J. Plasma Fusion Res. SERIES, Vol. 4 (2001) 487–491

Stability and Radial Structure of Drift-Waves in the Presence of Dust

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(Received: 5 December 2000 / Accepted: 16 August 2001)

Abstract

It is shown that the well-known stability of the drift wave in a sheared slab geometry does not hold in the presence of dust particles. Due to the presence of dust particles in tokamaks, the magnetic shear damping is reduced drastically. As a result, both the collisionless and collisional (dissipative) drift modes become unstable under typical parameter regimes of tokamak. Consequently, drift wave must still be considered as an underlying dynamic of anomalous transport in tokamak edges, where dust particles are found to be abundant.

Keywords:

dust, drift waves, transport, tokamak, shear

It is a common perception in tokamak plasma physics that drift-wave turbulence cannot be the agent behind energy and particle transport in tokamak edges [1]. This is, because the sheared slab modes are linearly stable [2] some interplay with toroidal effects is needed to provide a nonadiabatic electron response sufficient to drive them [3] and because the toroidal driving term (being proportional to $\frac{L_n}{R}$, where L_n is the density scale lenth and R is the major radius) falls towards the edge regions in contrast to the observed rise in saturated fluctuation amplitudes [4].

Before closing judgement on the viability of drift waves as an agent for transport, however, one should take into account the presence of dust particles in the plasma, since, the presence of dust has been confirmed in the edge of fusion devices like TEXTOR-94 [5]. Although the existence of dust has been known for a long time, only recently their presence in fusion devices and its possible consequences on plasma operation and performance have begun to be addressed [5-9]. Dust can be formed by evaporation and sublimation of wall material which is thermally overloaded, for example in the course of a disruption [5], or by spallation and flaking of films grown for wall conditioning (carbon-, boron- or siliconization) [10]. Large molecular ions have indeed been identified during carbonization [11]. Future fusion devices operating with D-T will have ^{3}He (due to decay of T), ^{4}He (due to neutron induced spallation reactions of low-Z wall material) and/or alpha-particles, all of these may induce embrittlement of the near-surface and lead to the ejection of grains. Furthermore, in the future long-pulse fusion devices, the growth and agglomeration of dust particles may occur via sputtering processes in the edge regions of plasma. It is believed that conditions prevailing at the edges of a detached limiter and a detached divertor are appropriate for such sputtering process [5]. Dust is therefore an important safety issue for ITER and for other future fusion reactors as many deleterious effects of dust have recently been predicted [5,6]. It is therefore extremely important to investigate what effects these dust particles

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have on the stability of microinstabilities and consequent plasma transport.

In this letter, we revisit the theory of drift waves in a sheared slab geometry in the presence of dust. We demonstrate that the well-known stability of drift waves in a sheared slab geometry does not hold in the presence of dust particle. Due to the presence of dust, the magnetic shear damping is reduced drastically and as a result, both collisionless and collisional drift waves become unstable for typical parameter regimes of tokamak operation. Consequently, drift wave must still be considered as an underlying dynamics of anomalous transport in tokamak edges.

We will first develop a nonlocal theory of *collisionless* drift waves in the presence of dust. We consider a multicomponent plasma of plane slab geometry with a uniform temperature but with a nonuniform density with density gradient in x direction. We also consider a sheared magnetic field, so that the magnetic fieldis represented by $\boldsymbol{B}_0(x) = B_0(\hat{e}_z + x/L_s\hat{e}_y)$, where L_s is the magnetic shear length and x is the distance from the mode rational surface defined by $\boldsymbol{k}\cdot\boldsymbol{B} = 0$. In this sheared magnetic field, the parallel wave number becomes $k_\parallel = k_y x/L_s$. Our plasma system consists of electrons, positive ions and negatively charged dust particles and the plasma is overall charge neutral, i.e., $n_\mathrm{i} = z_\mathrm{d} n_\mathrm{d} + n_\mathrm{e}$. A variety of competing processes such as photo-electric emission, secondary emission etc. determine whether a dust particle will be charged negatively or positively. In our case, keeping in mind the considerably higher mobility of the electrons with respect to the ions, we have considered dust particles to be negatively charged. We also assume that the variation of q_d with n_d is small in comparison to the variation of n_d with x [12]. We further assume that the positive ions and dust particles are cold and describe them by the usual fluid equations and the electrons are assumed to follow the Boltzmann relation [13]. To study the effect of electron Landau resonance and trapped electrons we consider the so-called $i\delta$ model. Dust particles are assumed to be unmagnetized. As the plasma is in homogeneous in the x direction only, the perturbation takes the form $\phi(\mathbf{x},t) = \phi(x)\exp[i(k_y y + k_z z - \omega t)]$. The continuity equation and the equation of parallel momentum can then be written as:

$$-i\omega n_\alpha + \nabla_\perp.[n_\alpha(x)\mathbf{v}_{\alpha\perp}] + ik_\parallel n_\alpha v_{\alpha\parallel} = 0 \qquad (1)$$

and

$$\omega v_{\alpha\parallel} = k_\parallel\left(\frac{q_\alpha}{m_\alpha}\right)\phi \qquad (2)$$

here

$$\nabla_\perp = ik_y\hat{e}_y + \hat{e}_x\frac{\mathrm{d}}{\mathrm{d}x} \qquad (3)$$

$$\mathbf{v}_{\alpha\perp} = \mathbf{v}_E + \mathbf{v}_{\alpha p} \qquad (4)$$

$$\mathbf{v}_E = -c\,\frac{(\nabla_\perp\phi\times\mathbf{B}_0)}{B_0^2} \qquad (5)$$

$$\mathbf{v}_{\alpha p} = i\left(\frac{c\omega}{B_0\omega_{c\alpha}}\right)\nabla_\perp\phi \qquad (6)$$

where, the subscript α represents the dust particles (d) and the positive ions (i). Here, $n_\mathrm{i}(n_\mathrm{d})$, $m_\mathrm{i}(m_\mathrm{d})$, $v_{\mathrm{i}\parallel}(v_{\mathrm{d}\parallel})$, $\omega_\mathrm{ci}(= eB_0/m_\mathrm{i}c)$, $\omega_\mathrm{cd}(= z_\mathrm{d}eB_0/m_\mathrm{d}c)$ are respectively the perturbed ion (dust particle) density, the ion (dust particle) mass, the parallel ion (dust particle) velocity, the ion and dust cyclotron frequency, whereas $v_{\alpha p}$ represents the polarization drift. Using the quasi-neutrality condition, we obtain the radial eigenvalue equation for the low frequency and long wavelength limit as [14-15]

$$\rho_\mathrm{H}^2\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - k_y^2\right)\phi - \left(1 - \frac{\omega_\mathrm{H}^\star + i\gamma}{\omega} - \frac{x^2}{x_s^2}\right)\phi = 0 \qquad (7)$$

where
$\rho_\mathrm{H}^2 = (1+\xi)\rho_\mathrm{s}^2 + \xi\rho_\mathrm{d}^2$, $\rho_\mathrm{s}^2 = c_\mathrm{s}^2/\omega_\mathrm{ci}^2$, $\rho_\mathrm{d}^2 = c_\mathrm{d}^2/\omega_\mathrm{cd}^2$, $c_\mathrm{H}^2 = (1+\xi)c_\mathrm{s}^2 + \xi c_\mathrm{d}^2$, $c_\mathrm{s}^2 = T_\mathrm{e}/m_\mathrm{i}$, $c_\mathrm{d}^2 = z_\mathrm{d}T_\mathrm{e}/m_\mathrm{d}$, $\omega_\mathrm{H}^\star = (1+\xi)\,\omega_\mathrm{e}^* - \xi\omega_\mathrm{d}^\star = \omega_\mathrm{e}^\star[(1+\xi) - \xi L_\mathrm{ni}/L_\mathrm{nd}]$, $\omega_\mathrm{e}^\star = -k_y c_s\rho_s/L_\mathrm{ni}$, $\omega_\mathrm{d}^\star = -k_y c_\mathrm{d}\rho_\mathrm{d}/L_\mathrm{nd}$, $\gamma = \omega_\mathrm{H}^\star\delta$, $x_\mathrm{s}^2 = \omega^2/k_\parallel'^2 c_\mathrm{H}^2$, $k_\parallel' = k_y/L_s$, $L_{ni}^{-1}(x) = |\mathrm{d}\ln n_\mathrm{i}/\mathrm{d}x|$, $L_\mathrm{nd}^{-1}(x) = |\mathrm{d}\ln n_\mathrm{d}/\mathrm{d}x|$, $\xi = z_\mathrm{d}N_\mathrm{d}/N_\mathrm{e}$ and $\mu = m_\mathrm{d}/m_\mathrm{i}$. Here, all symbols are assumed to have the usual meaning unless otherwise stated.

