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Near-Axis Investigations of Plasma Confinement in Linked-Mirror Type Stellarators

SHAFRANOV Vitaly*, COOPER Anthony W.[1], ISAEV Maxim,
MIKHAILOV Mikhail, NÜHRENBERG Jürgen[2] and SUBBOTIN Alexey
Russian Research Centre "Kurchatov Institute", 123182, Moscow, Russia
[1]Centre de Recherches en Physique des Plasmas, Association Euratom-Confederation Suisse, Ecole Polytechnique Federale de Lausanne, PPB, 1015 Lausanne, Switzerland
[2]Max-Planck-Institut für Plasmaphysik, Teilinstitut Greifswald IPP-EURATOM Association, D-17491 Greifswald, Germany

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Abstract

The approach for the optimization of systems with a poloidal direction of the lines B = constant on magnetic surfaces is considered. To understand the main peculiarities of the problem, the conditions of pseudosymmetry and quasi-isodynamicity are formulated in terms of the near-axis magnetic surfaces parameters as well as in terms of the Fourier spectrum of the magnetic field strength B in Boozer coordinates. Target functions are suggested for the numerical optimization toward pseudosymmetry and quasi-isodynamicity. The numerical optimization is now in progress. As an example, the first result of the numerical optimization is presented.

Keywords:

stellarator optimization, quasi-isodynamicity, pseudosymmetry, near-axis approximation

1. Introduction

The behavior of the B-lines (the B = constant contours on magnetic surfaces) determines essentially the confinement properties of the magnetic configurations. In the *quasisymmetrical* (QS) configurations discovered in Ref. [1], the B-lines move in a special invariant direction $\boldsymbol{q}$ on tori so that $\boldsymbol{q} \cdot \nabla B = 0$, providing conservation of the corresponding canonical momentum of the guiding centers. The condition of the *omnigeneity* [2] (the centers of banana orbits lie on the magnetic surfaces) formally is less restrictive than the QS condition. Nevertheless, the requirement of omnigeneity for all trapped particles is equivalent to the condition of QS except for cases of integer rotational transform in a system period, as it follows from Ref. [3]. The more realistic condition of *quasi-isodynamicity* (QI) [4] requires the fulfillment of the omnigeneity condition for a fraction of the trapped particles only (e.g., only the "banana" centers corresponding to the deeply to moderately-deeply trapped particles must lie on the magnetic surfaces) and can be satisfied in configurations in which the B = constant lines point in the direction of the poloidal coordinate. The condition of *pseudosymmetry* (PS) [5] means the absence of locally trapped particles but allows the centers of banana orbits to move off the magnetic surfaces.

In present paper the possibilities to fulfill different conditions of improved particle confinement are

Corresponding author's e-mail: SHAFRAN@nfi.kiae.ru

analysed for systems with poloidal direction of B-lines.

In the "ideal" QS configurations with poloidal direction of B-lines, the poloidal invariant direction on tori is determined by the vector $\boldsymbol{q} = J\boldsymbol{B} + \boldsymbol{B} \times \nabla\Phi$, where J and Φ are the toroidal current and magnetic flux functions, respectively.

The corresponding poloidal canonical momentum is $P_\theta = e(\Phi + \rho_\| J)$, thus for $J = 0$ an integral of the motion is Φ = constant (all drift trajectories lie on magnetic surfaces, so that there is no banana or neoclassical effects at all). It is seen that for $J = 0$, the invariant direction is perpendicular to that of the B-lines (the orthogonal systems [6], zero secondary currents).

Despite the fact that poloidal QS cannot be satisfied everywhere near a closed magnetic axis, the systems with poloidal direction of the B-lines look attractive [4]. Furthermore, the poloidal PS is compatible with the QI condition. Another property of systems with poloidal direction of B-lines is the possibility to have an absolute minimum of B whereupon the drift trajectories of deeply trapped particles are enclosed within an ellipsoidal surface inside the plasma column.

