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The MHD Simulations of 3D Magnetic Reconnection near Null Point of Magnetic Configurations

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Abstract

We investigate 3D plasma flow in the vicinities of critical points of magnetic configurations. The study is based on the analysis of exact self-similar solution of the MHD equations and 3D computer simulations. Both the analytical solution and 3D MHD simulations demonstrate appearance of singular distribution of the electric current density near the magnetic field separatrix surfaces of the form of the current and vortex sheets.

Keywords:
magnetic reconnection, current sheets, 3D MHD simulation

The studies of the magnetic reconnection are relevant to a broad range of problems of space and fusion plasmas (see [1,2] and the references in). In the limit when the nonlinearity effects dominate over dissipation, self-similar solutions for the MHD equations can be used in order to describe the self-consistent evolution of the plasma flow and of the magnetic field. The process when in the plasmas it takes a finite time to form the singularity in the electric current distribution near critical points is called the magnetic collapse. The magnetic collapse corresponds to the self-pinching of the electric current carrying plasma in inhomogeneous external magnetic field when the gradient of the external magnetic field is larger or of order of that of the magnetic field associated with the plasma electric current. In order to investigate non-self-similar plasma motions, we use 3D MHD computer simulations.

In the vicinity of $\boldsymbol{x} = 0$, the magnetic field is equal to $\boldsymbol{B}(\boldsymbol{x}, t) = \boldsymbol{B}(0, t) + (\boldsymbol{x}\cdot\nabla)\boldsymbol{B}(\boldsymbol{x}, t) + \ldots$. We introduce the notation $b_i = B_i(0, t)$ for the uniform component of the magnetic field, and $A_{ij} = \partial B_i/\partial x_j|_{x_k=0}$ for the matrix of the magnetic field gradients. If the uniform part vanishes $b_i = 0$, a null point of the magnetic field occurs at $x_i = 0$, where $B_i = A_{ij}x_j$. Further we assume that A_{ij} is not zero. By virtue of the condition div $\boldsymbol{B} = 0$, the trace of the matrix A_{ij} is zero ($A_{kk} = 0$) and the sum of the eigenvalues vanishes, $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (λ_α $\alpha = 1, 2, 3$ are eigenvalues) of the matrix A_{ij}.

In the present paper the magnetic configuration with three nonvanishing eigenvalues is considered. We assume that the initial configuration of the magnetic field is current free with the matrix A_{ij} of the diagonal form:

$$A_{ij} = \mathrm{diag}\,\{a, b, -(a+b)\}. \tag{1}$$

If a and b have the same sign, the separatrix surface is $z = 0$.

In dimensionless form the MHD equations can be written as

$$\partial_t \rho + \nabla(\rho \boldsymbol{v}) = 0, \tag{2}$$

$$\partial_t \boldsymbol{v} + (\boldsymbol{v}\nabla)\boldsymbol{v} = -\tilde{\beta}\nabla(\rho T)/2\rho + \boldsymbol{j}\times\boldsymbol{B}/\rho\,, \qquad (3)$$

$$\rho(\partial_t T + (\boldsymbol{v}\nabla)T)/(\gamma-1) + \rho T(\nabla\cdot\boldsymbol{v}) = -\tilde{\kappa}\Delta T + 2\tilde{\nu}_m\, \boldsymbol{j}^2/\tilde{\beta}, \qquad (4)$$

$$\partial_t \boldsymbol{B} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) - \tilde{\nu}\Delta\boldsymbol{B}, \quad (\nabla\cdot\boldsymbol{B}) = 0, \qquad (5)$$

where the adiabatic index is equal to $\gamma = 5/3$.

As usual $\tilde{\beta} = 8\pi p_0/B_0^2$, where p_0 is a pressure at t_0. The inverse Lundquist number is $\tilde{\nu}_m = c^2/4\pi\sigma v_a s$, $\tilde{\kappa} = 0.01$.

The MHD equations (2-5) have self-similar solutions when $T = 0$ [3]. The discussions of the plasma pressure effects are presented in [3,4]. In these solutions $\rho = \rho(t)$, $v_i(\boldsymbol{x}, t) = w_{ij}(t)x_j$, $B_i(\boldsymbol{x}, t) = A_{ij}(t)x_j$.