We consider the spatial variation of the diamagnetic drift frequency $\omega_\mathrm{H}^\star$ and consider the simple case in which $\omega_\mathrm{H}^\star$ is peaked at the mode rational surface at $x = 0$ and has a parabolic profile (thereby considering the most unstable situation), i.e. $\omega_\mathrm{H}^\star = \omega_\mathrm{0H}^\star(1 - x^2/L_\mathrm{0H}^2)$ where L_0H is the density variation scale length and will be assumed to be of the order of L_ni. With this, eq. (7) becomes

$$\rho_\mathrm{H}^2\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} + (\Lambda - Px^2)\phi = 0 \qquad (8)$$

where

$$\Lambda = \left(\frac{\omega_\mathrm{0H}^\star}{\omega} - k_y^2\rho_\mathrm{H}^2 + \frac{i\gamma}{\omega} - 1\right) \qquad (9)$$

$$P = \frac{1}{L_\mathrm{0H}^2} - \frac{L_\mathrm{ni}^2[(1+\xi) + \xi z_\mathrm{d}/\mu]}{\rho_s^2 L_s^2[(1+\xi) - \xi L_\mathrm{ni}/L_\mathrm{nd}]^2} \qquad (10)$$

In deriving eq. (8) we have assumed that $\omega \approx \omega_\mathrm{0H}^* =$

$|k_y \rho_s c_s [(1+\xi) - \xi L_{ni}/L_{nd}]/L_{ni}|$, which is usually true for drift type waves. Eq. (8) is a simple Weber equation. Depending on the sign of P, we have two types of solutions.

For $P > 0$, i.e., for

$$\frac{1}{L_{0H}^2} > \frac{L_{ni}^2[(1+\xi)+\xi z_d/\mu]}{\rho_s^2 L_s^2[(1+\xi)-\xi L_{ni}/L_{nd}]^2} \qquad (11)$$

the solution satisfying the physical boundary conditions, i.e., $\phi \to 0$ at $x = \pm\infty$ is given by

$$\phi(x) = \phi_0 \exp\left[-\frac{\sqrt{|P|}}{2\rho_H}x^2\right]$$

The mode therefore decays with x, i.e., it does not propagate and hence, is intrinsically undamped.

Now, for the opposite limit, when $P < 0$, i.e., for

$$\frac{1}{L_{0H}^2} < \frac{L_{ni}^2[(1+\xi)+\xi z_d/\mu]}{\rho_s^2 L_s^2[(1+\xi)-\xi L_{ni}/L_{nd}]^2} \qquad (12)$$

The solution is $\phi(x) = \phi_0 \exp\left[-i\frac{\sqrt{|P|}}{2\rho_H}x^2\right]$

Thus, in this case we get a nonlocalized mode with outgoing energy flux at $x = \pm\infty$. In the absence of any energy source feeding the wave, the perturbation will decay in time, because of convective wave energy leakage. The overall stability of the system is determined by the balance between this intrinsic damping and destabilizing effects modeled by the $i\delta$ term and is given by the dispersion relation

$$\Lambda = \rho_H \sqrt{P},$$

i.e.,

$$\left(\frac{(\omega_{0H}^\star + i\gamma)}{\omega} - 1 - k_y^2\rho_H^2\right) = i\rho_H\left(\frac{L_{ni}^2[(1+\xi)+\xi z_d/\mu}{\rho_s^2 L_s^2[(1+\xi)-\xi L_{ni}/L_{nd}]^2} - \frac{1}{L_{0H}^2}\right)^{1/2}$$

which yields the stability criterion

$$\gamma < \frac{\rho_H\left(\frac{L_{ni}^2[(1+\xi)+\xi z_d/\mu}{\rho_s^2 L_s^2[(1+\xi)-\xi L_{ni}/L_{nd}]^2} - \frac{1}{L_{0H}^2}\right)^{1/2}\omega_{0H}^\star}{(1+k_y^2\rho_H^2)} \qquad (13)$$

To analyze the stability criterion we will first consider a plasma without any dust particle. Without the dust contribution, the condition for stability of the mode is given by

$$\frac{1}{L_{0H}^2} < \frac{L_{ni}^2}{\rho_s^2 L_s^2} \qquad (14)$$

Putting representative values of the plasma parameters from TEXTOR-94, e.g., $L_s \sim R \sim 175$ cm., $L_{0H} \sim L_{ni} \sim a \sim 46$ cm., $\rho_s \sim 0.1$ cm. [15-16], it is easy to see that the above inequality is easily satisfied. In other words, we have recovered the well-known result that the drift wave is stable in a sheared slab geometry [2].