In the present paper, such properties of systems with a poloidal direction of B-lines are analyzed in the near-axis approximation.

2. The Pseudosymmetry and Quasi-Isodynamicity Conditions in the Near-Axis Approximation

Geometrical approach. Some characteristic features of the configurations with poloidal directions of B-lines can be easily understood in the near-axis approximation. The zero-order term in the B-expansion with respect to the distance from the magnetic axis, $B_0(\zeta)$, determines the poloidal direction of the B-lines close to the axis except for points at the extrema, $B_0'(\zeta) = 0$. Here ζ is the toroidal angle coordinate. At these points, the dependence of B on the poloidal angle could lead to the formation of islands of B = constant lines with width magnitudes proportional to the axis curvature and minor radius of the toroidal surface. To conserve the poloidal topology of the B-lines here, one should have zero curvature of the magnetic axis at the points $B_0'(\zeta) = 0$. In addition, the amplitude of the bumpy magnetic field should be larger than the poloidal inhomogeneity of B in the cross-sections with maximal magnetic axis curvature. If the PS condition is fulfilled near the region with minimal longitudinal magnetic field, the line $B = B_{min}$ is closed and goes around the magnetic axis. This line is not necessarily straight in Boozer coordinates. In this case one can try to improve the confinement of the deeply trapped particles by fulfilling the condition of QI ($J_\| = \int v_\| dl = J_\|(a)$). Thus, as one of possible ways to improve the particle confinement, the combination of the PS and the QI conditions can be considered. In comparison with the QI configuration, the fulfillment of the PS condition near the region with the maximal magnetic field permits to avoid the trapped-passing transitional orbits. If a PS configuration is considered initially, the further fulfillment of the QI condition translates into an improvement of the confinement of both the deeply trapped and moderately-deeply trapped particles.

In the conventional near-axis approximation, the PS and QI condition were derived in [7], proving the possibility to fulfill them in the first and the second orders of the expansion.

Fourier spectrum approach. For numerical optimization it is more important to express the PS and QI conditions in terms of the B_{mn} components of the Fourier spectrum of B in Boozer coordinates,

$$B = \sum_{m,n} B_{m,n} \cos(m\theta_B - n\zeta_B)\,. \tag{1}$$

Let us consider that the bumpy magnetic field has only one minimum ($\zeta = 0$) and one maximum ($\zeta = \pi$) on the field period, where $0 \le \zeta \le 2\pi$. In the near-axis approximation $B_{m,n} \sim a^m$, where a is proportional to the average radial distance from the magentic axis, so that the first poloidal harmonic can be considered as a small value of first order and the second poloidal harmonic is of second order. Then the PS and QI conditions can be derived in the same manner as in the conventional near-axis approximation [7].

Necessary PS conditions.

In the first approximation, the PS condition requires

$$\sum_n B_{1,n} = 0 \qquad \sum_n B_{1,n}(-1)^n = 0\,. \tag{2}$$

These conditions mean that the magnetic axis curvature is equal to zero at the extrema (maximum and minimum) of the bumpy magnetic field, $\zeta = 0$ and $\zeta = \pi$.

From the second order consideration, the amplitude of the second poloidal harmonic is determined,

$$\left(\sum B_{1,n}\, n\right)^2 / \sum B_{0,n}\, n^2 = 4\sum B_{2,n}, \tag{3}$$

$$\left(\sum B_{1,n}(-1)^n n\right)^2 / \sum B_{0,n}(-1)^n n^2 = 4\sum B_{2,n}(-1)^n \tag{4}$$

in the bumpy magnetic field extremum points. It is worth to note that the conditions (2)–(4) are obtained for systems with two extremum points of the longitudinal magnetic field on a field period. The presence of additional extrema of $B_0(\zeta)$ can violate the PS condition even if the necessary PS conditions (2)–(4) are fulfilled.

QI conditions.

