J. Plasma Fusion Res. SERIES, Vol. 2 (1999) 470–473

Alfvén Wave Dissipation in the Dipole Axisymmetric Magnetosphere

GRISHANOV Nikolay I., DE AZEVEDO Carlos A. and DE ASSIS Altair S.[1]
Universidade do Estado do Rio de Janeiro, RJ 20550-013, Brazil
[1]Universidade Federal Fluminense, Niterói 24024-000, Brazil

(Received: 8 December 1998 / Accepted: 11 June 1999)

Abstract

Some physical features of the kinetic Alfvén wave dissipation by the trapped electrons are analyzed in an axisymmetric magnetosphere with dipole magnetic field lines, taking into account the bounce-resonant wave-particle interaction. It is shown that the electron Landau damping of kinetic Alfvén waves in a dipole magnetosphere depends substantially on the geomagnetic field nonuniformity and the plasma energy/temperature.

Keywords:

kinetic Alfvén wave, Landau damping, dipole magnetosphere

1. Magnetospheric Plasma Model

To study the wave processes in magnetospheric plasmas in the frequency range of Alfvén waves it is necessary to solve the Maxwell's equations with a "nonlocal" dielectric tensor [1-3]. For these (as well for other magnetohydrodynamic) waves, this tensor can be derived by solving the drift-kinetic equation for the trapped particles, taking into account a two-dimensional inhomogeneity of the geomagnetic field and plasma parameters [3,4]. In this paper, we analyze some physical features of a Landau damping of the kinetic Alfvén waves by the trapped electrons in an axisymmetric magnetosphere with dipole magnetic field lines:

$$B(R, \phi) = B_0\left(\frac{R_0}{R}\right)^3 \sqrt{1+3\sin^2\phi}.$$

Here R_0 is the radius of the Earth (or another planet), R is the geocentric distance, ϕ is the geomagnetic latitude, and B_0 is the magnetic field in an equatorial plane on the Earth's surface, where $R = R_0$, $\phi = 0$. All magnetospheric particles (both the electrons and protons, core and energetic) are trapped, performing the bounce oscillations along the geomagnetic field line near the minimum of $B(R, \phi)$. To describe these particles we introduce the new variables

$$v = \sqrt{v_\parallel^2 + v_\perp^2},\ \mu = v_\perp^2 B(L, 0)/v^2 B(L, \phi),$$

and

$$L = R/R_0 \cos^2\phi$$

instead of $v_\parallel$, $v_\perp$, R, which are associated with the conservation integrals of energy: $v_\perp^2 + v_\parallel^2 = \text{const}$, magnetic moment: $v_\perp^2/2B = \text{const}$, and the equation of $\boldsymbol{B}$-field line: $R/\cos^2\phi = \text{const}$.

Depending on μ, the domain of perturbed distribution functions is defined by the inequalities

$$L^{-2.5}/\sqrt{4L-3} \le \mu \le 1 \text{ and } -\phi_t \le \phi \le \phi_t(\mu),$$

where $\pm\phi_t(\mu)$ are the local mirror points for the trapped particles at a given (by L) magnetic field line, which are defined by the zeros of parallel velocity:

$$\cos^6\phi_t - \mu\sqrt{1+3\sin^2\phi_t} = 0.$$

Corresponding author's e-mail: grishano@uerj.br

Due to the Earth's atmosphere, the trapped particles will be thermalized by the collisions with atmospheric molecules and atoms before they reach the Earth's surface. Any particle with

$$\mu < \mu_0 = L^{-2.5} / \sqrt{4L-3},$$

will not survive more than one half of the bounce time and will be precipitated into the atmosphere.

As is well known, the density growth of energetic particles, e.g., during the magnetic storm, can be a reason of the wide class of the drift-wave instabilities (see, for example, Ref. 5 and the bibliography there). In particular, the pressure gradient of energetic protons in the direction perpendicular to $\boldsymbol{B}$ can be as a source of energy to excite the geomagnetic pulsations corresponding to the low-frequency Alfvén waves. The most important characteristic of these instabilities is the threshold of critical density of energetic protons (or their critical pressure), at which the wave excitation is beginning. To find the instability thresholds it is necessary to compare the growth rate and damping rate of Alfvén waves in magnetospheric plasmas.

