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A Magnetohydrodynamic Model of MRX Discharges

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Abstract

A magnetohydrodynamic (MHD) model of the Magnetic Reconnection Experiment (MRX) has been established. It can reproduce both of the "pull" and "push" mode operations with co-helicity injection. Time evolutions of the MRX configuration, such as magnetic island formation, can be explained by quasi-static transition of the Taylor state according to change of poloidal field coil current. MHD simulations agree well not only with the theoretical model but also with experimental evidences.

Keywords:

magnetohydrodynamics, magnetic reconnection, magnetohydrodynamic relaxation

1. Introduction

Magnetic reconnection found in fusion, space, and astrophysical plasmas [1-3] has been attracted many researcher's attention in the last two decades. This is because reconnection could play a crucial role in magnetohydrodynamic (MHD) relaxation processes. In relaxation of an MHD plasma, there are two characteristic features, such as conversion of magnetic energy and topology change of field lines [4], for which magnetic reconnection is responsible [5]. Magnetic Reconnection Experiment (MRX) in Princeton Plasma Physics Laboratory is the first laboratory experiment focusing on reconnection physics under a controllable condition [6]. Field configuration in MRX is varied in time by changing currents in a pair of flux-core (FC) coils which consist of poloidal (PF) and toroidal field (TF) coils. Then, a plasma flow is induced, which drives reconnection at a separatrix point between FC coils. MRX has two types of operation modes, so-called "pull" and "push" modes. In the "pull" mode, PF coil currents are reduced in time, while it is increased in the "push" case. In addition, two types of toroidal field configurations are available in MRX, that is, co- and counter-helicity injections. In the co-helicity case, the same toroidal field is induced by the FC coils into the plasma region, while anti-parallel toroidal fields are given in the counter-helicity one. Changing the operation scheme (four modes in total), Yamada and his co-workers have found a morphological feature of field lines during reconnection. During the "pull" operation, a Y-shaped current sheet is formed in the counter-helicity case, while a magnetic island is created in a diffusion region of reconnection in the co-helicity injection [7,8]. Our previous work based on the Taylor's relaxation theory and MHD simulations revealed a physical mechanism of the island formation during the "pull" mode [9]. A main purpose of the present paper is to study time evolutions of the MRX configuration during the "push" operation with co-helicity injection and to compare the results with the "pull" mode.

2. Theoretical Model

Here, we consider the Taylor state in a rectangular plasma container with a pair of FC coils, neglecting toroidal effects in z direction. Since no center rod is used in the MRX, one of the side boundaries at $x = \pm L_x$ corresponds to the main (symmetry) axis in comparison

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of the model configuration with the MRX device. The model is located in the poloidal plane of $-L_x \le x \le L_x$ and $-L_y \le y \le L_y$ ($L_x = 1$ and $L_y = 1.4$). A pair of FC coils with their radii of 0.2 are, respectively, placed at $(x, y) = (0, \pm 0.6)$. Fig.1 schematically shows the model configuration.

In a system with symmetry in z direction, the Taylor state can be given by

$$\nabla^2 \Psi + \mu^2 \Psi = 0 \qquad (1)$$

in terms of the poloidal flux Ψ. Without loss of generality one may set $\Psi = 0$ on the outer boundary and $\Psi = \Psi_0 = 1$ on the inner (FC coil) boundary. Solving Eq.(1) for given μ and Ψ_0, one finds a Taylor state in the model configuration. Here, it is meaningful to mention a definition of magnetic helicity in a multiply-connected region. According to Taylor [10], the magnetic helicity in a torus is defined as $K = \int \boldsymbol{A} \cdot \boldsymbol{B} \mathrm{d}v - \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l} \oint \boldsymbol{A} \cdot \mathrm{d}s$ so that K is indeed invariant to a multi-valued gauge potential χ ($\boldsymbol{B}$ and $\boldsymbol{A}$ are magnetic field and its vector potential). Here, $\mathrm{d}\boldsymbol{l}$ and $\mathrm{d}s$ denote loop integrals the long and short way around the toroidal surface. In the model configuration of MRX, the gauge-invariant helicity is written as follows:

$$K \equiv \int \boldsymbol{A} \cdot \boldsymbol{B} \,\mathrm{d}x\mathrm{d}y - \sum_i \Psi_0 \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}_i , \qquad (2)$$

where $\mathrm{d}\boldsymbol{l}_i$ means the loop integral around the i-th FC coil surface, while $\mathrm{d}x\mathrm{d}y$ denotes the integral in the multiply-connected region between the outer and inner boundaries.

