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Intensive Nonlinear Plasma Acceleration by Viscosity: A Possible Root of the Energetic Solar Wind

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Abstract

A study is done on the entropy acceleration of the plasma through a nozzle. It has been found that, if the flow velocity is set into the super thermal state, the viscous force couples with the inertial force to build up the positive feedback loop. The plasma ions can then be accelerated by the potential induced across the thin sheath formed by the feedback. Such process works out so long as the flow velocity is less than the electron thermal velocity. Hence, the ions can be accelerated along the flow up to the value of the order of the thermal velocity of electrons; the ion energy can be quite a high value of about electron energy times to the mass ratio of the ion to the electron. The two examples of the acceleration process are presented: a straight arc channel case and the solar wind case.

Keywords:

entropy acceleration, plasma acceleration, viscosity, Navier-Stokes, arcjet, plasma source, solar wind

1. Introduction

The study is given on the plasma source to be used for the electromagnetic (EM) plasma accelerator. The source has to eject out the plasma with the velocity greater than the thermal speed, since the EM force chokes up the subthermal plasma flow [1,2]. Hence, the source must be of the entropy acceleration type, none of which has ever shown to eject out the super thermal plasma stably. Here, the problem is reduced to the design of the nozzle, which turns the subthermal flow into the super thermal one at the exit muzzle. The theory is developed in a general form to cover the solar wind gravitational nozzle [3] with the aim using the observed data of the solar wind for to check up the model.

2. Formulation of the Basic Equations Set

A generalized nozzle flow including that of gravity is assumed to follow the momentum balance of the form below at steady state:

$$mn(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p - \nabla\cdot\pi + \mathbf{j}\times\mathbf{B} - GM_{\Theta}\, mn\mathbf{z}/z^3 , \quad (1)$$

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where m is the plasma ion mass, n the density, $\mathbf{v}$ the fluid velocity, p the pressure, π the stress tensor, $\mathbf{j}$ the arc current density, $\mathbf{B}$ the induction by the arc current, G the gravitational constant, M_{Θ} the solar mass and z the distance along the nozzle. The total particle flux along the nozzle is given by the ion current equivalent I, and is assumed to conserve along the nozzle:

$$I = envA , \quad (2)$$

where A is the nozzle cross-section changing slowly enough along the flow, and n and v are assumed to be only a function along the flow.

It is possible now to rewrite Eq. (1) using Eq. (2) in a more convenient form of a quasi 1-D model:

$$\left[\left(\frac{v}{v_{th}}\right)^2 - 1\right]\frac{v'}{v} = -\frac{T'}{T} + \frac{A'}{A} - \frac{1}{2p\,\mu_0 A}(<B^2>A)' + \frac{4}{3p}(\eta v')' - \frac{GM_{\Theta}}{(z\,v_{th})^2} , \quad (3)$$

where T is the plasma temperature, the thermal velocity of the plasma is given by $v_{th} = (2kT/m)^{1/2}$, η the plasma viscosity and the prime denotes d/dz. In Eq. (3), each term of RHS reflects the force acting on the flow. It is noted that if the flow velocity is increased to pass over the thermal point at $v = v_{th}$, then all the forces on the RHS begins to work opposite direction since v' must change its sign.

3. Plasma Acceleration through a Straight Arc Channel (SAC)

A schematic drawing of a typical SAC is given in Fig. 1. As is seen, the working D_2 is injected into the system from the gas inlet on the LHS end, and is heated up by a pre-heater into downstream to be fully ionized before it reaches to the cathode. The main intensive arc is assumed to develop between the cathode and the anode with uniform T. For acceleration through an SAC, the fourth viscosity term in RHS of Eq. (3) is only important if the fringing field near the two electrodes were ignored. It is possible now to rewrite Eq. (3) into Eq. (4) below of a normalized form using the parameters, $u = v/v_{th}$, $\varsigma = z/l_p$ and $g = c_\eta(A/I/l_p)^{5/2}$, where l_p is the SAC length and c_η the quantity of approximately constant involving the Coulomb logarithm:

$$(u^2 - 1)\,u' = g u^2 u'', \tag{4}$$

where the prime gives $d/d\varsigma$. Equation (4) is the Navier-Stokes and clearly shows that viscosity is quite important in a simple straight tube. Actually, if η were ignored, then Eq. (4) becomes trivial.

