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Revised Boundary Condition for the $E \times B$ Drift Dominated Plasma Flow in the Scrape-off Layer

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Abstract

In the case of a strongly magnetized plasma interacting with a conducting surface, the $E \times B$ flow generated by the self consistent presheath electric field plays a terminal role in determining the nature of the parallel flows at the magnetized presheath edge [1]. Presence of the cross-field drifts thus modifies the boundary condition for the plasma flow to be used at the ends of the scrape-off layer [2]. Detailed study of the presheath processes reveals a new kind of mechanism for perpendicular transport in the region of magnetized presheath which is observed to have a resonant behavior close to the targets. This shows that the perpendicular and parallel plasma flows are strongly coupled and can not be assembled non-interactively to achieve a condition for the entrance into the region of magnetized presheath. The entrance condition should therefore be determined in terms of self consistent electric field which effectively couples the parallel and the perpendicular flows. The form of such a coupling has been obtained and the revised boundary condition for the plasma flow in the scrape-off layer is worked out which is found to allow sub-sonic parallel flows at the magnetized presheath edge in presence of an $E \times B$ flow.

Keywords:

scrape-off layer, boundary condition, parallel flow, oblique incidence

1. Introduction

The Boundary condition used at the ends of scrape-off layer (SOL), typically at the divertor and limiter surfaces, generally have contribution from various flows generated in parallel and perpendicular directions. These flows include a parallel flow from the bulk plasma and $E \times B$ flows both in parallel as well as cross field direction. At the entrance of the magnetized presheath which is typically in the region of SOLwhere the plasma connects to the solid surface, a crucial role is played by the inertia in the $E \times B$ direction, in determining the nature of flows parallel to magnetic field [1]. A more general boundary condition was worked out by Stangeby and Chankin [3] by incorporating an extra effect of the cross field flow generated due to a radial electric field. If the radial electric field is absent, this boundary condition reduces in the its conventional form $v_{\parallel} = c_s$ obtained by Chodura [2], which is true in an ideal case of perfectly magnetized plasma. In a magnetized presheath region, mechanisms are present which are responsible for deflecting the parallel flows into the direction normal to wall and provide an extra contribution to the boundary condition. This deflection mainly occurs as there is a shear in the $E \times B$ flow generated because of a nonzero divergence of the self-consistent presheath electric field $\mathbf{E}$ (not a radial or external field). The transfer of energy from a parallel to normal flow is expected to be very effective close to the solid surface and needs to be incorporated in the

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boundary conditions. In this paper this effect is included by allowing a finite derivative of the self consistent electric field in the direction normal to the wall, which results into an effective polarization drift of ions and modifies the moment equations.

2. Modified Fluid Equations

According to the geometry shown in the Fig. 1, there is a self consistent presheath electric field along y, i.e. poloidal direction and a radial electric filed due to variation in electron temperature along radial, i.e. x direction. The magnetic field **B** lies in the toroidal-poloidal plane, making an angle θ ($\sin\theta = B/|B_\theta|$) with the solid surface. A new set of unit vectors ($\hat{e}_\parallel$, $\hat{e}_\perp$) can be defined for the simplicity (see Fig. 1), where $\hat{e}_\parallel$ is along the direction of magnetic field, in y-z plane, $\hat{e}_\perp$ is perpendicular to $\hat{e}_\parallel$ in the same plane and $\hat{e}_x$ in unchanged.

The total flow velocity vector in terms of flows along parallel $\hat{e}_\parallel$, perpendicular $\hat{e}_\perp$ and x, as below

$$\mathbf{v} = \{ v_x \hat{x} + (v_\parallel \sin\theta + v_\perp \cos\theta)\hat{y} + (v_\perp \sin\theta - v_\parallel \cos\theta)\hat{z} \} \tag{1}$$

Every ion gyrating and drifting in the y direction, should experience an acceleration in the perpendicular direction due to presheath electric field E_y changing implicitly with time in the gyrocentre frame as there is no explicit time dependence in the steady state plasma. This is equivalent to a polarization drift [4] when there is a time varying electric field in the transverse direction (here $E_\perp$). This drift is given by

$$v_p = \frac{1}{\Omega B}\frac{\mathrm{d}E_\perp}{\mathrm{d}y}\frac{\mathrm{d}y}{\mathrm{d}t} \tag{2}$$

