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Time-Domain Self-Consistent Plasma Equilibrium in Damavand Tokamak

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Abstract

In this context, the time-domain equilibrium of plasma in Damavand tokamak subject to high elongation ($K \sim 3.5$) and high aspect ratio ($A \sim 5$) is studied. A self-consistent simulation of plasma evolution is obtained by mixed numerical solution of equilibrium and transport equations, in two-dimensions and one-dimension, respectively. At each time step, first the equilibrium equation is solved by variational axisymmetric finite element method under a prescribed plasma scenario, then flux surface averaged transport functions of mass, energy, and magnetic field are calculated. Here, an exact variational approach to the finite element method using first order elements has been proposed. The formulation permits simultaneous solution of plasma and vacuum regions without consideration of an isolation boundary. Ionization of Deuterium plasma is also included in the model to study the breakdown and pre-breakdown stages.

Keywords:

tokamak, plasma, equilibrium, transport, finite element method, simulation

1. Introduction

During past decades the problem of self-consistent plasma transport in tokamak plasmas has been considered through many works. Theoretical investigations were led by a pioneering work of the so-called neoclassical theory of transport in [1]. In [2] a detailed review of neoclassical transport theory has been presented and a complex system of transport equations for plasma density and temperature of species have been derived. An equivalent MKS representation of this set of equations is given in [3]. Another early review [4] has considered the methods of numercially self-consistent solving the transport and equilibrium equations. In [5] numerical solution of these equations was reported. It was found there that the solutions of neoclassical transport equations are subject to large errors, up to two orders of magnitude in estimations of the energy confinement time τ_E. This problem had been associated with similar codes, so that it was become clear that the neoclassical theory of transport could not provide a proper understanding of the anomalous plasma transport mechanism.

It took some 10 years of more research and efforts to explain reasons for these discrepancies. Although many related the misunderstanding to the neglect of turbulence, plasma edge interactions and impurity transport mechanisms, no successful unified theoretical formulation was put forward. As a result, extensive particle codes relying on massively parallel computing were started to develop. However, neoclassical transport codes remained as a valuable tool for design stage and study of tokamaks.

Self-consistent solution of plasma transport in axisymmetric toroidal plasmas was considered in [6] with an extensive two-dimensional transport model

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described in [7]. This formulation exploited two diffusion time scales at each time step. In a more recent study [8] a $1^1/_2$ dimensional code named DINA has been developed. It uses two-dimensional equilibrium and one-dimensional flux-surface-averaged transport equations to represent the tokamak plasma and is able to track the plasma evolution in time. To improve the simulation results, the authors have modified the electron heat conduction coefficient in each time step, so that the consequent temperature profile is according to one of the existing scaling rules for energy confinement time.

In [9] the much complex code ASTRA has been presented with the possibility for presetting transport equations and coefficients. It has been concluded that the ASTRA system had provided an adequate representation of the discharges for present experimental conditions. Using ASTRA code authors of [10] demonstrated the possibility of self-consistent simulation of L and H modes with a single model, using modified transport coefficients in accordance to a previously reported anomalous reported estimates [11]. However, equilibrium analysis in ASTRA relies on pre-assumption of geometric configuration of magnetic surfaces.

It should be pointed out that there also exist newer plasma transport models such as [12] which demonstrate a high degree of complexity. Numerical solution of these systems are generally too difficult.

Here we report a self-consistent solution of flux-surface-averaged transport equations and equilibrium for Damavand tokamak. Damvand is a small tokamak with a highly elongated plasma ($K \sim 3.5$) and high aspect ratio ($A \sim 5$). We use the axisymmetric finite element method (FEM) [13] with first-order triangular elements to solve equilibrium, in its variational approach. Our code solves the equilibrium with no special limitation on the geometry of magnetic surfaces within the plasma. Plasma and vacuum regions are treated as a single ensemble. The ionization and recombination of Deuterium is involved to study the breakdown and pre-breakdown evolution in a single model.