The first integral of Eq. (4) yields

$$u_T = \frac{1}{u_0} + u_0 - g u_0' = \frac{1}{u} + u - g u', \tag{5}$$

where u_0 and u_0' denote the values on the cathode surface and u_T is a constant. Equation (5) may be integrated again to give

$$\begin{aligned} \varsigma(u)/(g c_f) = {} & \tan^{-1}[c_f(2u/u_T - 1)] \\ & - \tan^{-1}[c_f(2u_0/u_T - 1)] \\ & + 1/(2c_f)[\ln(1 - u_T u + u^2) \\ & - \ln(1 - u_T u_0 + u_0^2), \end{aligned} \tag{6}$$

where c_f is a constant defined by $c_f = u_T/(4 - u_T^2)^{1/2}$. The form of c_f suggests that free choice of the boundary values on the cathode surface are not allowed, since u_T must be in the range $-2 < u_T < 2$ for the solution to exist. Hence, Eq. (5) gives the permissible range of u_0' below:

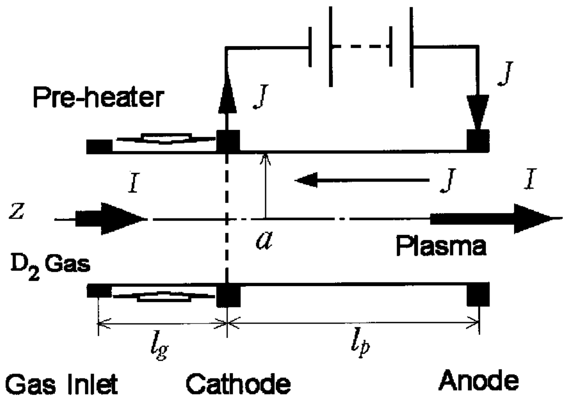

Fig. 1 The ideal arcjet system of a straight tube with a radius *a*. It is composed of the two major parts: the pre-heater of an injected cold gas and the intensive arc container. The pre-heater equips an RF oscillator or else. Cold D_2 gas is injected into the gas inlet port at a rate equal to ion current equivalent *I*. The gas is heated up and fully ionized toward downstream by the pre-heater. The plasma in the arc container keeps a high temperature, which is maintained by the arc current *J* as fed by the external power source.

$$u_0' > (u_0')_{min} = (u_0 + 1/u_0 - 2)/g\,, \tag{7}$$

$$u_0' < (u_0')_{max} = (u_0 + 1/u_0 + 2)/g\,, \tag{8}$$

For the purpose of drawing up a concrete picture of the system, a model arcjet system of $I = 1000$ A, $l_p = 10$ mm and the channel radius $a = 1.75$ mm is assumed. Since the fully ionized state is to be studied, $T > 3$ eV is postulated. Under the parameters above, $(u_0')_{max}$ in Eq. (8) does not give realistic restriction, and hence only $(u_0')_{min}$ is considered. The relation between u_0 and $(u_0')_{min}$ are depicted in Fig. 2 by taking T as a parameter. It is seen that larger the deviation of u_0 from unity and lower the T larger u_0' is required for the solution to exist.