Thus v_p represents a coupling between an $E_y \times B$ flow parallel to the wall and a drift generated in the direction normal to the wall due to shear in this flow. This is evident as the polarization drift is proportional to the rate of change of E_y/B with y. As the drift along y has the contribution from $v_\parallel$, an $E \times B$ drift v_D and the polarization drift v_p (later two along $\hat{e}_\perp$), the total polarization drift becomes

$$v_p = \frac{1}{\Omega B}\frac{\mathrm{d}E_\perp}{\mathrm{d}y}[v_\parallel \sin\theta + (v_D + v_p)\cos\theta] \tag{3}$$

$$v_p = \alpha v_\parallel + \beta v_D \tag{4}$$

where,

$$\alpha = \left[\frac{\frac{1}{\Omega B}E'_\perp \sin\theta}{1 - \frac{1}{\Omega B}E'_\perp \cos\theta}\right] \tag{5}$$

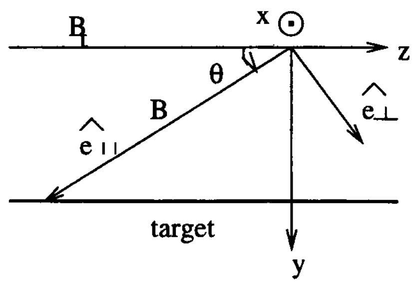

Fig. 1 The geometry of the region, magnetic field B is in the y-z plane, making an angle θ with the toroidally continuous target surface.

$$\beta = \frac{\alpha}{\tan\theta} \tag{6}$$

and Ω is the ion gyro-radius and a prime denotes a differentiation with respect to y.

2.1 Equation of continuity

The equation of continuity, which comes from the conservation of flux along all the three directions can now be written as below, assuming that all the derivatives of electric field higher than $E'_\perp$ are zero

$$\sin\theta\frac{\mathrm{d}}{\mathrm{d}y}(nv_\parallel) + \alpha\cos\theta\frac{\mathrm{d}}{\mathrm{d}y}(nv_\parallel) + \beta\cos\theta \times \frac{\mathrm{d}}{\mathrm{d}y}(nv_D) = -\cos\theta\frac{\mathrm{d}}{\mathrm{d}y}(nv_D) - \frac{\mathrm{d}}{\mathrm{d}x}(nv_x) \tag{7}$$

A general dependence of electric field E_x and therefore of $v_D = E_x/|B|$ on x, can be adopted directly from the formulation of Sta-ngeby and Chankin [3]. The E_x here arises due to shear in the electron isotherms along the radial direction close to the separatrix, which results into a variation of the floating potential along x at the magnetic presheath edge. The v_D therefore is given by

$$v_D(\phi) = v_D(0) - \frac{\phi}{|B|\, l_T} \tag{8}$$

where ϕ is the electrostatic potential and

$$l_T = \left(\frac{1}{T_e}\frac{\mathrm{d}T_e}{\mathrm{d}x}\right)^{-1}. \tag{9}$$

The term $\mathrm{d}v_D/\mathrm{d}y$ can therefore be calculated, using the Boltzmann distribution of the electrons ($n_e = n_0 \exp(e\phi/kT_e)$) and the quasi-neutrality ($n_i = n_e$) to obtain an effective E_y as follows

$$\frac{\mathrm{d}v_D}{\mathrm{d}y} = \frac{\mathrm{d}v_D}{\mathrm{d}\phi}\frac{\mathrm{d}\phi}{\mathrm{d}y} = -\frac{1}{|B|\, l_T}\frac{kT_e}{ne}\frac{\mathrm{d}n}{\mathrm{d}y}. \tag{10}$$

The flux nv_x along x, is written using a general x dependence of density, $n = n_0(1 + x/l_n)$ and of electric field E_y. Thus

$$-\frac{\mathrm{d}}{\mathrm{d}x}\Gamma_x = \frac{\cos\theta n E_y(0)}{|B| l_p} = -\frac{\cos\theta k T_e}{e|B| l_p}\frac{\mathrm{d}n}{\mathrm{d}y} \tag{11}$$

where, $l_p = [(l_T)^{-1} + (l_n)^{-1}]^{-1}$.

