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Effect of Ion Viscosity on Neoclassical Tearing Mode

YOSHIDA Shigeki, ITOH Sanae-I.[1], YAGI Masatoshi[1] and AZUMI Masafumi[2]
Kyushu University, Interdisciplinary Graduate School of Engineering Sciences, Kasuga, Japan
[1]Kyushu University, Research Institute for Applied Mechanics, Kasuga, Japan
[2]Japan Atomic Energy Research Institute, Ibaraki, 311-0193, Japan
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Abstract

Linear stability analysis of neoclassical tearing mode (NTM) is performed on the basis of four-field reduced magnetohydrodynamic (MHD) model which takes account of fluctuating ion parallel flow and ion neoclassical viscosity. The dependence of the growth rate on the kinetic effects is investigated. It is shown that the linear NTM is stabilized by ion neoclassical viscosity and that the stabilizing effect of ion parallel compressibility is weak in the banana-plateau regime. It is found that not only ion neoclassical viscosity but also both ion and electron diamagnetic effects are important for the stabilization of NTM.

Keywords:
neoclassical tearing mode, free energy source, ion neoclassical viscosity, ion neoclassical flow, plasma β value, pressure gradient, four-field model

1. Introduction

The magnetic islands are often observed in high β (the ratio of plasma pressure to magnetic pressure) tokamak plasmas [1]. To achieve high performance in fusion plasmas, it is necessary to understand the physical mechanism of island excitation and its saturation level in high β plasma and the associated collapse phenomena. Several works on neoclassical tearing mode (NTM) have been done [2-4] and it is found that saturated island width, which is determined by the competition between the driving term due to the fluctuating bootstrap current and the change in the logarithmic derivative of the outer solution across the resistive layer $\varDelta'$ is not inconsistent with experimental observations [5]. However, the onset condition or these dynamics are not fully understood from the view point of conventional approach. The previous analysis of NTM is essentially based on three-field model [2-4], in which the electron neoclassical viscosity is only taken into account assuming that the collision frequency is larger than the rotation frequency of the NTM, i.e. $\omega < \mu_i$. This assumption is inappropriate for the case of the high temperature collisionless plasma and in the time scale of the island evolution, where $\omega > \mu_i$ is held. In addition, the parallel compressibility is also neglected. In this paper, we analyze the linear stability of NTM based on the four-field reduced MHD model which includes the ion parallel flow and ion neoclassical viscosity [6,7]. Here, we call linear NTM for the tearing mode with flucutating bootstrap current and neoclassical flow damping independent of the sign of the tearing parameter $\varDelta'$ [2]. The dependence of the growth rate on $\varDelta'$, plasma β value, kinetic effects is systematically investigated. The numerical analysis is described in Sec. **2**. Summary and discussions are given in Sec. **3**.

Corresponding author's e-mail: shigeki@riam.kyushu-u.ac.jp

2. Four-field reduced MHD model equation and numerical analysis

We consider a cylindrical plasma of major and minor radii R_0 and a with a toroidal magnetic field B_0 in the coordinate (r, θ, z). To analyze the linear stability of NTM, the four-field reduced neoclassical MHD model [6,7] is introduced. This model consists of the vorticity equation:

$$\begin{aligned}&\frac{\partial}{\partial t}\nabla_\perp^2 F + [F, \nabla_\perp^2 F] - \alpha_i \nabla_\perp \cdot [p, \nabla_\perp F] \\ &= -\nabla_\parallel \nabla_\perp^2 A + \mu_i^{cl} \nabla_\perp^4 F \\ &- \frac{q}{\epsilon}\mu_i^{neo}\frac{\partial U_{pi}}{\partial r} - \frac{q}{\epsilon}\frac{m_e}{m_i}\mu_e^{neo}\frac{\partial U_{pe}}{\partial r},\end{aligned} \tag{1}$$