We will now consider the situation when dust particles are present. To facilitate comparison with the experiments, condition (13) is simplified (for $L_{nd} \sim L_{ni}$) to the condition for instability

$$\frac{1}{L_{0H}^2} > \frac{L_{ni}^2}{\rho_H^2 L_s^2} \qquad (15)$$

The experimental data of dust particles from fusion devices are, however, not in general available. We will, therefore, take the dust density as the same with the impurity density found in TEXTOR-94. The drastic reduction in the stabilizing role of the magnetic shear is, however, clear because of $\rho_H \gg \rho_s$. For an example, we assume the following representative values for a graphite (carbon) dust in the hydrogen plasma in TEXTOR-94 like tokamak (graphite grains have indeed been detected in TEXTOR-94 [5]), $L_s = 175$ cm., $L_{ni} \sim L_{nd} \sim L_{0H} \sim a \sim 46$ cm., $\rho_s \sim 0.1$ cm., $N_e = 10^{13}$ cm.$^{-3}$, $N_d = 10^{11}$ cm.$^{-3}$, $m_d = 10^8 m_i$ [15-19]. With these data, we find that the inequality (15) *is* satisfied! This, therefore, shows that due to the presence of dust particles the well-known stability of the slab drift wave is destroyed and the mode becomes unstable due to the drastic reduction in the magnetic shear damping. The exact growth rate is given by the dispersion relation

$$\omega = \frac{\omega_{0H}^\star + i\gamma}{(1+k_y^2\rho_H^2) + \rho_H\left(\frac{1}{L_{0H}^2} - \frac{L_{ni}^2[(1+\xi)+\xi z_d/\mu]}{\rho_s^2 L_s^2[(1+\xi)-\xi L_{ni}/L_{nd}]^2}\right)^{1/2}} \qquad (16)$$

In the following section, we will investigate the similar effect on the *collisional* (dissipative) drift wave. Following the same notations as before, the linearized radial eigenvalue equation can be written as [20]

$$\rho_H^2\left(\frac{d^2}{dx^2} - k_y^2\right)\phi - \frac{1}{1-i\eta}\left(1 - \frac{\omega_H^*(x)}{\omega} - \frac{k_\parallel^2 c_H^2}{\omega^2}\right)\phi = 0 \qquad (17)$$

where, $\eta = \nu_{ei} m_e \omega / k_\parallel^2 T_e$. Note with $\eta = 0$, the above equation reduces to that for the collisionless case eq. (7)

with $\gamma = 0$. Now choosing $\omega_H^*(x)$ profile as before the above equation reduces to

$$\rho_H^2\left(\frac{d^2}{dx^2}-k_y^2\right)\phi-\frac{x^2}{x^2-ix_{Rd}^2}$$
$$\left[1-\frac{\omega_H^*}{\omega}+\left(\frac{1}{L_{0H}^2}-\frac{L_{ni}^2}{\rho_H^2L_s^2}\frac{[(1+\xi)+\xi z_d/\mu]}{[(1+\xi)-\xi L_{ni}/L_{nd}]^2}\right)x^2\right]\phi=0 \quad (18)$$

where $x_{Rd}^2 = \omega\nu_{ei}/k_{\parallel n}'^2 v_{Te}^2$, v_{Te} being the electron thermal velocity. In order to solve the equation (17) we use the perturbation treatment. By using the independent variable $\tau_d = x/\lambda_d$ equation (17) transforms to

$$\left(\frac{d^2}{d\tau_d^2}+\gamma_d-\frac{\tau_d^2}{4}-\frac{\Lambda_d}{\tau_d^2-i\tau_{Rd}^2}\right)\phi(\tau_d)=0 \quad (19)$$

In deriving equation (18) we have used

$$\lambda_d^2=\frac{\rho_H x_{0d}}{2},\ \frac{1}{x_{0d}^2}=\frac{1}{L_{0H}^2}-\frac{L_{ni}^2}{\rho_H^2L_s^2}\frac{[(1+\xi)+\xi z_d/\mu]}{[(1+\xi)-\xi L_{ni}/L_{nd}]^2},$$

$$\gamma_d=-\frac{\lambda_d^2}{\rho_H^2}\left(1-\frac{\omega_{0H}^*}{\omega}+k_y^2\rho_H^2+i\frac{x_{Rd}^2}{x_{0d}^2}\right),$$

$$\Lambda_d=i\frac{x_{Rd}^2}{\rho_H^2}\left(1-\frac{\omega_{0H}^*}{\omega}+i\frac{x_{Rd}^2}{x_{0d}^2}\right),\ \tau_{Rd}^2=\frac{x_{Rd}^2}{\lambda_d^2}.$$