Solving Eq.(1) by the second-order finite difference and the conjugate-gradient method, we have plotted the helicity and energy of the Taylor state in Fig.2, showing that a magnetic island appears between the FC coils when $|\mu|$ exceeds a critical value ($\simeq 1.6$). As $|\mu|$ increases further (but less than the lowest eigenvalue of Eq.(1), $\simeq 3.2$), the island with a spheromak-like configuration grows and finally covers the whole system. The result gives us an important suggestion on the co-helicity injection of the MRX discharge. Since the MRX plasma is low temperature (say, lower than 20 eV), it may be approximate to the force-free state, that is, a solution of Eq.(1) for $|\mu| < 1.6$ before decreasing the PF coil current. As the PF coil current is reduced in a time scale longer than the Alfvén transit time, Ψ is "pulled" into the FC coils, namely, Ψ_0 becomes smaller. This means that more plasma current is induced in the system, namely, $|\mu|$ is increased by decreasing Ψ. On the other hand, in the "push" mode, the PF current is increased resulting reduction of the plasma current. Thus, $|\mu|$ is decreased during the "push" operation. The "pull" and "push" operation, respectively, correspond to injection and reduction of the helicity normalized by Ψ_0^2/L_x. Therefore, the magnetic island will appear when $|\mu| > 1.6$ in the "pull" mode, while the configuration tends to the vacuum state in the "push" case.

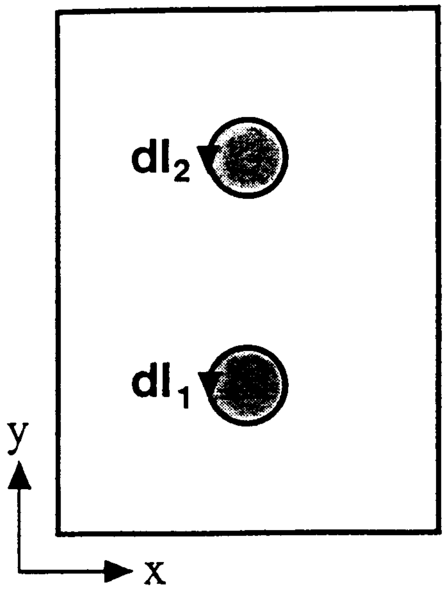

Fig. 1 A schematic plot of MRX configuration showing a path of integration defined in Eq.(2). Shaded regions indicate cross-sections of flux-core coils.

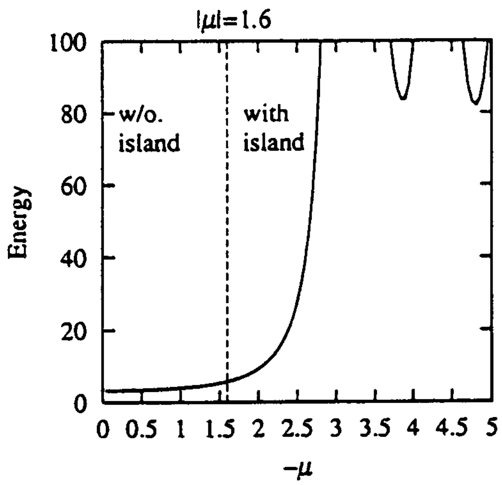

Fig. 2 Magnetic energy versus μ of the Taylor state given by Eq.(1). For $|\mu| > 1.6$, a magnetic island appears.