2. Kinetic Alfvén Waves

Of course, in order to have the full comprehensive description of both the eigenfunctions and eigenvalues for the perturbed electromagnetic fields in the Earth's magnetosphere it is necessary to solve the two-dimensional wave equations varying in L and ϕ directions. However, we believe that some important wave characteristics can be analyzed into the scope of geometric optics approximation for monochromatic waves in magnetized nonuniform plasmas, if/when

$$E_\parallel \sim \exp(-\mathrm{i}\omega t + \mathrm{i}\int k_R\, \mathrm{d}R + \mathrm{i}\pi n\phi/\phi_0 + \mathrm{i}m\theta).$$

So, to estimate the electron Landau damping of kinetic Alfvén waves we are going to use the well-known dispersion relation of ones, coupling the wave frequency, ω, perpendicular and parallel (relative to $\mathbf{B}$) components of the wave vector, $k_\perp$, $k_\parallel$, and the transverse and longitudinal dielectric permittivity, $\varepsilon_\perp$, $\varepsilon_\parallel$:

$$\frac{k_\perp^2 c^2}{\omega^2} = \frac{\varepsilon_\parallel}{\varepsilon_\perp}\left(\varepsilon_\perp - \frac{k_\parallel^2 c^2}{\omega^2}\right), \qquad (1)$$

where

$$k_\perp = k_\mathrm{R} + \frac{m}{LR_0}, \quad k_\parallel = \frac{2\pi}{l(L)} n,$$

and

$$l(L) = \frac{LR_0}{2\sqrt{3}}\left[\sqrt{3}\sin\phi_0(L)\sqrt{1+3\sin^2\phi_0(L)} + \ln\left(\sqrt{3}\sin\phi_0(L) + \sqrt{1+3\sin^2\phi_0(L)}\right)\right]$$

is the half-length of a given (by L) magnetic field line. The points

$$\pm\phi_0(L) = \pm\arccos(1/\sqrt{L})$$

are the beginning and the end of a considered magnetic field line on the Earth's surface; the mode numbers n and m are integer.

In Eq. (1), $\varepsilon_\perp$ is defined by the local contribution of protons to the real part of the transverse permittivity:

$$\varepsilon_\perp = \frac{\omega_\mathrm{pi}^2}{\omega_\mathrm{ci}^2 - \omega^2} \approx \frac{c^2}{c_\mathrm{A}^2}, \quad \omega_\mathrm{pi}^2 = \frac{4\pi N_0 e^2}{M_\mathrm{i}},$$
$$\omega_\mathrm{ci} = \frac{eB_0}{M_\mathrm{i} c L^3}, \quad c_\mathrm{A}^2 = \frac{B_0^2}{4\pi N_0 M_\mathrm{i} L^6}. \qquad (2)$$

The contribution of trapped electrons to the longitudinal permittivity elements of an axisymmetric magnetosphere with dipole magnetic field lines we define by the expressions evaluated in Ref. 4:

$$\varepsilon_\parallel^{n,n'}(L) = \frac{\omega_\mathrm{pe}^2 L^2 R_0^2}{2\pi^2 v_\mathrm{Te}^2 \phi_0}\sum_{p=1}^{\infty}\frac{1}{p^2}\int_{\mu_0}^{1}\tau_\mathrm{b} D_p^n \hat{D}_p^{n'}$$
$$\times\left\{1 + \frac{2\omega^2}{p^2\omega_\mathrm{b}^2}\left[1 + \mathrm{i}\sqrt{\pi}\frac{\omega}{p\omega_\mathrm{b}} W\left(\frac{\omega}{p\omega_\mathrm{b}}\right)\right]\right\}\mathrm{d}\mu, \qquad (3)$$

