J. Plasma Fusion Res. SERIES, Vol. 4 (2001) 173–179

On Viscous Damping and Rotation in Low-Shear Stellarators

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(Received: 5 December 2000 / Accepted: 16 August 2001)

Abstract

One of the main results of the Wendelstein 7-AS stellarator (major radius 2 m, average plasma radius 0.18 m, magnetic field 2.5 T, low shear) is the achievement of the H-mode confinement in ECR- and NBI-heated plasmas. The features of the H-mode are similar to those of a tokamak: reduction of the H_α-emission, onset of poloidal rotation and a 30% improvement of the energy confinement time, which is acompanied by a reduction of the turbulence level. However, the critical dependence on the rotational transform is a particular feature of this low shear stellarator and in the following paper an attempt will be made to understand this phenomenon by investigating the specific influence of the magnetic geometry on the onset of poloidal shear flow. In the frame of a dissipative model of a steady-state plasma equilibrium with flow the poloidal and toroidal force balance on magnetic surfaces are investigated. The neoclassical poloidal viscosity has a strong dependence on the external rotational transform. The H-modes windows found in Wendelstein 7-AS can be identified with regions of high order rational magnetic surfaces, where viscous damping is small. The paper discusses the conditions for convective plasma flow, which in conjunction with the inertial forces is providing the spin-up mechanism of poloidal shear flow.

Keywords:
stellarator, plasma rotation, viscosity, convection, bifurcation

1. Introduction

L-H transitions in the low-shear Wendelstein 7-AS stellarator exhibit all characteristic features of L-H transitions found in tokamak experiments: a sudden decay of H_α-emission, increase of plasma confinement accompanied by a quiescent phase without significant MHD-activity and plasma fluctuations [1]. In the phase prior to the L-H transition ELMs have been observed which also exhibit intermittency leading to a steepening of the electron temperature gradient in the quiescent phase between ELMs. However, in contrast to tokamak experiments these L-H transitions only occur above a density threshold [2] and only in some narrow gaps on the iota-scale [3].

A particular feature of the H-mode windows is that these exist in the neighbourhood of low-order rational magnetic surfaces close to $\iota = 1/2$ and $\iota = 5/6$. The regions where in Wendelstein 7-AS L-H transition have been observed are around $\iota(a) \approx 0.48$, ≈ 0.52 and ≈ 0.55. In the H-mode window neither $\iota = 1/2$-surface nor $\iota = 5/6$ surface exists inside the plasma body. The upper boundary of the H-mode window is characterised by $\iota = 1/2$, $\iota = 10/19$ and $\iota = 5/6$ at the plasma boundary. On these magnetic surfaces magnetic islands exist, which are the natural result of the 5-fold symmetry of the magnetic field and the toroidal curvature. At the lower boundary we find the limit at $\iota(0) = 5/11$, $\iota = 5/10$ and $\iota = 10/19$. Magnetic surfaces inside the H-mode window have a rotational transform $\iota = m/n$ with $m \gg 1$, $n \gg 1$. Such surfaces are called „high order rational surfaces", while those surfaces with low m and n are „low order

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rational surfaces". Magnetic islands on the plasma boundary provide an enhanced mechanism of radial particle and momentum transfer and thus inhibit the build-up of a poloidal shear flow, which is considered as a basic feature of the H-mode physics. The effect of rotational transform and magnetic field line topology on poloidal viscosity has been investigated elsewhere [4]. Braginskii viscosity exhibits only a weak dependence on the rotational transform [5]. The viscosity in the plateau regime, however, does strongly depend on the rotational transform. On low order rational surfaces strong maxima arise which may lead to strong damping of a poloidal shear flow. The plateau viscous damping is small in those regions where the H-mode windows exist. In the following the model of a non-ideal plasma in the boundary region will be discussed. The H_α-emission indicates the presence of neutral hydrogen in the L-phase, while the H-phase the neutrals apparently have been reduced to a low level. Neutrals in the boundary region affect the parallel momentum balance and leads to small pressure gradients along the field lines. A similar effect arises due to parallel viscous forces and the inertial forces of the plasma rotation. Although these forces are small the resulting pressure gradients along fields lines can lead to large perpendicular gradients in poloidal direction and thus to large convective cells. Such a convective state can be laminar or turbulent, showing all modes of transition between these states. The H-mode is characterised by the absence of a large convective state, leading to a steepening of density and temperature gradients.