Substituting $\mathrm{d}v_D/\mathrm{d}y$ and $\mathrm{d}\Gamma_x/\mathrm{d}x$ from Eqs. (10) and (11) respectively, continuity equation (7) yields

$$n\frac{\mathrm{d}v_\parallel}{\mathrm{d}y} = -\frac{1}{(\beta+1)\sin\theta}\left[(\beta+1)\times(\sin\theta v_\parallel + \cos\theta v_D) + \frac{kT_e\cos\theta}{e|B|}\left(\frac{1}{l_p} - \frac{\beta+1}{l_T}\right)\right]\frac{\mathrm{d}n}{\mathrm{d}y}. \tag{12}$$

2.2 Momentum equation

We now write the momentum balance in the direction parallel to magnetic field **B** as

$$\sin\theta m n v_\parallel \frac{\mathrm{d}v_\parallel}{\mathrm{d}y} + (\sin\theta + \alpha\cos\theta)\frac{\mathrm{d}p_i}{\mathrm{d}y} - neE_\parallel = S_m - m v_\parallel S_p \tag{13}$$

where S_m and S_p are the momentum and particle sources respectively, generated due to the cross field flow contributions, and given by

$$S_m = -\cos\theta\frac{\mathrm{d}}{\mathrm{d}y}\{m v_\parallel n(v_p + v_D)\} - \frac{\mathrm{d}}{\mathrm{d}x}(m v_\parallel n v_x) \tag{14}$$

and

$$S_p = -\cos\theta\frac{\mathrm{d}}{\mathrm{d}y}\{n(v_p + v_D)\} - \frac{\mathrm{d}}{\mathrm{d}x}(n v_x). \tag{15}$$

Substituting Eqs. (14) and (15) in the right hand side of Eq. (13) and using the equation of continuity, quasi-neutrality and Boltzmann electrons, Eq. (13) can be written as

$$n\frac{\mathrm{d}v_\parallel}{\mathrm{d}y}[(\beta+1)(\sin\theta v_\parallel + \cos\theta v_D)] = -\frac{\mathrm{d}n}{\mathrm{d}y}\times\left[\sin\theta\left(c_s^2 + \beta\frac{\gamma T_i}{m}\right) + \frac{\cos\theta k T_e}{e|B|}\frac{\mathrm{d}v_\parallel}{\mathrm{d}x}\right] \tag{16}$$

where all the derivatives of electric field E_y, higher than E'_y are neglected.

We can now eliminate $\mathrm{d}v_\parallel/\mathrm{d}_y$ from Eqs. (12) and (16), and equate the coefficient of $\mathrm{d}_n/\mathrm{d}_y$ to zero, since $\mathrm{d}_n/\mathrm{d}_y \neq 0$ as a finite electric field is required at the magnetic presheath edge. This yields

$$\left(v_\parallel + \frac{v_D}{\tan\theta}\right)^2 + \left(\frac{kT_e}{(\beta+1)\tan\theta e|B| l_n}\right)\times\left(v_\parallel + \frac{v_D}{\tan\theta}\right) = \frac{1}{(\beta+1)}\left[c_s^2 + \beta\frac{\gamma T_i}{m} + \frac{kT_e}{\tan\theta e|B|}\frac{\mathrm{d}v_\parallel}{\mathrm{d}x}\right] \tag{17}$$

We now define, the Mach number, $M = v_\parallel/c_s$, the strength of drift,

$$g = \frac{v_D}{2c_s\tan\theta} \tag{18}$$

and the other quantities

$$\xi = M + 2g \tag{19}$$

$$2a = \left[\frac{kT_e}{(\beta+1)\tan\theta c_s e|B| l_n}\right] \tag{20}$$

$$b = \left[\frac{kT_e}{\tan\theta(\beta+1)e|B|}\frac{\mathrm{d}v_\parallel}{\mathrm{d}x}\right] \tag{21}$$

$$c = \left[\frac{1}{(\beta+1)} + \frac{\beta\gamma T_i}{(\beta+1)mc_s^2}\right]. \tag{22}$$

Thus the revised equation which should be solved to find the correct value of boundary condition (the value of $v_\parallel$ at $y = 0$) while integrating the SOL equations in each iteration, becomes

$$\xi^2 + 2a\xi - b - c = 0 \tag{23}$$

where, l_n (i.e. l_T and l_p) and $\mathrm{d}v_\parallel/\mathrm{d}x$ are the parameters to be iterated on while converging to an exact SOL solution.

3. Analysis and Discussion

The Eq. (23) has some additional effects due to the inclusion of polarization drift. Since the factor β is proportional to E'_y, these effects are present if the derivative of the electric field E_y is finite at the magnetic presheath edge. This can be readily seen that Eq. (23) reduces into the equivalent equation obtained by Stangeby and Chankin [3] in the limit ($\beta \to 0$). In order to compare the results we adapt the similar procedure to find the solution of Eq. (23) by redefining the quantities a and b in terms of g using the definitions of v_D which is an $E_x \times B$ drift given by

$$v_D = -\frac{kT_e}{e|B| l_T}|\psi_w| \tag{24}$$

where, ψ_w is the total potential drop between magnetic presheath edge and the wall. The a and b therefore take the form

$$2a = \frac{2g}{(\beta+1)} b_0 \tag{25}$$

$$b = -\frac{2g(\xi - 2g)}{(\beta+1)} a_0 \tag{26}$$

where,

$$a_0 = \frac{1}{(\beta+1)} \frac{l_T}{|\psi_w| l_v} \tag{27}$$

and

$$b_0 = -\frac{1}{(\beta+1)} \left(\frac{l_T}{|\psi_w| l_n} - \frac{\beta}{|\psi_w|} \right). \tag{28}$$