where

$$\hat{D}_p^n = \int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} - 2\pi p\frac{\tau(\phi)}{\tau_\mathrm{b}}\right)\cos\phi\sqrt{1+3\sin^2\phi}\,\mathrm{d}\phi$$
$$+ (-1)^{p-1}\int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} + 2\pi p\frac{\tau(\phi)}{\tau_\mathrm{b}}\right)\cos\phi\sqrt{1+3\sin^2\phi}\,\mathrm{d}\phi,$$

$$D_p^n = \int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} - 2\pi p\frac{\tau(\phi)}{\tau_\mathrm{b}}\right)\mathrm{d}\phi$$
$$+ (-1)^{p-1}\int_0^{\phi_t}\cos\left(\pi n\frac{\phi}{\phi_0} + 2\pi p\frac{\tau(\phi)}{\tau_\mathrm{b}}\right)\mathrm{d}\phi.$$

$$\tau_{\rm b} = 4\int_0^{\phi_t} \cos\phi \, \frac{\sqrt{1+3\sin^2\phi}}{\sqrt{1-\mu b(\phi)}}\, {\rm d}\phi, \quad \omega_{\rm b} = \frac{2\pi v_{\rm Te}}{R_0 L \tau_{\rm b}},$$

$$\tau(\phi) = \int_0^{\phi} \cos\eta \, \frac{\sqrt{1+3\sin^2\eta}}{\sqrt{1-\mu b(\eta)}}\, {\rm d}\eta,$$

$$b(\phi) = \frac{\sqrt{1+3\sin^2\phi}}{\cos^6\phi}, \quad \omega_{\rm pe}^2 = \frac{4\pi N_e e^2}{M_{\rm e}},$$

$$W(z) = e^{-z^2}\left(1 + \frac{2{\rm i}}{\sqrt{\pi}}\int_0^z e^{x^2}{\rm d}x\right).$$

The expressions (3) give us the contribution of the full spectrum of the perturbed electric field (by $\Sigma_{n'=\pm\infty}$) to the given n-Fourier harmonic of the perturbed current density component:

$$\frac{4\pi {\rm i}}{\omega} j_{\parallel}^{(n)}(L) = \sum_{n}^{\pm\infty} \varepsilon_{\parallel}^{n,n'} E_{\parallel}^{(n')}.$$

However, for the purpose of our paper, we use only the diagonal ($n = n'$) elements of the longitudinal permittivity; therefore further the umbral indexes ^{n,n} can be omitted. Note, evaluating the longitudinal dielectric permittivity, to simplify the problem, we used the isotropic Maxwellian distribution function for the trapped electrons with density $N_{\rm e}$, temperature $T_{\rm e}$, charge e and mass $M_{\rm e}$; $v_{\rm Te}^2 = 2T_{\rm e}/M_{\rm e}$. Moreover, in $\varepsilon_{\parallel}(L)$ we neglect the drift corrections assuming that the wave frequency ω is much larger than the drift frequency, that is valid when

$$m v_{\rm Te} R_0^{-1} L^{-1} \omega_{\rm ce}^{-1} << 1,$$

where $\omega_{\rm ce} = eB_0/L^3 M_{\rm e} c$, and m is the azimuthal wave number over θ (east-west) direction.

As one can see, by Eq. (3), there is a possibility to carry out analytically the Landau integration over particle energy (in velocity space), for the longitudinal permittivity elements, and to express $\varepsilon_{\parallel}(L)$ by the p-summation of bounce resonant terms including the well-known plasma dispersion function W(z). This feature simplifies substantially the estimations of both the real and imaginary parts of such an important plasma characteristic as its longitudinal permittivity, in the wide range of wave frequencies and plasma parameters.