2. Plasma Equilibrium

To describe the plasma equilibrium in the boundary region we consider the momentum balance of a single-fluid model taking into account dissipative effects as viscosity, resistivity and plasma neutral interaction. The momentum balance and Ohm's law are

$$\nabla p = \boldsymbol{j} \times \boldsymbol{B} - \boldsymbol{F}_n - \nabla \cdot \pi(\boldsymbol{v}) - \nabla \rho \boldsymbol{v} : \boldsymbol{v} \tag{1}$$

$$-\nabla \Phi + \boldsymbol{v} \times \boldsymbol{B} = \eta \boldsymbol{j} \tag{2}$$

Futhermore, we retain the equation of continuity $\nabla \rho \cdot \boldsymbol{v} = S$ and $\nabla \cdot \boldsymbol{j} = 0$. The energy equation is

$$\nabla \cdot \left\{ \boldsymbol{q} + \frac{3}{2} p \boldsymbol{v} \right\} + p \nabla \cdot \boldsymbol{v} = Q \tag{3}$$

The ideal gas law $p = kT\rho/m_I$ correlates density to temperature. The heat flux is poportional to the temperature gradient $\boldsymbol{q} = -\chi \cdot \nabla \boldsymbol{T}$ with a strong anisotropy between parallel and perpendicular thermal conductivity χ. S is the mass source and Q is the energy source. The viscous forces are given by the Braginskii viscosity or by the neoclassical viscosity depending on the collisionality. The interaction with the neutral background is $\boldsymbol{F}_n = m_I n_0 N_0 <\sigma v> \boldsymbol{v}$, n_0 is the plasma density and N_0 the density of neutral hydrogen, σ is the charge exchange cross section. Here we have assumed that the neutrals are at rest. The role of neutral gas background in the physics of L-H transition of tokamaks has been reviewed by Fukuda [6], experimental studies have been reported in [7-9], however there seems to be be no consensus on how important the interaction with neutrals is. Theoretical studies on the effect of neutral background on L-H transition are described in refs. [10-12] The main result is that small neutral interaction has little effect on the L-H transition, however it was shown in ref. [13], that a small friction parallel to magnetic field lines can give rise to bifurcation and convective solutions. There is no continuous transition of a model with parallel momentum loss by charge exchange to a model without charge exchange, on rational magnetic surfaces singularities occur.

The model outlined in eqs. 1–4 is appropriate to describe a quiescent plasma with small diffusive velocity, which has no feedback on the momentum balance. Laminar convective solutions are possible and a selfconsistent rotating plasma can also be described by this model. Reduced versions of this model have been analysed in ref. [14], in particular existence and uniqueness of solutions have been investigated. In ref. [14] it was shown that the friction model (neglect of viscosity and inertial terms) has a unique solution if resistivity and plasma neutral are large enough. Convergence of an iterative scheme can be proven under these conditions. The iterative procedure starts with a magnetic fiels $\boldsymbol{B}_0$ and computes pressure, density, temperature, electric potential and the plasma current from eqs. 1–5. The current generates a magnetic field $\boldsymbol{B}_1$ and the procedure is repeated. The sequence of magnetic fields $\{\boldsymbol{B}_0\}$ converges if the sequence of current densities $\{\boldsymbol{j}_n\}$ is uniformly bounded and satisfies some continuity conditions (ref. [14]). In the following we consider the magnetic field as given and analyse how the solutions of the eqs. 1–3 depend on the structure of the magnetic field.

In order to describe stability and turbulence the first order derivatives have to be added in eqs. 1, 3,4 together with Faraday's law. This will introduce the turbulent Reynolds stresses $<\rho \boldsymbol{v}:\boldsymbol{v}>$ and the turbulent fluxes $<\rho \boldsymbol{v}>$ and $<p\boldsymbol{v}>$. The brackets $< >$ denote the average over the turbulent time scale.