Ohm's law:

$$\begin{aligned}&\frac{\partial}{\partial t}(A - \alpha^2 \frac{m_e}{m_i}\nabla_\perp^2 A) \\ &= -\nabla_\parallel [F - (\alpha_i + \alpha_e) p] + \alpha^2 \frac{m_e}{m_i}[\phi, \nabla_\perp^2 A] + \eta_\parallel^{cl} \nabla_\perp^2 A \\ &- 4\mu_e^{cl}\alpha^2 \frac{m_e}{m_i}\nabla_\perp^4 A + \alpha h_{BS}\frac{m_e}{m_i}\mu_e^{neo} U_{pe},\end{aligned} \tag{2}$$

ion parallel momentum equation:

$$\frac{\partial}{\partial t} v_\parallel + [\phi, v_\parallel] = -\nabla_\parallel p + 4\mu_i^{cl}\nabla_\perp^2 v_\parallel - \mu_i^{neo} U_{pi} - \frac{m_e}{m_i}\mu_e^{neo} U_{pe}, \tag{3}$$

and the electron continuity equation:

$$\frac{\partial}{\partial t} p + [\phi, p] = \hat{\beta}\left[-\nabla_\parallel (v_\parallel + \alpha\nabla_\perp^2 A) + \eta_\perp^{cl}\nabla_\perp^2 p - \alpha\frac{m_e}{m_i}\frac{q}{\epsilon}\mu_e^{neo}\frac{\partial U_{pe}}{\partial r}\right], \tag{4}$$

where $F = \phi + \alpha_i p$ is the generalized potential.
The closed set of equations (1)-(4) is called as 'four field model' in this paper. On the other hand, the approximations $\nabla_\parallel v = 0$ and $U_{pi} = 0$ give the three-field closed set of equations, where

$$U_{pi} = v_\parallel + \frac{q}{\epsilon}\frac{\partial F}{\partial r}, \tag{5}$$

$$U_{pe} = v_\parallel + \alpha\nabla_\perp^2 A + \frac{q}{\epsilon}\frac{\partial}{\partial r}(F - \alpha p). \tag{6}$$

U_{pi} and U_{pe} are ion and electron fluctuating neoclassical flow which include the trapped particle effect in the tokamak plasma. The poloidal Alfven time $a/(\epsilon v_A)$ and the minor radius a are used for normalization [7]. The variables of four-field $\{\phi, A, v_\parallel, p\}$ are the fluctuating electrostatic potential, vector potential parallel to the magnetic field, parallel velocity and electron density, respectively. In this model, the ion and electron temperature T_i and T_e are assumed to be constant and uniform. The coefficients $\{\mu_i^{cl}, \mu_e^{cl}, \eta_\parallel^{cl}, \eta_\perp^{cl}\}$are classical ion viscosity, electron viscosity (hyper-resistivity), parallel and perpendicular resistivity. μ_e^{neo} and μ_i^{neo} are neoclassical electron and ion viscosity expressed by the interpolation formula [6,7]. α is the normalized ion skin depth $(c/a\omega_{pi})$, $\alpha_e = \alpha/(1+\tau)$ and $\alpha_i = \alpha\tau/(1+\tau)$ with $\tau = T_i/T_e$. The last term of right hand side of equation (2) is the fluctuating bootstrap current term, where h_{BS} is a parameter to control the absolute value of flucutating bootstrap current arbitrarily. We set $h_{BS} = 1.0$ as a default value. β is the local plasma beta value and $\hat{\beta} = \beta/(1+\beta)$. q and ϵ indicate safety factor and inverse aspect ratio respectively. On the basis of the liniearized four-field model, the dependence of the growth rate of NTM on various physical effects is investigated solving eigen value problem. To check the validity of the code, we also developed the initial value code and compared the growth rate obtained by two codes.

At first, tearing mode is analyzed changing the absolute value of fluctuating bootstrap current.

The model safty factor profile and the pressure profile are given by

$$q(r) = \frac{q(r_s) - q(0)}{2^b - 1}[1 + (r/r_s)^a] + q(0) - \frac{q(r_s) - q(0)}{2^b - 1}, \qquad p(r) = \frac{\beta}{\epsilon}(1 - r^2)^2, \tag{7}$$

where r_s is the rational surface. We set $a = 3.0$, $b = 1.0$, $q(0) = 1.2$ and $r_s = 0.6$. In this case we have $\Delta' = 10.6$.
Figure 1 shows collisionality dependence of the growth rate γ of (2,1) mode in cases with $h_{BS} = 1.0, 2.0, 3.0, 4.0$. It is confirmed that the fluctuating bootstrap current destabilize the tearing mode in the banana-plateau regime (NTM), while the destabilization effect is weak for the classical tearing mode in the high collisionality regime.