The QI condition imposes an additional limitation on the $B_{m,n}$ spectrum. It can be expressed also through the $B_{m,n}$ values.

In the first approximation, this condition requires [4]

$$\sum B_{1,n} n^2 = 2\iota_1 \sum B_{1,n} n. \tag{5}$$

In the second approximation, the QI condition has the form:

$$\begin{aligned} \sum B_{2,n} n^2 &= 4\iota_1 \sum B_{2,n} n \\ &-\left(\sum B_{1,n} n\right)^2 \sum B_{0,n} n^4 / 4\left(\sum B_{0,n} n^2\right)^2 \\ &+\sum B_{1,n} n \sum B_{1,n} n^3 / \sum B_{0,n} n^2 \\ &-5\left(\sum B_{1,n} n\right)^2 \iota_1^2 / \left(2\sum B_{0,n} n^2\right). \end{aligned} \tag{6}$$

Here ι_1 is the rotational transform in one period of the system.

In an optimized QI configuration [4], the curvature of the magnetic axis is equal to zero in the cross-section with minimal magnetic field, but the maximum of longitudinal magnetic field is located at the cross-section with maximal curvature. Thus, there is no line $B = B_{max}$ going around the magnetic axis. Instead, we have here a single point $B = B_{max}$ surrounded by islands of B-lines on the magnetic surface. The particles with high enough longitudinal velocity can be the trapped on some part of the trajectory while they can transit on another part. The radial excursion after one transition from trapped to passing trajectories is not very large. Nevertheless, a sequence of such transitions can be considered as some kind of diffusion (without collisions) and can lead to the loss of the particles.

The combination of the QI and PS conditions can be realized, for example, if the 6-fold periodic bumpy field component of the initial QI configuration similar to that described in Ref. [8] is replaced by a corresponding component with 3-fold periodic structure. As a result, the system will have a more complex periodic structure. The preliminary result of such optimization is shown in Fig. 1.

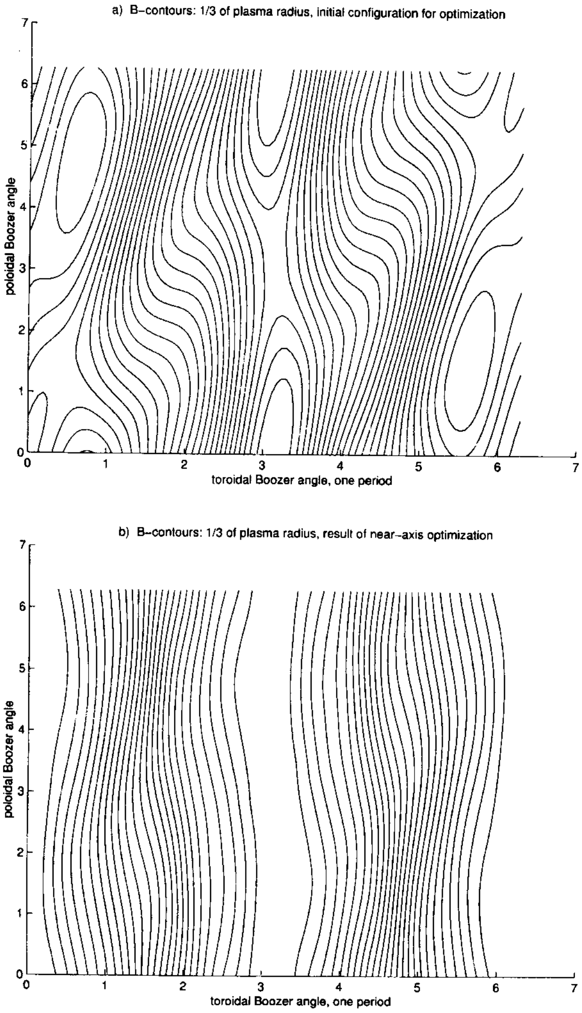

Fig. 1 Contours of B = constant on the magnetic surface $a = 1/3a_{pl}$ in Boozer coordinates for 3-fold periodic configurations with aspect ratio A = 12. (a) initial configuration; (b) result of the optimization toward pseudosymmetry based on near-axis conditions (2)–(4).