3. Momentum Balance

In the limit of ideal magnetohydrodynamics resistivity, charge exchange interaction and the inertial forces are neglected. This is the standard model to compute stellarator equilibria. Since there is no feedback of a plasma motion on the momentum balance an arbitrary rotation within the magnetic surfaces can be superimposed. In this case there are two free surface functions $\Phi_0(\psi)$ and $\Lambda_0(\psi)$ and the rotation velocity is

$$\boldsymbol{V}_0 = \frac{\nabla\Phi_0 \times \boldsymbol{B}}{B^2} + \lambda_0 \boldsymbol{B} + \Lambda_0 \boldsymbol{B} \; ; \; \nabla\cdot \boldsymbol{V}_0 = 0 \tag{4}$$

In lowest order pressure surfaces, density surfaces, temperature surfaces and magnetic surfaces coincide. In this limit the plasma motion is incompressible. However, even under these conditions several stationary solutions can exist, in addition to the classical case of a slowly diffusing plasma also convective solutions without net toroidal or poloidal fluxes can exist. Furthermore a superposition of rotation, convective and diffusive solutions can exist.

In order to clarify the force balance in more detail we assume that a steady state solution exists and that the dominating part of the plasma velocity is a tangential velocity given by eq. 4. We make the following ansatz $\boldsymbol{v} = \boldsymbol{V}_0 + \delta\boldsymbol{v}$. Pressure surfaces and magnetic surfaces no longer coincide. The force balance on pressure surfaces determines the size of the rotation velocity, which can be seen by averaging the momentum balance equation on pressure surfaces. This averaging process on a surface $P = c$ is defined as a surface integral with the weight function $|\nabla p|$. For any periodic scalar function g we get $\langle \boldsymbol{B}\cdot\nabla g\rangle = 0$ and $\langle \boldsymbol{j}\cdot\nabla g\rangle = 0$. If the magnetic surfaces and the pressure surfaces coincide, this averaging is taken over magnetic surfaces. To proceed further we introduce a formal expansion with respect to dissipative and inertial forces. Introducing the expansion parameter ε we consider the friction and viscous forces as of order ε^2 and the inertial forces of the order ε. The magnetic field is a fixed field, which has closed magnetic surfaces labeled by the flux function ψ.

The plasma functions are expanded in powers of ε

$$\boldsymbol{j} = \boldsymbol{j}_0 + \varepsilon \boldsymbol{j}_1 + \ldots \; ; \; \boldsymbol{v} = \boldsymbol{V}_0 + \varepsilon\delta\boldsymbol{v} + \ldots$$
$$p = P(\psi) + \varepsilon\delta p + .. \; ; \; \rho = \rho_0(\psi) + \varepsilon\delta\rho + .. \tag{5}$$

In lowest order the current is the current of an ideal equilibrium and the velocity is a linear combination of the lowest order plasma current and the magnetic field

$$\boldsymbol{j}_0 = P'(\psi)\boldsymbol{e}_p \; ; \; \boldsymbol{V}_0 = E(\psi)\boldsymbol{e}_p + \Lambda(\psi)$$
$$\boldsymbol{e}_p \times \boldsymbol{B} = \nabla\psi \tag{6}$$

$\boldsymbol{e}_p$ is the poloidal Hamada vector on the magnetic surface, this vector is divergence-free. $E\nabla\psi$ is the radial electric field in lowest order. In this order in ε we have no tangential forces on magnetic surfaces, also in first order the average inertial forces are zero. In second order of ε the averaged poloidal force balance is

$$0 = \langle \boldsymbol{e}_p \cdot \boldsymbol{F}_n \rangle + \langle \boldsymbol{e}_p \cdot \nabla\cdot \pi(\boldsymbol{V}_0)\rangle + \left\langle \boldsymbol{e}_p \cdot \nabla\cdot \left(\delta\rho \boldsymbol{V}_0 : \boldsymbol{V}_0 + \rho_0 \delta\boldsymbol{v} : \boldsymbol{V}_0 + \rho_0 \boldsymbol{V}_0 : \delta\boldsymbol{v} \right) \right\rangle \tag{7}$$

and the parallel force balance becomes Eq. 8

$$0 = \langle \boldsymbol{B} \cdot \boldsymbol{F}_n \rangle + \langle \boldsymbol{B} \cdot \nabla\cdot \pi(\boldsymbol{V}_0)\rangle + \left\langle \boldsymbol{B} \cdot \nabla\cdot \left(\delta\rho \boldsymbol{V}_0 : \boldsymbol{V}_0 + \rho_0 \delta\boldsymbol{v} : \boldsymbol{V}_0 + \rho_0 \boldsymbol{V}_0 : \delta\boldsymbol{v} \right) \right\rangle \tag{8}$$