The dependence of the growth rate on Δ' is analyzed and the result is compared with that of the three-field model. Δ' is calculated by changing $q(0)$ and fixing a, b, r_s in the model q profile. Figure 2 shows the dependence of the growth rate on Δ' in cases with collision frequency $\nu_e = 2.4$ (plateau regime) and $h_{BS} = 1.0$. The relation between $\mu_{i,e}^{neo}$ and ν_e is shown in ref. [7]. The dashed line and solid line indicate the results of the three-field model and the four-field model respectively. In the four-field case, the threshold value of the instability $\Delta'_c (\approx 10.2 > 0)$ is a finite positive number while $\gamma > 0$ holds for $\Delta' > 0$ in the three-field case. Although the absolute value of collision frequency where the growth rate is equla to zero, depends on the value of Δ', ion neoclssical viscosity is an essential factor to the stabilization of the NTM, which does not include in the three-field model.

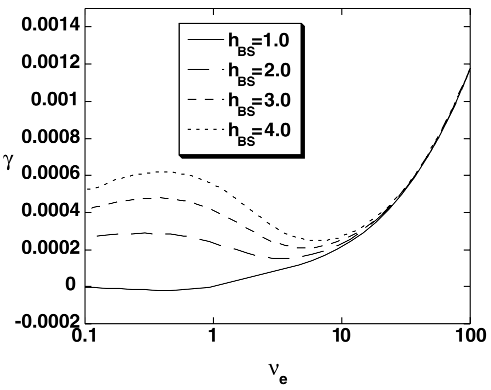

Fig. 1 The dependence of γ on collisionality in cases with $h_{BS} = 1.0, 2.0, 3.0, 4.0$.

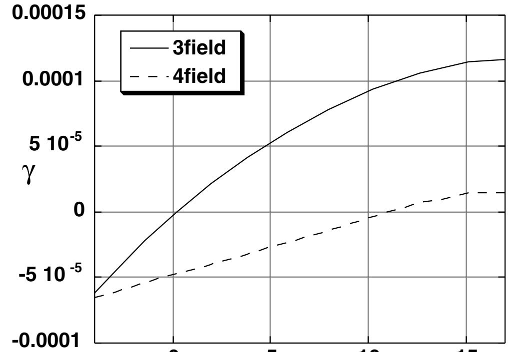

Fig. 2 The dependence of γ on Δ' in cases with three-field and four-field models. Parameters: $\beta = 0.01, \alpha = 0.01$, $\alpha_i = \alpha_e = \alpha/2$ are used.

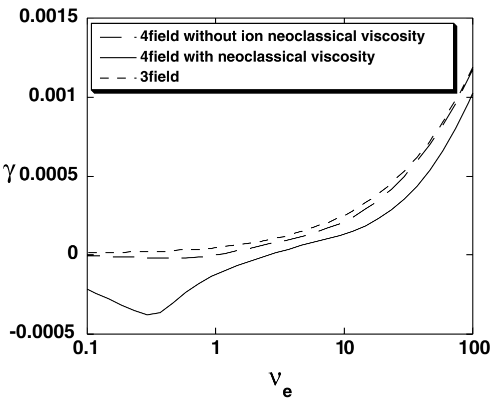

Fig. 3 The dependence of γ on collisionality in cases with three and four-field models. The solid line, dashed line and long dashed line indicate the results of four-field model, three-field model and four-field model without ion neoclassical viscosity, respectively.

The effect of ion neoclassical viscosity on the growth rate is analyzed. Figure 3 shows the dependence of the growth rate on collisionality. The solid line, dashed line and long dashed line indicate the results of four-field model, three-field model and four-field model without ion neoclassical viscosity, respectively. It is found that the NTM is strongly stabilized by ion neoclassical viscosity and effect of parallel compressibility is rather weak.