2. Theoretical Model

In our code, the basic set of transport equations that have been presented in [3] is employed with modifications in ion and electron heat fluxes according to [11]. The effects of ionization and recombination are included in the source terms to provide the main mechanism in breakdown and pre-breakdown stages. This also enhances the possibility for studying plasma edge interactions.

The heat flux in transport equations are given as [11]:

$$q_l = \chi_l n_l \nabla T_l + 2.5\, T_l \Gamma_l \tag{1}$$

where $l = e$, i stands for electrons or ions, n and T are the density and the temperature of species, and Γ is the convective flux caused by particle transport defined as:

$$\Gamma_l = D_l \nabla n_l + V n_l \tag{2}$$

The anomalous values for the electron transport coefficients χ_e and D_e in metric units are defined as:

$$\chi_e = 1.25 \times 10^{28} \left(\frac{T_e}{A_i}\right)^{0.5} \left(\frac{r}{R}\right)^{1.75} (n_e q R)^{-1} \tag{3}$$

$$D_e = 0.5 \chi_e$$

while for χ_i and D_i the neoclassical values are used. Here, the electron temperature T_e is given in energy units. This system of transport equations has demonstrated good agreement with experiments in many small and large tokamaks.

The ionization and recombination processes in plasma are taken into account by adapting proper source terms in the continuity equations of plasma species as follows:

$$S_l = \sigma_i v_e n_e n_i - \sigma_a v_e n_e n_i \approx (\sigma_i - \sigma_a) v_e n_l^2 \tag{4}$$

where v_e is the electron thermal speed, σ_i and σ_a are ionization and recombination cross-sections, being local functions of temperature. An extensive data with approximate interpolating functions for the major atomic processes in hydrogen plasmas is already reported [14].

For equilibrium, the variational form of Grad-Shafranov's equation is used, to be solved by the axisymmetric variational FEM described in the next section:

$$I(\Psi) = \iint \left\{ \frac{1}{2r} \left[\left(\frac{\partial \Psi}{\partial r}\right)^2 + \left(\frac{\partial \Psi}{\partial z}\right)^2 \right] - 2\pi \mu_0 J_t \Psi \right\} dr dz \tag{5}$$

where r and z are radial and axial components of cylindrical coordinates, Ψ is the poloidal flux and J_t is toroidal current density, including the poloidal magnetic system of tokamak.

The toroidal current density J_t in (5) is obtainable from the relation:

$$J_t = \frac{1}{2\pi r} \left(r^2 \frac{dp}{d\Psi} + \frac{1}{\mu_0} \frac{dI}{d\Psi} \right) + J_t \big|_{ps} \tag{6}$$

where $J_t|_{ps}$ is the toriodal current density maintained by the poloidal system. The plasma pressure p and the flux function $I = rB_t$ are found from transport equations at each time step. Since plasma density and thus plasma toroidal current in the low density or vacuum region are nearly zero, the validity of (6) is attained.

3. Numerical Method

The scheme of solving equilibrium and transport equation is as follows: with the initial values for poloidal system currents, plasma density n, electron temperature T_e, and ion temperature T_i given, the toriodal current density J_t is calculated. Then the poloidal flux Ψ is found through minimization of (5). Then the circuit equations are solved to find the parallel inductive electric field in plasma and poloidal field coils. Finally transport equations are integrated in time domain to find next values of n, T_e, and T_i.

Minimization of the functional in (5) is realized by using the finite element method (FEM). The classes of FEMs fall into two categories: Ritz-Galerkin methods, and variational methods. Despite the existing literature [15] on Ritz-Galerkin FEM, due to its attractive features we formulate exactly and employ a variational approach with first order axisymmetric triangular elements. Besides its simplicity, this approach permits accurate minimization of (5), so that in the limit of small elements the solution would converge to the exact one [13]. As discussed below, variational methods lead to a symmetric coefficient matrix which reduces the necessary storage and improves the efficiency. Also, variational methods are superior in terms of error distribution, since as a special case of moment method, variational FEM may be shown to be equivalent to least square minimization of error [16].