These relation state that in case of an rotating equilibrium the dissipative forces in direction of the magnetic field and the plasma current are balanced by the inertial forces. If the convective velocity is known these relations can be used to compute the rotation velocity $\boldsymbol{V}_0$. In a stellarator without net toroidal current the current lines are poloidally closed if they stay on closd surfaces. Thus eq. 7 basically describes a poloidal force balance while eq. 8 describes the toroidal force balance. The friction and viscous forces tend to slow down the plasma rotation, the driving mechanism has to be provided by the inertial forces. This driving mechanism is the Stringer spin-up mechanism [15]. If the plasma becomes turbulent the averaged momentum balance eqs. 7 and 8 will be slightly modified and the time-dependent part of $\delta\boldsymbol{v}$ leads to an additional driving mechanism, which has become known as the Reynolds stress mechanism.

The quadratic term of the inertial forces is the centripetal forces while the othe two are the Coriolis forces. The centripetal forces are important at large velocities, in L-mode they can be neglected. Inserting V0 into the driving forces yields these forces as a linear combination of E, Λ and its radial derivatives [4]. The convective velocity is written as a sum of a perpendicular and parallel component

$$\delta\boldsymbol{v} = \delta\boldsymbol{v}_\perp + \lambda\boldsymbol{B} \tag{9}$$

The parallel velocity usually is larger than the perpendicular velocity [4]. The poloidal force due to the parallel velocity is

$$\left\langle \rho_0 \delta \boldsymbol{v} \cdot \left(\boldsymbol{e}_p \times \boldsymbol{w}_0 \right) \right\rangle = R_{11} E$$

$$R_{11} = \left\langle \rho_0 \lambda \boldsymbol{B} \cdot \left(\boldsymbol{e}_p \times \nabla \times \boldsymbol{e}_p \right) \right\rangle \tag{10}$$

All other coefficients depend on the perpendicular part of dv. This parallel velocity associated to any diffusive or convective plasma motion is the main driving term of the Stringer spin-up. Taking into account the perpendicular convective velocity would lead to driving forces, which are also proportional to the first order radial drivatives of E and Λ. So far a time-independent convective motion has been assumed. However, if the convection is unstable and a turbulent state arises, the averaging procedure must be extented over the time scale of the turbulence. The extension of the previous analysis to the turbulent plasma is described in [16], the classical Stringer spin-up of a quiescent plasma is replaced by the anomalous Stringer spin-up [17]. The poloidal spin-up term in this case is

$$\left\langle \overline{\rho \delta \boldsymbol{v}} \cdot \left(\boldsymbol{e}_p \times \boldsymbol{w}_0 \right) \right\rangle = R_{11} E$$

$$R_{11} = \left\langle \overline{\rho \lambda} \boldsymbol{B} \cdot \left(\boldsymbol{e}_p \times \nabla \times \boldsymbol{e}_p \right) \right\rangle \tag{11}$$

The overbar denotes the time average.

4. Viscous Forces

The friction forces do not depend on the structure of the magnetic field, therefore the H-mode windows can be affected only indirectly by the neutral gas background. Viscosity – and in particular neoclassical viscosity – exhibits strong dependence on the topology of the magnetic field. For a stellarator plasma the surface averaged viscous forces are

$$\left\langle \boldsymbol{e}_p \cdot \nabla \cdot \pi \right\rangle = \left\langle \left(p_{\parallel} - p_{\perp} \right) \boldsymbol{e}_p \cdot \frac{\nabla B}{B} \right\rangle$$

$$\left\langle \boldsymbol{B} \cdot \nabla \cdot \pi \right\rangle = \left\langle \left(p_{\parallel} - p_{\perp} \right) \boldsymbol{B} \cdot \frac{\nabla B}{B} \right\rangle \tag{12}$$