Next, ion and electron diamagnetic effects on the NTM are analyzed. Essentially, two terms contribute the stability. One is the ion diamagnetic effect proportional to α_i which appears in the vorticity equation and the other is electron diamagnetic effect proportional to α_e in the Ohm's law. Figure 4 shows collisionality dependence of the growth rate in cases with various values of α_i and α_e fixing $\alpha = 0.01$. In this case, the relation $\alpha = \alpha_i + \alpha_e$ is not held except $\alpha_i = \alpha_e$. The solid line, long dashed line and dashed line indicate the results of $\alpha_i = \alpha_e = 0.005, \alpha_i = 0, \alpha_e = 0.005$ and $\alpha_i = 0.005$, $\alpha_e = 0$ respectively. It is found that synergetic effect of ion neoclassical viscosity and both ion and electron diamagnetic effects give rise to the stabilization of the NTM.

Finally, the finite-β effect on the NTM is examined. Figure 5 indicates the dependence of the growth rate on the collisionality in cases with $\beta = 0.005, 0.01, 0.02$. The solid line, long dashed line and dashed line indicate the results of $\beta = 0.005, 0.01, 0.02$, respectively. It is found that the growth rate becomes small as the β increases in collisional and plateau regime. The NTM in the plateau regime is stabilized by finite-β effect. In the banana regime, this effect is week.

3. Summary and discussion

The linear stability of the NTM is investigated based on four-field reduced MHD equations in which ion neoclassical viscosity and fluctuating ion parallel flow are taken into account. The results are compared with those from conventional three-field model which only includes the electron neoclassical viscosity.

At first, we confirm that the NTM is destabilized by the fluctuating bootstrap current in the banana-plateau regime.

It is found that the stable regime of the NTM exists even if $\Delta' > 0$ in four field model which implies the careful examinations are necessary for experimental interpretations.

The threshold value, Δ'_c, depends on the collisionality and we obtain $\Delta'_c \approx 10.2$ for the case with $\nu_e = 2.4$ (plateau regime). To clarify whether ion neoclassical viscosity or ion parallel flow play the more important role on the stabilization of the NTM, the dependence of the compressibility on the growth rate without ion neoclassical viscosity is investigated. It is found that the ion neoclassical viscosity stabilizes the NTM rather than the ion parallel compressibility does in the banana-plateau regime. The stability effect is weak for tearing mode in the collisional regime. It is concluded that the synergetic effect of ion neoclassical viscosity and both ion and electron diamagnetic effects stabilize the NTM. Finally, the finite-β effect is shown to stabilize the tearing mode and NTM in plateau regime.

More detailed analysis is necessary to discuss the

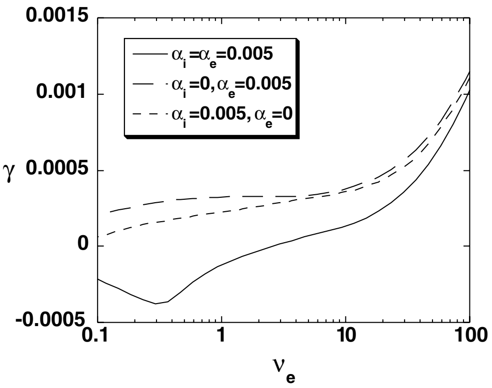

Fig. 4 The dependence of γ on collisionality in cases with/without ion and electron diamgnetic effects. The solid line, long dashed line and dashed line indicate the results of $\alpha_i = \alpha_e = 0.005, \alpha_i = 0, \alpha_e = 0.005$ and $\alpha_i = 0.005, \alpha_e = 0$, respectively.

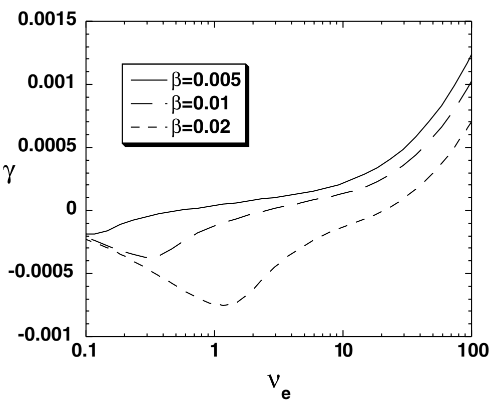

Fig. 5 The dependence of γ on collisionality in cases with various values of β. The solid line, long dashed line and dashed line indicate the results of $\beta = 0.005, 0.01, 0.02$, respectively.

threshold value quantitatively for various current and pressure profiles. In addition, to clarify the onset of magnetic island or subcritical bifurcation in high β experiments, the nonlinear simulation of the NTM is necessary. It is left as a future work.

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