First order triangular elements are defined using the standard interpolation shape functions:

$$\mathbf{N}^T = [\,1 \;\; r \;\; z\,]\begin{bmatrix} 1 & r_i & z_i \\ 1 & r_j & z_j \\ 1 & r_k & z_k \end{bmatrix}^{-1} \equiv [\,1 \;\; r \;\; z\,]\mathbf{D} \equiv [\,\mathbf{N}_i \;\; \mathbf{N}_j \;\; \mathbf{N}_k\,] \tag{7}$$

where r_l and z_l are coordinates of element vertices, or nodes, and i, j, and k represent indices of each of nodes belonging to an element, numbered in a clockwise manner. The linear approximation to any function $f(r,z)$ on the element e is done as:

$$f^e(r,z) \equiv N^{e^T}\begin{bmatrix} f^e(r_i,z_i) \\ f^e(r_j,z_j) \\ f^e(r_k,z_k) \end{bmatrix} \equiv N^{e^T}\begin{bmatrix} f_i \\ f_j \\ f_k \end{bmatrix} \equiv N^{e^T}\boldsymbol{f}^e \tag{8}$$

where f_l are the values of the function f on the nodes. These values for each function are to be determined on a grid of nodes, which constitute a network of triangular meshes. This way of discretization of functions produces a continuous piecewise linear approximation. On each element e, the gradient of $f^e(r,z)$ is thus expressed as:

$$\nabla f^e \equiv \nabla N^{e^T}\boldsymbol{f}^e = \begin{bmatrix} \partial/\partial r \\ \partial/\partial z \end{bmatrix} N^{e^T}\boldsymbol{f}^e = \begin{bmatrix} D_{ji} & D_{jj} & D_{jk} \\ D_{ki} & D_{kj} & D_{kk} \end{bmatrix}\boldsymbol{f}^e \tag{9}$$

which is a constant vector.

Discretizing (5) to element integrals results in:

$$I(\Psi) = \sum_e \int_{S^e} \frac{1}{2r}|\nabla\Psi|^2 - 2\pi\mu_0 J_t \Psi \, dS^e \tag{10}$$

where the summation is performed on all elements represented by the index e and the gradient is expressed in Cartesian coordinates (r,z). Inserting (8) for the toroidal current density J_t and the poloidal flux Ψ, and taking partial derivatives with respect to nodal values Ψ_l we get finally:

$$\frac{\partial I}{\partial \boldsymbol{\Psi}} = \sum_e \int \frac{1}{r}\nabla N^e \nabla N^{e^T}\boldsymbol{\Psi}^e - 2\pi\mu_0 N^{e^T} N^e J_t^e \, dS^e \equiv 0 \tag{11}$$

which transforms into the set of linear equations:

$$\left[\sum_e \int \frac{1}{r}\, dS^e \,\nabla N^e \nabla N^{e^T}\right]\Psi = \left[2\pi\mu_0 \sum_e \int N^{e^T} N^e \, dS^e \, J_t^e\right] \tag{12}$$

where Ψ is the array of unknown nodal values. The element integrals in the left hand side are simple and straightforward to evaluate, however the integrands of right hand side are higher order functions of coordinates and required special integration scheme. It is possible to expand them on elements and directly evaluate them, however, this process is time consuming and very difficult to implement. A simple mathematical formula for evaluation of this integral is given in [17] and a general approach for evaluation of element intergals in axisymmetric variational FEM is found in [18].

Finally, the coefficients matrix in the left hand side of (12) is singular unless a zero reference point for the poloidal flux Ψ is assigned. This may be simply such chosen to coincide with the plasma magnetic center. Also, the above minimization of (5) leads to erroneous results due to a boundary integral resulting in the standard variational approach. This may be expressed as:

$$\iint \nabla \cdot \left(\frac{\partial \Psi}{r} \nabla \Psi\right) drdz = \oint \frac{1}{r} (\nabla \Psi \times dl)_t \qquad (13)$$

where $\delta\Psi$ is the first-order variation in the poloidal flux, and the closed integral is in the counter-clock-wise sense. The above must vanish identically for obtaining correct results.