The formulation of the viscous forces given in eqs. 12 is valid in any regime of collisionality. In a collision-dominated plasma the Braginskii viscosity leads to forces, which show no strong dependence on the rotational transform (ref. [5]). In a neoclassical regime, however, the viscous forces are dependent on the rotational transform and the radial electric field [5]. Solving the linearized kinetic equations allows one to compute the anisotropic pressure in eq. 12. The linearized equation is

$$L f_1 = -\frac{m}{kT} F_0 \left(v_{\parallel}^2 - \frac{1}{2} v_{\perp}^2 \right) \boldsymbol{V}_0 \cdot \frac{\nabla B}{B}$$

$$L = v_{\parallel} \frac{\boldsymbol{B}}{B} \cdot \nabla + \boldsymbol{v}_D \cdot \nabla - C \tag{13}$$

where L is the operator and $\boldsymbol{v}_D$ the particle drift velocity including the electric drift. The evaluation of the viscous forces yields

$$\begin{pmatrix} \left\langle \boldsymbol{e}_p \cdot \nabla \cdot \pi \right\rangle \\ \left\langle \boldsymbol{B} \cdot \nabla \cdot \pi \right\rangle \end{pmatrix} = \begin{pmatrix} \mu_p & \mu_b \\ \mu_b & \mu_t \end{pmatrix} \begin{pmatrix} E \\ \Lambda \end{pmatrix} \tag{14}$$

where the viscous coefficients are

$$\mu_p = \frac{m^2}{kT} \left\langle \int \omega_p L^{-1} F \omega_p d^3 v \right\rangle$$

$$\mu_t = \frac{m^2}{kT} \left\langle \int \omega_b L^{-1} F \omega_b d^3 v \right\rangle$$

$$\mu_b = \frac{m^2}{kT} \left\langle \int \omega_p L^{-1} F \omega_b d^3 v \right\rangle \tag{15}$$

and

$$\omega_p = \left(v_{\parallel}^2 - \frac{1}{2} v_{\perp}^2 \right) \boldsymbol{e}_p \cdot \frac{\nabla B}{B}$$

$$\omega_b = \left(v_{\parallel}^2 - \frac{1}{2} v_{\perp}^2 \right) \boldsymbol{B} \cdot \frac{\nabla B}{B} \tag{16}$$

In axisymmetric tokamaks there is only a poloidal variation of the magnetic field strenght, which leads to a proportionality of all viscous coefficients

$$\mu_t = A^2 \mu_p \quad ; \quad \mu_b = A \mu_p \tag{17}$$

There is no viscous damping force in toroidal direction. In this case the matrix of viscosity coefficients in eq. 15 is singular. The same property is valid in any quasi-symmetric system. In standard and advanced stellarators the matrix is non-singular and any poloidal and toroidal rotation will be slowed down by viscosity.

In summary we may conclude that at small radial electric field the poloidal viscous force grows linearily with the plasma rotation velocity and exhibits a strong resonant character on rational magnetic surfaces. Strong radial electric fields reduce the neoclassical viscous damping and the poloidal force becomes a decreasing function of the electric field. In this case the resonance on rational magnetic surface has nearly vanished. The effect of the electric field grows with poloidal mode number n, which implies that the E-field is more effective on high order rational surfaces than on low order rationals.

5. Plasma Convection

It must be expected that the poloidal and toroidal

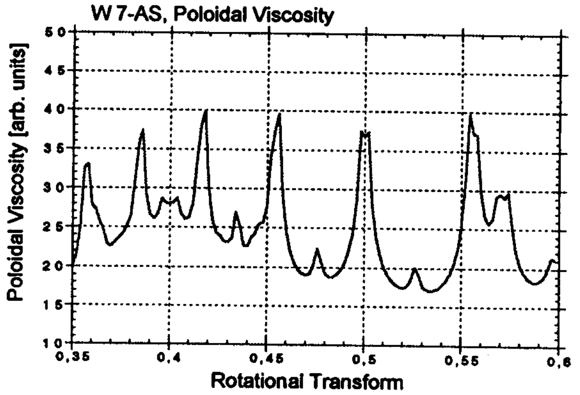

Fig. 1 Poloidal viscous coefficient μ_p vs rotational-transform. Collisionality and electric field are kept fixed. Additional resonant Fourier coefficients at ι = 10/19 and 10/21 have been introduced