3. The Global Formulation of the PS Condition

In the paper [9], the mathematical formulation of the PS condition was given.

$$f = \frac{[\boldsymbol{B}\,\nabla\Psi]\cdot\nabla B^2}{\boldsymbol{B}\cdot\nabla B^2} \tag{7}$$

should be finite. Thus, as the target function for the optimization towards PS, minimization of the integral

$$T=\int\frac{(\partial B^2/\partial\theta_B)^2}{(\iota_1\partial B^2/\partial\theta_B+\partial B^2/\partial\zeta_B)^2+a^2\varepsilon^2}\,\mathrm{d}a\,\mathrm{d}\theta_B\,\mathrm{d}\zeta_B \quad (8)$$

can be used. The term $a^2\varepsilon^2$ is introduced to avoid a singular denominator in the integrand. In such a formulation, the PS condition requires that the extrema of B along the magnetic field line can only exist in regions where the gradient of B in the direction perpendicular to $\boldsymbol{B}$ on the magnetic surface is equal to zero also.

4. The Conditions of Absolute Minimum-B Existence in Mirror-Type Stellarators

The systems with poloidal direction of B-lines can be considered as some kind of nontrivial combination of a mirror-type system and a stellarator. Adding rotational transform in an open trap can change the properties of the system significantly. For example, a new possibility arises to fulfill the conditions %in such combined system becomes possible to fulfill the condition of isodynamicity in near-axis approximation in the periodic system.% of isodynamicity in the near-axis approximation in such combined periodic systems [10].

Another difference arises if we consider the possibility to create an absolute minimum of B in a mirror-type system. It is well known that in the open magnetic traps with zero rotational transform, it is possible to create regions with enclosed ellipsoidal surfaces B = constant in which the absolute minimum of B lies inside these topological spheres (see, e.g. [11]). Adding rotational transform (rotating elliptical cross-sections) provides another possibility to create an absolute minimum of B in systems with bumpy magnetic fields. For the system with a straight magnetic axis, the condition of the minimum of B in directions perpendicular to the magnetic axis (where the magnetic field is required to be minimal) can be expressed as

$$\eta'' th\eta > 2b''/b; \quad (9)$$

$$\frac{(\eta''+b''/b)(ch\eta-sh\eta)}{2sh\eta(1+1/ch^2\eta-th\eta)}<\delta'^2,$$

$$\delta'^2<\frac{(\eta''-b''/b)(sh\eta+ch\eta)}{2sh\eta(1+1/ch^2\eta+th\eta)}. \quad (10)$$

Here $\exp(\eta)$ corresponds to the ratio of the semiaxes of the elliptic cross-section, ($'$) denotes the derivative along the magnetic axis, δ' is the rotation velocity of the cross-section and b is the normalized magnetic field at the magnetic axis, where $\langle b\rangle = 1$.

In closed systems, the condition of the existence of an absolute minimum of B can be fulfilled also if the curvature is equal to zero in the cross-section with minimal longitudinal magnetic field, but the requirement on the η'' can be stricter.

The possibility of the existence of surfaces B = constant with the topology of a sphere opens a new avenue of optimization. The practical value of this approach can be ascertained with further computations.

5. Conclusions

The near-axis consideration of the PS and QI conditions in mirror-type stellarators has shown that it is possible to fulfill both of them and has permitted to identify the main geometric characteristics of such configurations. The PS and QI conditions (2)–(6) derived in the near-axis approximation and the global PS condition (7) can be used to construct the target functions for the corresponding optimization. The simultaneous fulfillment of the PS and QI conditions requires a more complex structure of the system period. The numerical optimization towards the PS and QI system is in progress now.

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