The inverse function $u(\varsigma)$ of $\varsigma(u)$ is evaluated and depicted in Fig. 3 for three u_0' values as a function of ς by taking $u_0 = 0.9$ and $T = 6$ eV. Quite a rapid acceleration is seen to appear for all the cases. It is noted in the region of $u \gg 1$ that Eq. (5) gives $gu' \approx u$, which shows that u increases exponentially with ς so that the fluid is said to achieve a high energy by the viscous force. Exponential increase of u suggests that the positive feedback loop is set up between the viscous force and the inertial one. Setting up of the feedback loop is done via the particle flux conservation; the increase of the velocity decreases density to steepen the gradient, and hence the fluid is further accelerated. In an arc channel, the external power is injected through the electron channel, and the collision process heats up both ions and electrons. The viscous force is the built-in force in the ion channel and hence the characteristic scale length of acceleration $l_{acc}(\sim l_{pu}/u')$ is unable to be smaller than the mean free path λ_i of ion-ion collisions. Suppose that if $l_{acc} \approx \lambda_i$ is achieved, then electrons cross over the front to form a space potential. If that were the case, ions are further accelerated by the potential. However, if ions achieve the value of the order of the electron thermal velocity, no potential may be developed. This indicates that the upper limit of the beam energy must be limited to $\sim (m/m_e)T$, where m_e denotes the electron mass.

It may be of worth to study the role of the pressure force for the acceleration. To do so, the forces are normalized by the factor p/l_p. Then, Eq. (4) suggests the forms: the pressure force $f_p = u'/u$ and the viscous force $f_v = guu''$. If Eqs. (4), (5) and (7) are utilized, it is possible to have

$$f_p = \frac{1}{gu}\left\{u + \frac{1}{u} - 2 + g[\,u_0' - (u_0')_{min}\,]\right\}, \tag{9}$$

$$f_v = (u^2 - 1) f_p\,. \tag{10}$$

As is seen, f_p always takes a positive value while f_v changes its sign at the thermal point. Here, if Eq. (3) is recalled, negative f_v in the subthermal region is said to accelerate the flow. This concludes that Eq. (4) gives the only solution to accelerate the flow.

Lastly, it is pointed out that the principal driving force of the region in the left hand side of the cathode must be due to the temperature gradient toward the cathode.

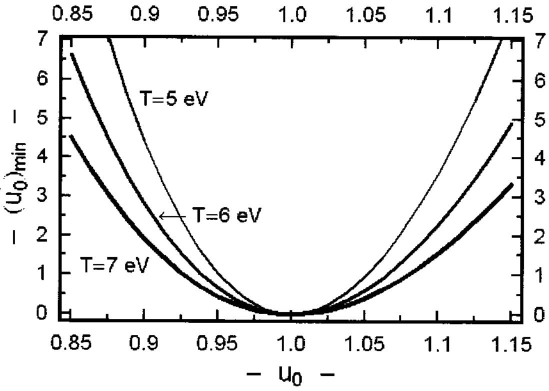

Fig. 2 The minimal initial acceleration $(u_0')_{min}$ of the plasma for three different temperatures as a function of the initial velocity u_0 on the cathode surface of the arc container. The plasma is accelerated downstream if u_0' is grater than $(u_0')_{min}$.

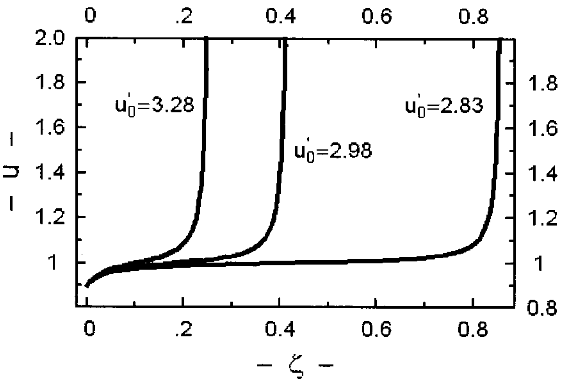

Fig. 3 The three traces of the plasma velocity u are shown along ς from the cathode surface under the plasma temperature of $T = 6$ eV. The three different u_0' are given for the same sub-thermal initial velocity at $u_0 = 0.9$.