The general solution of Eq. (1) gives us the following expressions for real ($\omega_{\rm KAW} = {\rm Re}\omega$) and imaginary ($\gamma_{\rm KAW} = -{\rm Im}\omega$) parts of the frequency of kinetic Alfvén waves:

$$\omega_{\rm KAW}^2 \approx k_{\parallel}^2 c_{\rm A}^2, \quad \gamma_{\rm KAW} = \omega_{\rm KAW} \frac{c_{\rm A}^2 k_{\parallel}^2 k_{\perp}^2 v_{\rm s}^4}{2c^2 \omega_{\rm ci}^4} {\rm Im}\,\varepsilon_{\parallel}, \quad (4)$$

where $v_{\rm s}^2 = 2T_{\rm e}/M_{\rm i}$ is the ion-sound velocity. Of course, Eq. (1) can be also applied to estimate the damping length of Alfvén waves, $\kappa_{\rm KAW} = {\rm Im}k_{\perp}/{\rm Re}k_{\perp}$, excited by the resonance transformation of the fast magnetosonic wave (with the given ω and mode numbers m and n) into the kinetic Alfvén wave at the magnetic shell where $\omega = k_{\parallel}(L) c_{\rm A}(L)$. In this case,

$$\kappa_{\rm KAW} = \frac{{\rm Im}\,\varepsilon_{\parallel}}{2{\rm Re}\,\varepsilon_{\parallel}} = \sum_{p=1}^{\infty} \frac{\sqrt{\pi}}{p^5} \tau_{\rm b} D_p^n \hat{D}_p^n \frac{\omega^3}{\omega_{\rm b}^3} \exp\left(\frac{\omega^2}{p^2\omega_{\rm b}^2}\right) {\rm d}\mu. \quad (5)$$

Thus we see that the dispersion characteristics (in particular, such as $\gamma_{\rm KAW}$ and $\kappa_{\rm KAW}$) of the kinetic Alfvén waves are defined by the bounce-resonant interaction between the wave and the trapped magnetospheric electrons.

For the low-frequency waves with $\omega/\omega_{\rm b} << 1$, the first ($p = 1$) bounce-resonant term is main in $\varepsilon_{\parallel} = \Sigma_{p=1}^{\infty} \varepsilon_{\parallel,p}$, see Eq. (3). In particular, this means that the damping length, $\kappa_{\rm KAW} \sim {\rm Im}\,\varepsilon_{\parallel}$ of kinetic Alfvén waves in the dipole magnetospheric plasmas depends on the plasma temperature as $T^{-1.5}$ in contrast to $T^{-0.5}$ for the straight magnetic field case (or in the local approximation).

3. Numerical Results

Now we present the results of numerical calculations of the imaginary part of the longitudinal permittivity:

$${\rm Im}\,\varepsilon_{\parallel}(L) = \sum_{p=1}^{\infty} {\rm Im}\,\varepsilon_{\parallel,p}(L) = \frac{\omega_{\rm pe}^2 \omega^3}{\sqrt{2}\pi^{4.5}\omega_{\rm bo}^5 \arccos\sqrt{1/L}} \times \sum_{p=1}^{\infty} \frac{1}{p^5}\int_{\mu_0}^{1} \tau_{\rm b}^4(\mu) D_p^n \hat{D}_p^n \exp\left[-\frac{\omega^2}{p^2\omega_{\rm bo}^2}\frac{\tau_{\rm b}^2(\mu)}{2\pi^2}\right] {\rm d}\mu. \quad (6)$$

For the wave processes in magnetospheric plasmas with the given temperature of ions and electrons, it is interesting to know how the Landau damping of waves depends on the wave frequencies. The contribution of trapped electrons with the temperature $T_{\rm e}$ to ${\rm Im}\,\varepsilon_{\parallel}$ as a function of ω (by the parameter $\omega/\omega_{\rm bo}$) is presented in Fig. 1, where $\omega_{\rm bo} = \sqrt{2} v_{\rm Te}/R_0 L$. The concrete calculations are carried out for the oscillations with longitudinal mode numbers n = 1, 2, 3, which can be excited in the region of the Earth's radiation belts, L = 5. The results here are normalized to $\omega_{\rm pe}^2(L)/\omega_{\rm bo}^2$. This