Substituting these expressions into eqs. (2)-(5) we obtain a system of ordinary differential equations for $\rho(t)$, matrices $w_{ij}(t)$ and $A_{ij}(t)$

$$\dot{\rho} + w_{kk}\rho = 0, \qquad (6)$$

$$\dot{w}_{ij} + w_{ik}w_{kj} = -(A_{ik} - A_{ki})A_{kj}/\rho, \qquad (7)$$

$$\dot{A}_{ij} + w_{kk}A_{ij} + A_{ik}w_{kj} = w_{ik}A_{kj}. \qquad (8)$$

In the simplest spatially nonuniform magnetic configuration with a null point, the plasma velocity and magnetic fields are described by matrices [3]

$$w_{ij} = \begin{pmatrix} w_{11} & w_{12} & 0 \\ w_{21} & w_{22} & 0 \\ 0 & 0 & w_{33} \end{pmatrix}, \quad A_{ij} = \begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{pmatrix}. \qquad (9)$$

We assume that at the initial time the magnetic field is current-free and is given by the diagonal matrix given by eq. (1) which corresponds to the vicinity of the magnetic null point.

The results of numerical solution of system of eqs. (6)-(8) are presented in Figs. 1 and 2, where the time dependences of the density ρ, the vorticity $(w_{21} - w_{12})/2$ and the electric current density $(A_{21} - A_{12})/2$ are shown.

In Fig. 1 the time dependence of the density ρ, vorticity ω_z and electric current density j_z is shown for initial configuration described by the matrices: w^0_{ij} with $w^0_{12} = -0.025$, $w^0_{12} = 0.025$ and $w^0_{ij} = 0.$ for other ij; $A^0_{ij} = \mathrm{diag}\{0.5, 0.5, -1\}$. In this case the separatrix surface is in the plane $z = 0$, and excitation of the electric current is perpendicular to the separatrix surface. We see that at $t_0 = 35.25$ the ρ, ω_z, j_z tend to infinity. The density shows nonmonotonic dependence on time. At the initial stage the density of the plasma decreases and just then the magnetic collapse occurs.

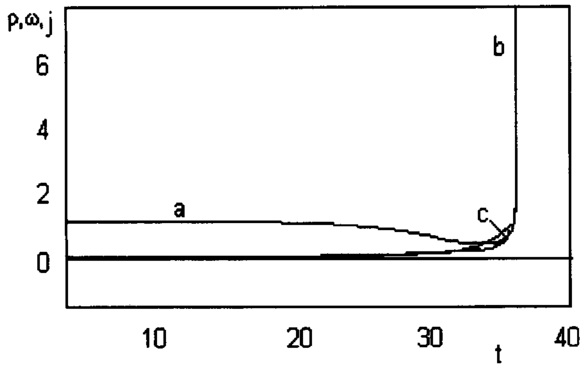

Fig. 1 Density ρ-a, electric current density j-b and vorticity ω-c versus time for $\omega^0_{12} = -0.025$, and $\omega^0_{12} = 0.025$; $A^0_{ij} = \mathrm{diag}\{0.5, 0.5, -1\}$.

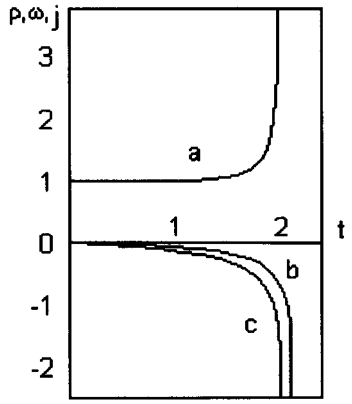

Fig. 2 Density ρ-a, electric current density j-b and vorticity ω-c versus time for $\omega^0_{12} = -0$, and $\omega^0_{12} = -0.025$; $A^0_{ij} = \mathrm{diag}\{2, -1, -1\}$.