Here, $\tau_{Rd}^2 = x_{Rd}^2/\lambda_d^2 = 2(\nu_{ei}/\omega_H^*)(m_e/m_i)(L_s/L_n) \ll 1$, and hence can be treated as a small parameter. With $\tau_{Rd} = 0$, we get the equation of a parabolic cylinder having the Hermite function solution

$$\phi_n(\tau_d)=2^{-n/2}H_n(\tau_d/\sqrt{2})\exp(-\tau_d^2/4) \quad (20)$$

The eigen value is $\gamma_n = n + 1/2$, where $n = 0, 1, 2...$ By first order perturbation technique,

$$\delta\gamma_n=\frac{\int_{-\infty}^{\infty}d\tau_d\phi_n^2(\tau_d)\left(\frac{\Lambda_d}{\tau_d^2-i\tau_{Rd}^2}\right)}{\int_{-\infty}^{\infty}d\tau_d\phi_n^2(\tau_d)} \quad (21)$$

With $n = 0$ we get the lowest order even ϕ mode as follows

$$\delta\gamma_0\approx\frac{\sqrt{\pi}\Lambda_d}{2\tau_{Rd}}(1+i)$$
$$\approx\frac{i\lambda_d^2\sqrt{\pi}\tau_{Rd}}{2\rho_H^2}\left(1-\frac{\omega_{0H}^*}{\omega}\right)(1+i) \quad (22)$$

The dispersion relation for $n = 0$ is given by $\gamma_0 - \delta\gamma_0 = 1/2$, i.e.,

$$\frac{1-\omega_{0H}^*}{\omega}\simeq-(k_y^2\rho_H^2+\rho_H/x_{0d})$$
$$\left[1+\frac{\sqrt{\pi}}{2}(1-i)\left(\frac{\nu_{ei}m_eL_s^2\rho_H}{\omega_{0H}^*m_iL_{ni}^2x_{0d}}\right)^{1/2}\right] \quad (23)$$

With $n = 1$ we obtain the odd ϕ mode as

$$1-\frac{\omega_{0H}^*}{\omega}\simeq-(k_y^2\rho_H^2+3\rho_H/x_{0d})$$
$$+2i(k_y^2\rho_H^2+5\rho_H/2x_{0d})\frac{\nu_{ei}m_eL_s^2\rho_H}{\omega_{0H}^*m_iL_{ni}^2x_{0d}} \quad (24)$$

So, from equations (22) and (23), when Re $(x_{0d}) > 0$, i.e., when

$$\mathrm{Re}\left(\frac{1}{L_{0H}^2}-\frac{L_{ni}^2}{\rho_s^2L_s^2}\frac{[(1+\xi)+\xi z_d/\mu]}{[(1+\xi)-\xi L_{ni}/L_{nd}]^2}\right)>0, \quad (25)$$

the mode is unstable. It is interesting to note that this condition is the same as in the case of collisionless drift wave (condition (11)). So, choosing experimental data as before, it is easy to see when dust particle is not present the mode is stable, i.e., we obtain the usual result that collisional drift waves are stable in a sheared slab geometry [21]. However, the scenario is, as in the collisionless case, entirely different in the presence of dust. Because of the drastic reduction in the shear damping, the collisional branch also gets destabilized due to the presence of dust in the plasma.

In summary, we have revisited the theory of drift waves in a sheared slab geometry in the presence of dust. We demonstrate that the well-known stability of drift waves in a sheared slab geometry does not hold in the presence of dust particle. Due to the presence of dust, the magnetic shear damping is reduced drastically and as a result, both collisionless and collisional (dissipative) drift waves become unstable for typical parameter regimes of tokamak operation. Since the presence of dust has recently been confirmed in the edge of TEXTOR-94 and since it is also likely to be formed during the D-T operation, wall conditioning (carbon-, boron- and siliconization) phases and its growth and agglomeration may occur via sputtering process in the edge regions of future long pulse fusion devices, this excitation of drift instability by dust may be a crucial issue for ITER and other future fusion reactors. Finally, it is interesting to note that, in contrast to the excitation of toroidal Alfven eigenmodes (TAE) by alpha particles which are formed only in an ignited tokamak, the excitation of drift waves by dust particles shown here will effect the tokamak operation at a *much earlier stage*. This is because, the dust can be formed even during the wall conditioning phases.

This work is supported by the Japan Society for the Promotion of Science.

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