rotation have a feedback on the onset of plasma convection. The simplest case occurs if $\delta \boldsymbol{v}$ is the classical diffusion velocity plus the Pfirsch-Schlüter effect. This velocity is independent of the superimposed $\boldsymbol{V}_0$ and it leads to the classical Stringer spin-up. However, this effect is computed without the dissipative and inertial terms in the equilibrium equation and only based on the resistivity in Ohm's law. Taking into account that neutral particle interaction and viscosity allow for a small pressure variation along magnetic field lines this can give rise to convective solutions, which do not exist in the limit without these dissipative terms. It is obvious that a convective thermal flux is minimized if the pressure stays constant on magnetic surfaces. The parallel pressure balance yields

$$\boldsymbol{B}\cdot\nabla p = -m_I n_0 f_0 \boldsymbol{B}\cdot\boldsymbol{v} - \boldsymbol{B}\cdot\boldsymbol{N}(\boldsymbol{v}) \qquad (18)$$

f_0 is the charge exchange collision frequency. The viscous term and the inertial term have been summarized in the term $\boldsymbol{N}(\boldsymbol{v})$. The perpendicular velocity is given by Ohm's law

$$\boldsymbol{v}_\perp = \frac{\nabla\Phi\times\boldsymbol{B}}{B} - \eta\frac{\nabla_\perp p}{B^2} \qquad (19)$$

The first term in eq. 19 is a convective velocity perperdicular to the magnetic field and the second term is the classical diffusion velocity. Inserting this into the equation of continuity provides one with an equation for the pressure

$$-\nabla\cdot\left(\eta\rho\frac{\nabla_\perp p}{B^2} + \frac{\rho}{m_I n_0 f_0}\nabla_\parallel p\right) + \nabla\cdot\rho\frac{\nabla\Phi\times\boldsymbol{B}}{B} = S \qquad (20)$$

The feedback of the pressure on the electric potential is obtained from the parallel component of Ohm's law and the perpendicular plasma current given by the perpendicular component of eq. 1.

$$j_\perp = -\frac{\nabla p\times\boldsymbol{B}}{B^2} - m_I n_0 f_0\frac{\nabla_\perp\Phi}{B^2} + \frac{N(\boldsymbol{v})\times\boldsymbol{B}}{B^2} \qquad (21)$$

The first term is the standard diamagnetic current and the second one the current caused by the electric field. In the following we neglect the contribution from viscosity and inertia and approximate the equation of the electric potential by

$$-\nabla\cdot\left(m_I n_0 f_0\frac{\nabla_\perp\Phi}{B^2} + \frac{1}{\eta}\nabla_\parallel\Phi\right) = \nabla\cdot\left(\frac{\nabla p\times\boldsymbol{B}}{B^2}\right) \qquad (22)$$

This is a second order elliptic equation for the electric potential. In the limit $f_0 \rightarrow 0$ this property is lost and the electric potential cannot be computed uniquely. Viscous and inertial forces would add another fourth order differential operator

$$M(\Phi) := \nabla\cdot\left(\frac{1}{B^2}\boldsymbol{N}\left(\frac{\nabla\Phi\times\boldsymbol{B}}{B^2}\right)\times\boldsymbol{B}\right) \qquad (23)$$

and instead of eq. 22 we get

$$-\nabla\cdot\left(m_I n_0 f_0\frac{\nabla_\perp\Phi}{B^2} + \frac{1}{\eta}\nabla_\parallel\Phi\right) + M(\Phi) = \nabla\cdot\left(\frac{\nabla p\times\boldsymbol{B}}{B^2}\right). \qquad (24)$$

In the limit of zero neutral density the term M(Φ) must be retained. As a non-linear equation eq. 24 can have more than one solution and a bifurcation point can exist, where these solutions merge. The bifurcation point is determined by linearizing the non-linear equation around one of the two solutions. Without the viscous and inertial terms the eq. 24 is linear and has only one solution at a given pressure gradient on the right hand side. The viscous operator in M would not change this result, however the inertial terms I(Φ) in N can give rise to multiple solutions. Expanding the potential in Fourier harmonics yields

$$\frac{1}{\eta}(m - n\iota)^2\Phi_{mn} - \left[\frac{1}{B^2}\nabla\cdot\left(m_I n_0 f_0\frac{\nabla_\perp\Phi}{B^2}\right)\right]_{mn} = \left[\frac{\nabla p\times\boldsymbol{B}}{B^2}\cdot\nabla\frac{1}{B^2}\right]_{mn} \qquad (25)$$