In Fig. 2 the time dependence of the same value as in Fig. 1 are shown for initial configuration described by the matrices: w^0_{ij} with $w^0_{21} = -0.025$ and $w^0_{ij} = 0$ for other ij; $A^0_{ij} = \mathrm{diag}\{2, -1, -1\}$. In this case the the separatrix surface is perpendicular to the plane $z = 0$, and excitation of the electric current directed along the separatrix surface. We see that the magnetic collapse developes much faster than in the previous case: at $t_0 = 1.9$ the ρ, ω_z, j_z tend to infinity.

We consider two initial configurations of the magnetic field.

In the first case the magnetic field is described by the vector potential:

$$\boldsymbol{A}(x, y, z) = -2\varepsilon x z \boldsymbol{e}_y + (1-\varepsilon)x y \boldsymbol{e}_z, \qquad (10)$$

where $\varepsilon \le 1$ characterizes the space inhomogeneity. The matrix A_{ij} given by eq. (1) has the components $a = 1 + \varepsilon$ and $b = -1 + \varepsilon$. The electric current is imposed from the boundary to be parallel to the magnetic separatrix.

The second magnetic configuration is described by expression

$$A(x, y, z) = -2\varepsilon x z \boldsymbol{e}_y - \varepsilon x y \boldsymbol{e}_z, \tag{11}$$

which corresponds to the separatrix perpendicular to the electric current imposed from the boundary. The matrix A_{ij} has the components $a = b = \varepsilon$ with $\varepsilon = 0.1$.

The system of MHD equations (2)-(5) has been solved numerically in cubic computational region: $-1 \leq x/s \leq 1$, $-1 \leq y/s \leq 1$, $-1 \leq z/s \leq 1$. We assume that, at t_0 a plasma with a uniform density and pressure is at rest in the current free magnetic field, described by the vector potential (10) or (11).

The boundary conditions correspond to the excitation of the electric current. To described magnetoacoustic waves we choose the vector potential at the boundaries $x = \pm 1$, $y = \pm 1$ of the form $A(x, y, t) = A_0(x, y) + f(t + lnr)$, where $A_0(x, y)$ is given by the (11). Function $f(\xi)$ is equal to $-E_1(\xi - 1)^2/\xi$ for $\xi > 1$ and 0 for $\xi < 1$. All the results of the computational simulations presented below are obtained for $\tilde{\nu}_m = 0.006$, $\tilde{\beta} = 0.012$, $\tilde{k} = 0.01$, $E_1 = 0.01$ and $\varepsilon = 0.01$ or $\varepsilon = 0.5$ in (11).

The first series of the numerical calculations was performed for the vector potential given by eq. (10) with $\varepsilon = 0.01$. The results of the simulations of the azimuthally symmetric magnetoacoustic perturbations are shown in Fig. 3 where a slice at $z = 0$ of the distribution of the electric current density, iso-surface of the electric current with iso-value equal 0.2 are shown. We see the formation of the current sheet in the vicinity

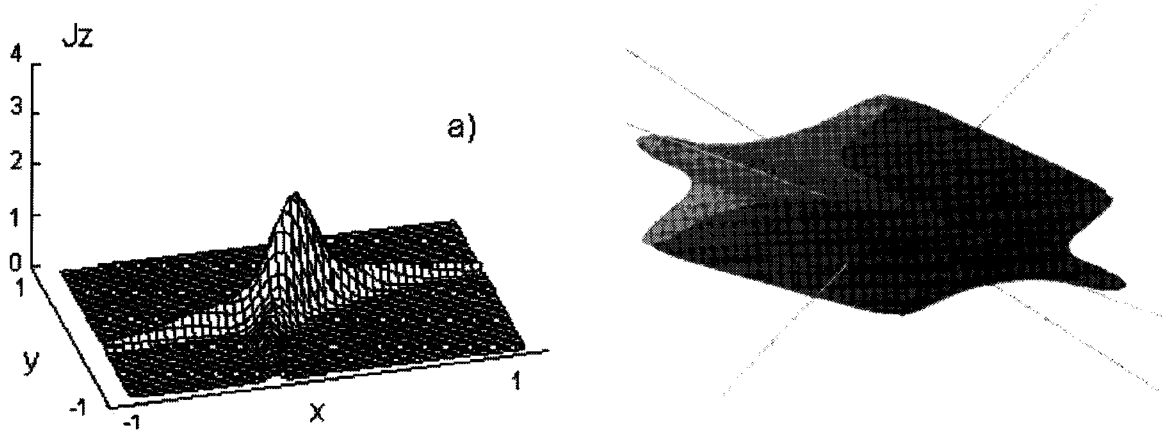

Fig. 3 Distribution a) and level surface with level value equal 0.2 of the current density for $\varepsilon = 0.01$.