3. Numerical Simulation

The conjecture derived in the previous section would be valid for slow variation of PF coil current (ideally, infinitesimal induction electric field). However, in real experiments, the PF coil currents are changed in a finite time (typically several tens of μsec). Thus, by means of numerical simulation, it is necessary to check the validity of the scenario in a realistic time scale. Using the same model described before, we have carried out two-dimensional MHD simulations, where the following equations are solved;

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{v}), \tag{3}$$

$$\rho \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\nabla p + \boldsymbol{j} \times \boldsymbol{B} + \nu(\nabla^2 \boldsymbol{v} + \frac{1}{3}\nabla(\nabla \cdot \boldsymbol{v})), \tag{4}$$

$$\frac{1}{(\Gamma-1)}\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{\Gamma}{\Gamma-1} p \nabla \cdot \boldsymbol{v} + \eta j^2 + \Phi, \tag{5}$$

$$\frac{\partial \Psi}{\partial t} = -E_z, \tag{6}$$

$$\frac{\partial B_z}{\partial t} = -\nabla \times \boldsymbol{E}_p. \tag{7}$$

Here, $\boldsymbol{j}$, $\boldsymbol{E}$, $\boldsymbol{B}$, Φ and e_{ij} are, respectively, given by

$$\boldsymbol{j} = j_z \hat{z} + \boldsymbol{j}_p = -\hat{z}\nabla^2 \Psi + \nabla B_z \times \hat{z}, \tag{8}$$

$$\boldsymbol{E} = E_z \hat{z} + \boldsymbol{E}_p = -\boldsymbol{v} \times \boldsymbol{B} + \eta \boldsymbol{j}, \tag{9}$$

$$\boldsymbol{B} = B_z \hat{z} + \nabla \Psi \times \hat{z}, \tag{10}$$

$$\Phi = 2\nu\left(e_{ij}\, e_{ij} - \frac{1}{3}(\nabla \cdot \boldsymbol{v})^2\right), \tag{11}$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right). \tag{12}$$

The suffix p denotes a poloidal (x, y) component of a vector. Velocity $\boldsymbol{v}$ is set to be zero on the boundaries. We have used a perfect conductor for the outer boundary, while $\boldsymbol{E}_p = 0$ and $E_z = \pm 0.01$ on the inner boundaries that correspond to the FC coil surfaces. The positive E_z means the "pull" operation, while the negative E_z is imposed on the FC coil surface in the "push" case. All of physical quantities are normalized by the typical length $L_x = 1$, a characteristic poloidal magnetic field $B_{p0} = \Psi_0/L_x = 1$, and the initial density $\rho_0 = 1$. The Alfvén velocity V_{A0} given by B_{p0} and ρ_0 is equal to 1. Time is measured by the Alfvén transit time $\tau_A = L_x/V_{A0}$. The initial pressure is set to be $p = 0.2$. Viscosity ν and ratio of the specific heats Γ are, respectively, $\nu = 1 \times 10^{-3}$ and $\Gamma = 5/3$. Furthermore, it is considered that the resistivity η might be larger nearby the FC coils than in the central region because of higher impurity density. Thus, we have employed an inhomogeneous η model such as

$$\eta = \eta_0 \left[1 + C_\eta \exp\left\{-(r - r_c)^2/r_c^2\right\}\right], \tag{13}$$

where r and r_c denote a distance from a center of the nearest FC coil and its radius. The previous simulation for the "pull" case using the inhomogeneous η has given a better agreement with the experiment than the constant resistivity case [9].

