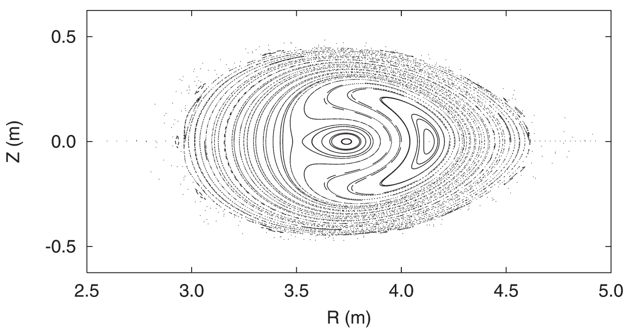

Fig. 4 Poincaré plots of field lines at the horizontally elongated poloidal cross section in the LHD equilibrium with $\beta \approx 3.8$ %. The total net toroidal current I_t is –100 kA/T.

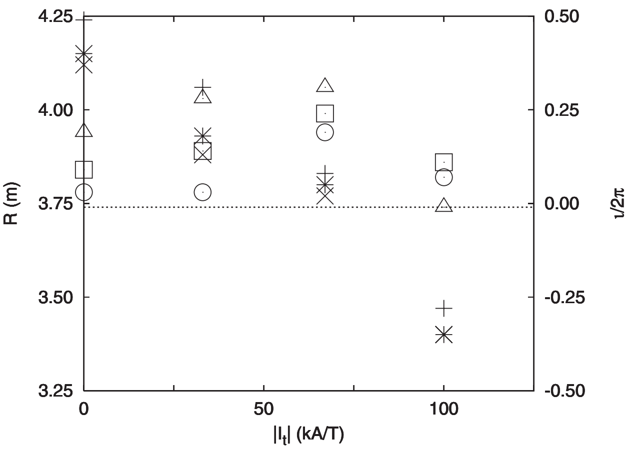

Fig. 5 The Shafranov shift of the axis for 1) open circle: $\beta \approx$ 0.56 %, 2) open square: $\beta \approx$ 1.7 %, and 3) open triangle: $\beta \approx$ 3.8 %. A dotted line represents the positions of the axis in the vacuum. The central rotational transforms for 1) cross (×): $\beta \approx$ 0.56 %, 2) asterisk (∗): $\beta \approx$ 1.7 %, and 3) plus (+): $\beta \approx$ 3.8 % are also shown.

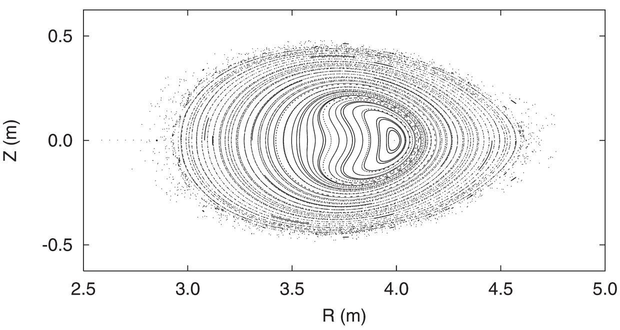

Fig. 6 Poincaré plots of field lines at the horizontally elongated poloidal cross section in the LHD equilibrium with $\beta \approx$ 1.7 %. The total net toroidal current I_t is –(100/1.5) kA/T.

total net toroidal current $|I_t|$ in Fig. 5. As $|I_t|$ increases, the axis shift to the outside of the torus increases and the rotational transform around the center decreases to zero. When the central rotational transform crosses zero, however, the Shafranov shift of the axis reduces. This is because of the topological change of the field line structure around the center; for example, see Figs. 3 and 6.

3. Conclusions

By using the HINT code, we have found that an LHD equilibrium having a zero rotational transform surface is possible to exist. And we have seen that the LHD equilibrium maintains the homoclinic-type structure composed by the islands near the center, when β increases. Let us consider comparison to the tokamak plasma with the current hole. Recently, an idea of an axisymmetric tri-magnetic-islands (ATMI) equilibrium has been proposed in ref. [12], where the ATMI equilibrium is one of the candidates for a tokamak equilibrium having a zero rotational transform surface. The ATMI equilibrium has three islands with $n = 0$ near the center, and they compose the heteroclinic-type structure [9,11]. As contrasted with ref. [12], in this article we have suggested that the homoclinic-type structure is also allowed. Note that the structure shown in this article is general in both helical and tokamak plasmas because of the islands having the toroidal mode number $n = 0$.

A numerical study of the nonlinear stability in a tokamak-like equilibrium having a negative central current [13] suggests that the equilibria obtained here require to be examined on an MHD stability of $n = 0$ modes. Then the nonlinear stability of the equilibria are studied now by using the nonlinear simulation code, which is a natural extension of the HINT code [14], and the results will be reported in near future.

References

[1] O. Motojima, N. Ohyabu, A. Komori *et al.*, *Proc. 19th IAEA Fusion Energy Conf.* Lyon, 2002, Paper OV/1-6.
[2] T. Ohkawa, Nucl. Fusion **10**, 185 (1970).
[3] D.F.H. Start, P.R. Collins, E.M. Jones, A.C. Riviere and D.R. Sweetman, Phys. Rev. Letters **40**, 1497 (1978).
[4] W.H.M. Clark, J.G. Cordey, M. Cox, R.D. Gill, J. Hugill, J.W.M. Paul and D.F.H. Start, Phys. Rev. Lett. **45**, 1101 (1980).
[5] H. Zushi, O. Motojima, H. Kaneko, M. Wakatani, A. Iiyoshi and K. Uo, J. Phys. Soc. Jpn. **57**, 3009 (1988).
[6] H. Shirai, M. Wakatani, H. Zushi, Y. Nakashima, O. Motojima, A. Iiyoshi and K. Uo, J. Phys. Soc. Jpn. **54**, 579 (1985).
[7] T. Hayashi, Theory of Fusion Plasmas (Varenna), 11 (1989).
[8] K. Harafuji, T. Hayashi and T. Sato, J. Comput. Phys. **81**, 169 (1989).
[9] T. Hayashi, T. Sato, H.J. Gardner and J.D. Meiss, Phys. Plasmas **2**, 752 (1995).
[10] R. Kanno, N. Nakajima, T. Hayashi and M. Okamoto, J. Plasma Phys. **61**, 213 (1999).
[11] E.A. Jackson, *Perspectives of Nonlinear Dynamics* (Cambridge University Press, Cambridge, 1991).
[12] T. Takizuka, J. Plasma Fusion Res. **78**, 1282 (2002).
[13] A. Ishizawa, M. Sato and M. Wakatani, Phys. Plasmas **10**, 3017 (2003).
[14] H. Miura, T. Hayashi and T. Sato, Phys. Plasmas **8**, 4870 (2001).