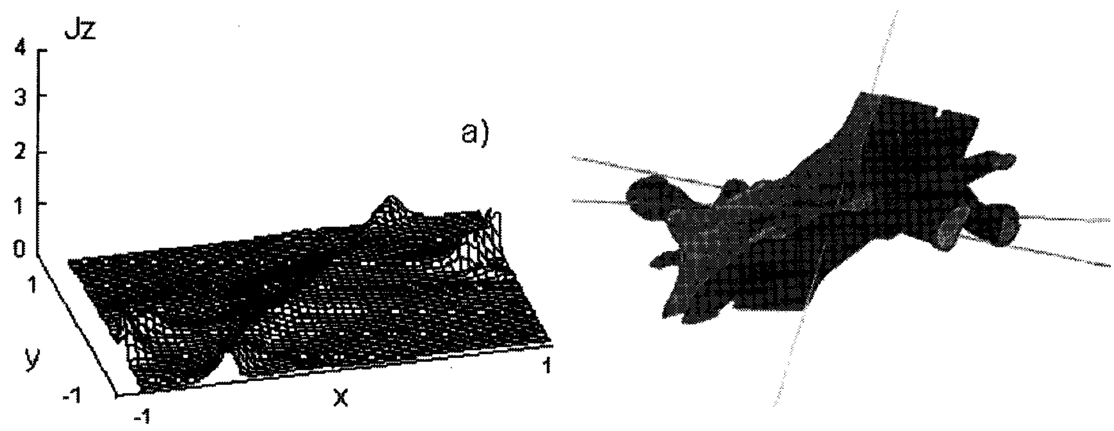

Fig. 4 Distribution a) and level surface with level value equal 0.2 of the current density for $\varepsilon = 0.5$.

of the separatrices.

The second series of the numerical calculations was performed for the vector potential given by Eq. (10) with $\varepsilon = 0.5$. In the Fig. 4 the distributions of the same functions as in Fig. 3 are presented. We see the formation of the current sheet and vortex sheets in the vicinity of the separatrices. The explicit dependence of the structure of the electric current sheet on z-axis is obtained.

In the case when the magnetic configuration is described by the expression eq. (11) we have not observed formation of the current and vortex sheets. The results of the computational simulation is shown in Fig. 5.

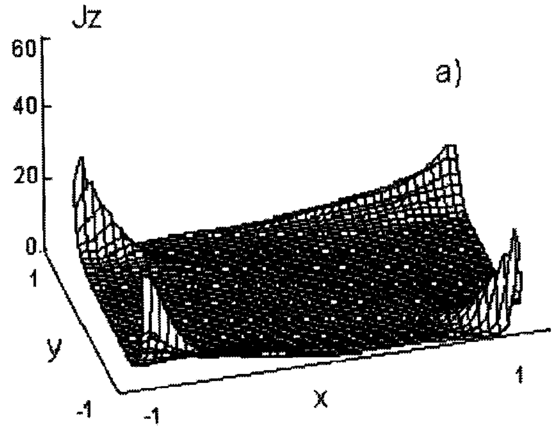

Fig. 5 Distribution of the electric current density for (11).

We have studied the process of the driven regimes of the magnetic reconnection in the 3D magnetic configurations. In the first case, when the electric current excited at the boundary of computational region is mainly parallel to the magnetic separatrix, the numerical simulations demonstrate the formation of the electric current sheet and vortex sheets in the vicinity of the separatrices. This qualitatively corresponds to analytical solutions describing the magnetic collapse. In second case, when the electric current excited at the boundary of computational region is orthogonal to the magnetic separatrix, the formation of the vortex and electric current sheets have not been observed in the computer simulations. Self-similar regimes of the magnetic configuration evolution show that for the magnetic collapse to appear in this case it takes a time much longer than in the previous case.

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