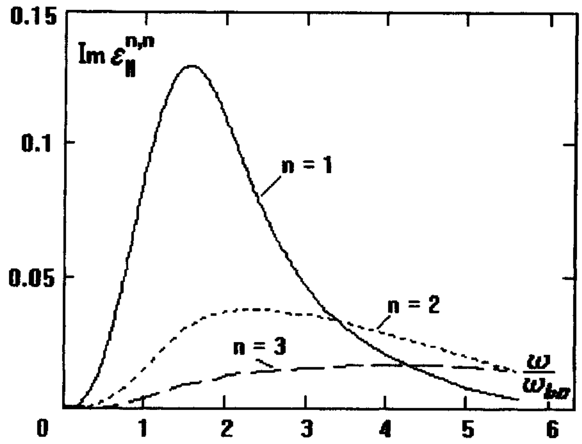

Fig. 1 Imaginary part of the longitudinal permittivity as a function of the wave frequency (by ω/ω_{bo}) for waves in an axisymmetric dipole magnetosphere when/where $L = 5$ and $n = 1, 2, 3$.

means that the bounce-resonant terms, Im $\varepsilon_{\parallel,p}$, can be approximated as $\sim z_p^3 \exp(-z_p^2)$, where $z_p = \omega/p\omega_{bo}$. As a result, $z_p^{max} = \sqrt{3/2}$ (or $p_{max} = \sqrt{2/3}\ \omega/\omega_{bo}$) corresponds to the maximum of $z_p^3 \exp(-z_p^2)$. The basic feature of these plots is that the maximum of Im $\varepsilon_{\parallel}$, in plasmas with a given temperature and $\omega/\omega_{bo} > \sqrt{3/2}$, is achieved for the waves with longitudinal mode numbers n larger than or of the order of p_{max}.

4. Conclusion

The electron Landau damping of the kinetic Alfvén waves in an axisymmetric dipole magnetosphere (as is for magnetospheric plasmas with circular magnetic field lines [6,7]) depends substantially on the geomagnetic field nonuniformity.

If $\omega << \omega_{bo}$, the imaginary part of the longitudinal permittivity decreases as $\sim v_{Te}^{-5}$. This decrease is stronger than $\sim v_{Te}^{-3}$ for plasmas in a straight magnetic field. It should be noted, since the bounce resonances are not effective in this frequency range, the drift corrections become substantial for the ultralow-frequency waves with large wave numbers $m \sim 100$. As was shown in Ref. 3, the excitation of the low-frequency geomagnetic pulsations in the range of Pc-3 and Pc-5 oscillations is associated with an effective bounce-drift interaction between the wave and energetic protons.

If $\omega \sim \omega_{bo}$, the numbers of the basic bounce resonances are defined by $p \sim \omega/\omega_{bo}$. In this case, Im $\varepsilon_{\parallel}(L)$ has a maximum for waves with longitudinal wave numbers $n \sim p$. This means that the resonant condition for the effective wave-particle interaction in magnetospheric plasmas should be understood as the condition where the wave performs the integer number (p) of oscillations during one bounce period $2\pi/\omega_{bo}$ of the trapped particles.

Acknowledgements

We are grateful to F.M. Nekrasov and A.G. Elfimov for useful discussions. This work was supported by Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ) of Brazil.

References

[1] D.J. Southwood, J.W. Dungey and R.J. Etherington, Planet. Space Sci. **17**, 349 (1969).
[2] V.I. Karpman, B.I. Meerson, A.B. Mikhailovsky and O.A. Pokhotelov, Planet. Space Sci. **25**, 573 (1977).
[3] L. Chen and A. Hasegawa, J. Geophys. Res. **96**, 1503 (1991).
[4] N.I. Grishanov, C.A. de Azevedo and A.S. de Assis, Phys. Plasmas **5**, 4384 (1998).
[5] O.A. Pokhotelov *et al.*, Planet. Space Sci. **34**, 695 (1986).
[6] N.I. Grishanov, C.A. de Azevedo and A.S. de Assis, J. Geophys. Res. **101**, 7881 (1996).
[7] N.I. Grishanov, A.G. Elfimov, C.A. de Azevedo and A.S. de Assis, Phys. Plasmas **3**, 3798 (1996).