4. The Gravitational Nozzle for the Solar Wind

The classical theory by Parker [3] is extended by taking the viscous force into account, since Eqs. (9) and (10) show that it is the principal plasma driving force in the super thermal region $u > \sqrt{2}$. Here, the same model with Parker's is adopted except that the viscous force is retained. Under this postulate, Eq. (3) is rewritten in the form below using the distance ς measured by the radius R_Θ of the sun:

$$(u^2 - 1)\frac{u'}{u} = \frac{2}{\varsigma}\left(1 - \frac{T_g}{2T}\frac{1}{\varsigma}\right) + C_{vis}\,\varsigma^2 uu'', \tag{11}$$

where $\varsigma = 1$ gives the surface of the sun, $T_g(= GM_\Theta m/2KR_\Theta)$ the temperature equivalent of the gravitational force, and $C_{vis}(= C_{\eta g}T^{5/2}/i_*R_\Theta)$ the coefficient giving the viscosity effect. The quantities $C_{\eta g}$ and i_* involved in C_{vis} have the following meanings: $C_{\eta g}$ is the similar quantity to C_η for the SAC flow, and $i_*(= \varsigma^2 env)$ is the proton flux through the unit base area of the cone on the surface of the sun, of which apex is at the center of the sun. It is noted that Eq. (11) derived by the quasi 1-D model gives exactly the same form to Parker, which assumes spherical symmetry of the system.

In the first place, the Parker curve is evaluated taking $C_{vis} = 0$. Then, Eq. (11) may be integrated. Among the solutions, the one giving the solar wind mode must be picked up. For that purpose, the requirement on the thermal point at $u = 1$ is available. Let the point giving the thermal position be ς_{th}, and then $\varsigma_{th} = T_g/2T$ must be held. If this requirement is applied, the integration of Eq. (11) is written as

$$u^2(1)/2 - \ln[u(1)] = T_g/T - 2\ln[T_g/2T] - 3/2. \quad (12)$$

The temperature T of the coronal hole is necessary for $u(1)$, and the value T = 130 eV cited in Ref. [4] is utilized to give $u(1)$= 0.0311. Since $u(1)$ is thus obtained, it becomes possible to draw up the solar wind velocity curve by Parker. Note that a typical one of such is given in Fig. 4.

The plasma density n_s of the coronal hole on the surface of the sun is required for the curve that takes viscosity into account. The data in Ref. [4] is also used and the value $n_s = 1.67 \times 10^{14}$ m^{-3} is assumed. Using the parker value of $u(1) = 0.0311$ and $u'(1) = 0.176$, Eq. (11) is numerically integrated and the curve is given in Fig. 4. As is seen, the difference is clear of the two. The "viscosity" curve shows rapid acceleration in the region $R_\Theta > 6$, which supports the observation shown in Ref [5].

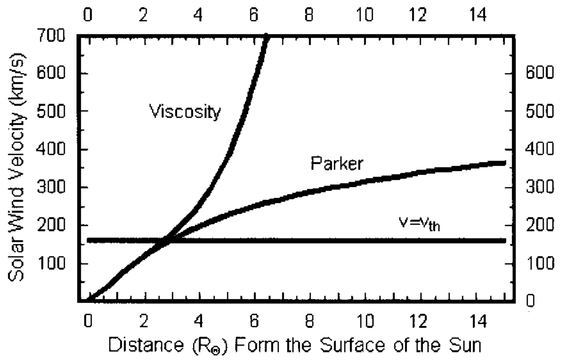

Fig. 4 The plasma acceleration by the viscous force is compared with the classical Parker mode.

5. Conclusion

A task of viscosity for the entropy acceleration of the plasma is shown. Viscosity is found to play the major role in accelerating the plasma under the flow of the super thermal state. The attainable limit of the beam energy is figured out to be ~ $(m/m_e)T$, which tells that even an arc plsama of ~ eV has a chance to be accerated up the beam of ~ keV range if the system is designed properly.

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