This is a system of coupled differential equations for the coefficients $\Phi_{mn}(\psi)$. Any Fourier mode describes a convective cell, which due to the boundary condition

$\Phi_{mn} = 0$ on both sides of the slab does not carry particles or heat across the boundary, but it strongly steepens the temperature and pressure gradients and thus leads to large radial transport. The first term in eq. 25 with the resistivity is the largest one and retaining only this term would yield an approximation, which is valid outside a singular layer. This singular layer occurs in the vicinity of rational surfaces with m-nι = 0, where the first term is zero and the second term with the radial derivatives of $\Phi_{mn}(\psi)$ provides an upper limit to the $\Phi_{mn}(\psi)$. Expanding the rotational transform around the resonant surface allows one to derive an estimate for the width of the singular layer. The result is

$$\left(n\iota'(\psi)\right)^2\left(\delta\psi\right)^4 \propto \frac{\eta m_I n_0 f_0}{B_0^2} \qquad (26)$$

At the boundary of the singular layer the first and the second term are of the same order. Retaining the viscosity term in the operator $M(\Phi)$ would lead to a similar effect: the width of the singular layer would be determined by the resistivity and the viscosity instead of the plasma neutral interaction. Eq. 26 shows that the singular layer may be large in low-shear systems; it also grows with plasma resistivity, which implies that any increase of the temperature in the boundary region would narrow the width of the singular layer. Furthermore, the width of the singular layer and the amplitude of the resonant components $\Phi_{mn}(\psi)$ decrease with rising poloidal mode number n.

Eqs. 20, 24 and the energy equation eq. 3 represent a closed system of elliptic equations, which together with appropriate boundary conditions allow one to compute plasma pressure, temperature and the electric potential. A basic issue of these non-linear equations is the problem of uniqueness. If only one solution of the system exists, bifurcaation and transition between different states cannot occur. However in anology to the classical Bénard problem several solutions can be expected. The conditions for unique solutions of equations 20 and 22 have been analysed leading to the result that large dissipative effects (resistivity and charge exchange losses) make the solution unique. However, in a real plasma these effects are small and more than one solution can exist. As a linear elliptic equation 22 has only one solution, the convective term in eq. 20, however, couples convective motion to the density variation and thus can give rise to convective solutions. This is the equivalent case to the classical thermal convection.

Retaining the inertial forces in eq. 26 would introduce another non-linearity, which can be the cause of another bifurcation. Furthermore, the non-linear terms in the energy equation can lead to bifurcations. In general, the bifurcation point is defined as the point where the linarized equation have a non-trivial solution. With respect to stability, this point is a point of marginal stability ($\omega = 0$). If a system approaches this point by a slow parameter variation, the plasma can jump from one stable solution to another stable solution.

6. Conclusions

The analysis has shown that a single fluid model of plasma flow in the boundary region can describe several features of the H-mode transition in

Wendelstein 7-AS. The force balance on magnetic surfaces requires that in L-mode – here defined as a state of small poloidal rotation– large convective motion occurs in order to compensate the viscous or charge exchange damping by means of the Coriolis forces. This convective motion also leads to enhanced plasma and energy losses. In particular, this convection can arise on low order rational magnetic surfaces, since the singular layer, caused by resistivity, viscosity and charge exchange interaction is large on these surfaces. The convection is driven by the pressure and temperature gradients. As in the case of the Bénard convection, when the viscosity is small, this solution can be unstable leading to a state of turbulent convection, which also drives the poloidal rotation.

Neoclassical viscosity is small on high order rational magnetic surfaces and it is further reduced by the radial electric field associated with the poloidal shear flow. Therefore in those regions with high order rational surfaces – the H-mode windows – small convective motion is needed to provide the necessary inertial drive. This also implies a reduction of plasma losses. This well-balanced equilibrium between driving and damping can be modified by the additional damping due to charge exchange losses. By reducing this damping term – which happens at density rise – this damping is reduced, thus facilitating the H-mode transition. Qualitavely this explains the experimental result of a threshold in density. In this model described above, the energy equation only plays a minor role. The shear flow is governed by the interplay of the continuity equation and the potential equation.

Since the present theory does not take into account time derivatives of plasma parameters and the magnetic field the issue of stability of these various stationary branches remains open.

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