We now write the solution of Eq. (23) giving the required boundary condition.

$$\frac{v_{\parallel MPSE}}{c_s} = -g(2 + a_0 + b_0) + \left[g^2 (a_0 + b_0)^2 + c + 4a_0 g^2 \right]^{\frac{1}{2}} \tag{29}$$

The value of $v_{\parallel MPSE}$ is plotted in the Fig. 2 for various values of parameter β which signifies the strength of the polarization drift at the magnetic presheath edge. This can be seen that the effect of the polarization drift which is the mechanism responsible for deflecting the parallel flow into the direction normal to the wall, is mostly to cancel off the effects produced by the v_D and bring the value of $v_{\parallel MPSE}$ to a constant sub-sonic value. The two cases present in the Fig. 2(a) and (b) are for values of parameters (a_0, b_0) equal to (1, 1) and (–1, 0) respectively, which represent maximum contrast in their behavior in the analysis of Ref. [3]. This is interesting to note that in both the cases the role of polarization drift remains unchanged and it keeps canceling the effect of $E \times B$ drift by pulling it to a constant sub-sonic value. This confirms that the presence of strong presheath mechanisms reduce the effects of drifts produced by an external electric field. This drift is modeled by an $E \times B$ drift present in the SOL due to a radial electric field E_x. As the parameter β has a resonant nature, it takes large values if the value of $E_\perp$ becomes larger at the magnetic presheath edge. In both of the cases presented here the value of v_{MPSE} is sub-sonic at $g = 0$ except the case when $\beta = 0$ which is the case of an ideal magnetic presheath where no drifts are present or the ions are fully magnetized.

4. Conclusion

The effect of an $E \times B$ drift may be reduced by the strong presheath mechanisms present inside a magnetic presheath, where the magnetic field is obliquely incident on the targets. The function of a polarization drift in a magnetic presheath is to deflect the parallel ion flows in the direction normal to the target thus to make the parallel flow velocity required by the ions at the magnetic presheath edge sub-sonic.

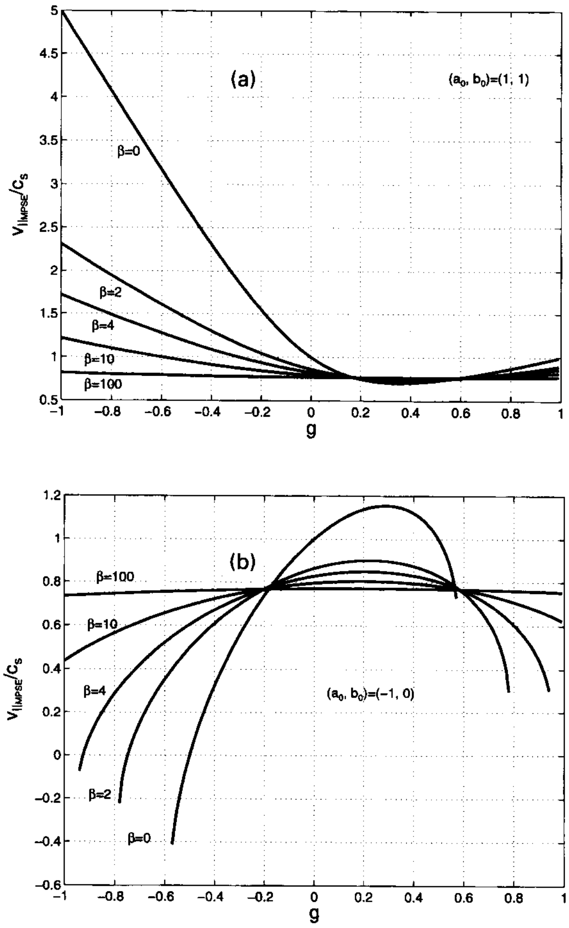

Fig. 2 The value of $v_{\parallel MPSE}/c_s$ plotted vs. the strength of $E \times B$ drift velocity g, for the values of β = 0.0, 2, 4, 10 and 100. Two cases presented are (a) (a_0, b_0) = (1, 1) and (b) (a_0, b_0) = (–1, 0).

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