Time evolutions of the magnetic helicity and energy obtained in the "pull" and "push" simulations are plotted in Fig.3 where solid line shows the Taylor state given by Eq.(1). Starting from the Taylor state with $\mu = 1$, in the "pull" case, both of the normalized energy and helicity are increased in time. One can see the time evolution follows the lowest energy branch of the Taylor state. After $t = 20\ \tau_A$, when the energy increases to 4.4, a small island appears in the diffusion region, and then, grows larger and larger. Comparing Figs.2 and 3 it is found that the energy at the first appearance of the island corresponds to $\mu = 1.2$ in the Taylor state. Although the estimated μ is smaller than the critical value of 1.6, the difference may be attributed to inhomogeneity of μ which is often observed in dynamic simulations of plasma relaxation. On the other hand, in

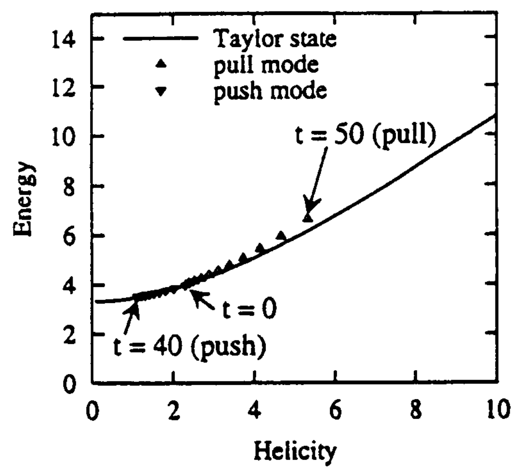

Fig. 3 Evolution of the magnetic energy and helicity found in MHD simulations for "pull" and "push" modes. Solid line shows the Taylor state. Marks representing simulation results are plotted at every 5 τ_A.

the "push" case, the normalized energy and helicity are decreased in time, while total values of them (not normalized by Ψ_0) are increased by the poloidal flux injection. Thus, the global configuration in the "push" case approaches to the vacuum solution, although a sharp peak of the plasma current is found at the separatrix point. Therefore, no magnetic island is formed in the "push" case as expected from the Taylor state analysis. The above results agree well with the theoretical prediction, and are also qualitatively consistent with more recent experiments where no island is observed in the "push" operation [11].

4. Conclusion

In this study we have investigated transition of field configuration in "pull" and "push" operations of the MRX discharge. The Taylor state analysis shows that the quasi-static change of the force free state can explain both of the operations with the co-helicity injection. In the "pull" case a magnetic island is formed, while the configuration tends to the vacuum solution in the "push" case. The simulation results under a realistic condition are consistent with the theoretical prediction, and can also successfully explain the experiments.

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References

[1] E.N. Paker, *Cosmical Magnetic Fields* (Clarendon Press, Oxford, 1979).

[2] T. Sato and T. Hayashi, Phys. Fluids **22**, 1189 (1979); H. Amo, T. Sato, A. Kageyama and the Complexity Simulation Group, Phys. Rev. E **51**, 3838 (1995).

[3] D. Biskamp, Phys. Rep. **237**, 179 (1993).

[4] J.B. Taylor, Phys. Rev. Lett. **33**, 1139 (1974).

[5] T. Sato and K. Kusano, Phys. Rev. Lett. **54**, 808 (1985).

[6] M. Yamada, H. Ji, S. Hsu, T. Carter, R. Kulsrud, N. Bretz, F. Jobes, Y. Ono and F. Perkins, Phys. Plasmas **4**, 1936 (1997).

[7] M. Yamada, H. Ji, T.A. Carter, S.C. Hsu, R.M. Kulsrud, N.L. Bretz, F.C. Jones, Y. Ono, M. Katsurai, T.-H. Watanabe, T. Sato and T. Hayashi, *Proc. 16th Int. Conf. Fusion Energy*, Montreal, 1996 (IAEA, Vienna, 1997), Vol.2, p.253.

[8] M. Yamada, H. Ji, S. Hsu, T. Carter, R. Kulsrud, Y. Ono and F. Perkins, Phys. Rev. Lett. **78**, 3117 (1997).

[9] T.-H. Watanabe, T. Hayashi, T. Sato, M. Yamada and H. Ji, Phys. Plasmas **6**, 1253 (1999); T.-H. Watanabe, T. Hayashi and T. Sato, Bull. Am. Phys. Soc. **43**, 1840 (1998).

[10] J.B. Taylor, Rev. Mod. Phys. **58**, 741 (1986).

[11] M. Yamada, H. Ji and Y. Ono, private communication (1998).