In order for (13) to vanish, it is necessary that either Ψ is kept fixed on the boundary, or its gradient is parallel to the boundary. The former is not physically realizable due to non-constant poloidal fluxes, while the latter imposes the flux lines to be normal to boundary. A convenient solution is to extend the solution region to infinity where both Ψ and $\nabla\Psi$ tend to zero. This is made possible by adding the so-called infinite elements [13] to the solution boundary as is justified in the next section.

4. Results

Different scenarios of elongated plasma equilibrium in Damavand are considered in [19]. In Fig. 1 the cross-section of elongated Damavand plasma is shown together with the poloidal field coils, the ohmical heating solenoid, the vaccum chamber and a toroidal field pancake. Here, the plasma current is about 40 KA, the elongation is about 3, and the triangularity is 0.2. The Damavand tokamak is symmetric with respect to its meditorial plane, and therefore only the upper half is involved in calculations. As it may be observed form the figure, the flux lines extend into the free space across the right and upper edges, by applying the infinite elements on boundaries. On the lower edge the symmetry condition is applied which result in normal flux lines to boundaries. On the z-axis, however, the Dirichlet's zero boundary condition is needed to be applied. It is noticed that without infinite elements, the coefficients matrix in the left hand side of (12) would be degenerate and the resulting solution is subject to severely large errors. This situation is illustrated in Fig. 2 where the infinite elements are not used on the boundaries:

The computation time for the developed code in MATLAB™ version 5.3 running on a 333 MHz Pentium platform is about 80 sec for each time-step iteration, when the total number of nodes and meshes are about 3000 and 11,200, respectively.

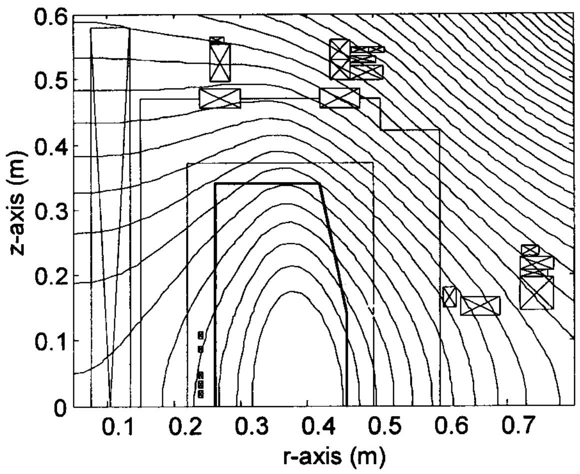

Fig. 1 Cross-section of Damavand plasma (infinite elements applied).

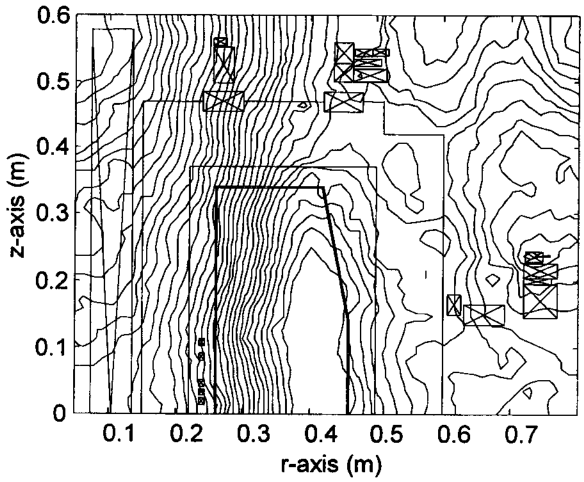

Fig. 2 Computed magnetic flux lines (without infinite elements).

5. Conclusions

A self-consistent simulation of plasma equilibrium and transport in tokamak was presented. The transport model exploited the magnetic-surface-averaged one-dimensional neoclassical theory, modified according to some more recent reported values for anomalous transport coefficient values which are based on empirical data. The numerical method solves the Grad-Shafranov equation for axisymmetric equilibrium in a free boundary configuration in which plasma and vaccum and poloidal coils are treated at once. The axisymmetric finite element method has been successfully adapted to the plasma equilibrium in variational form. Addition of infinite elements around the plasma boundary has prevented large errors in the solution of magnetic poloidal field flux, thus maintaining the stability of solution.

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