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Energy Transfer from Low-Frequency Magnetosonic Pulses to Particles in a Two-Ion-Species Plasma

TOIDA Mieko, DOGEN Daiju and OHSAWA Yukiharu
Department of Physics, Nagoya University, Nagoya 464-8602, Japan

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Abstract

The low-frequency mode of magnetosonic wave in a two-ion-species plasma is studied with a one-dimensional, electromagnetic simulation code based on a three-fluid model. It is found that a pulse of the low-frequency mode, as well as that of the high-frequency mode, is damped even if the plasma is collisionless and the pulse propagates perpendicular to a magnetic field. The damping is due to slight acceleration of heavy ions.

Keywords:
collisionless damping, magnetosonic wave, two-ion-species plasma, heavy ion acceleration, soliton

1. Introduction

Recently it has been found that in a plasma containing multiple ion species magnetosonic waves behave quite differently from those in a single-ion-species plasma [1-5]. First of all, the magnetosonic wave in a two-ion-species plasma is split into two modes: the high- and low-frequency modes [1]. The frequency of the low-frequency mode tends to zero as the wave number k goes to zero and approaches the ion-ion hybrid resonance frequency as $k \to \infty$ [6]. The high-frequency mode has a cut-off frequency of the order of the ion cyclotron frequency and has a resonance frequency of the order of the lower hybrid frequency. (The dispersion curves can be found in ref. 1).

The nonlinear behavior of the high frequency mode can be described by Korteweg-de Vries (KdV) equation [1], even though the high-frequency mode has the finite cut-off frequency. The characteristic soliton width is of the order of the electron skin depth. It was found that a nonlinear pulse of the high-frequency mode can accelerate minority heavy ions in a multi-ion-species plasma such as space plasma [3]. The heavy ions gain energy from the transverse electric field formed in the wave. Because of this energy transfer, nonlinear pulses of the high-frequency mode are damped, even when they propagate perpendicular to a magnetic field with small amplitudes [4-5]. This could be an important dissipation mechanism in a collisionless multi-ion-species plasma.

In this paper we study heavy ion acceleration in the low-frequency mode and the associated wave damping. In Sec. 2, the heavy ion motion in the low-frequency wave is analytically discussed. It is found that heavy ions can also be slightly accelerated by a nonlinear pulse of the low-frequency mode. This suggests that nonlinear pulses of the low-frequency mode are damped through the heavy ion acceleration. In Sec. 3, using the simulation based on a three-fluid model, we will show that the low-frequency pulse gives some energy to heavy ions and that the pulse is gradually damped.

2. Heavy Ion Motion

The low-frequency mode has weak dispersion,

$$\omega = v_{\mathrm{A}} k (1 - k^2 d_l^2 / 2), \tag{1}$$

in the long-wavelength region [1]. Here v_{A} is the Alfvén speed and d_l is defined as

$$d_l = \frac{v_A^3}{c^2}\left[\frac{\omega_{pa}^2\omega_{pb}^2}{\Omega_a^2\Omega_b^2}\left(\frac{1}{\Omega_a}-\frac{1}{\Omega_b}\right)^2 + \frac{\omega_{pb}^2\omega_{pe}^2}{\Omega_b^2\Omega_e^2}\left(\frac{1}{\Omega_b}-\frac{1}{\Omega_e}\right)^2 + \frac{\omega_{pe}^2\omega_{pa}^2}{\Omega_e^2\Omega_a^2}\left(\frac{1}{\Omega_e}-\frac{1}{\Omega_a}\right)^2\right]^{1/2}. \quad (2)$$

Here, ω_{pj} and Ω_j are the plasma and cyclotron frequencies of particle species j, respectively. The subscripts a and b refer to the lighter and heavier ion species, respectively. When two ion species are present with considerable densities, the first term in the square brackets, which is proportional to $(\Omega_a^{-1} - \Omega_b^{-1})^2$, is the dominant term, and d_l is of the order of the ion skip depth, c/ω_{pi}. The dispersion of the low-frequency mode is ~ (m_i/m_e) times as large as that of the magnetosonic wave in a single-ion-species plasma [7].

As can be expected from eq. (1), the nonlinear low-frequency wave can be described by the KdV equation [1]. The magnetic filed profile of the solitary wave propagating in the x direction in a magnetic field that points in the z direction can be given by

$$B_z/B_0 = 1 + B_n \operatorname{sech}^2(s). \quad (3)$$

Here, B_n is the normalized wave amplitude and the argument s is

$$s = [x - (1 + B_n/2)v_A t]/D, \quad (4)$$

with D being the soliton width

$$D = 2d_l/B_n^{1/2}. \quad (5)$$

The longitudinal electric field E_x and the transverse electric field E_y are

$$E_x/(v_A B_0/c) = v_A^3/(c^2 d_l)\sum_j (\omega_{pj}^2/\Omega_j^3) B_n^{3/2}\operatorname{sech}^2(s)\tanh(s), \quad (6)$$

$$E_y/(v_A B_0/c = B_n \operatorname{sech}^2(s). \quad (7)$$

We discuss single-particle motion of heavy ions in the solitary pulse, using the equation of motion

$$m_b \frac{dv_{bx}}{dt} = q_b\left(E_x + \frac{v_{by}}{c}B_z\right), \quad (8)$$

$$m_b \frac{dv_{by}}{dt} = q_b\left(E_y - \frac{v_{bx}}{c}B_z\right). \quad (9)$$

In order to analitically integrate these equations, we drop the perturbation of the magnetic field and neglect the change in x in E_x and E_y. We take the time derivative of Eq. (9) and eliminate v_{bx} using Eq. (8), then we obtain

$$\frac{d^2 v_{by}}{dt^2} + \Omega_b^2 v_{by} = F(t). \quad (10)$$

Here the function $F(t)$ is defined as

$$F(t) = \frac{\Omega_b v_A^4}{d_l c^2}\frac{\omega_{pa}^2}{\Omega_a^2}\left(1 - \frac{\Omega_b}{\Omega_a}\right) B_n^{3/2}\operatorname{sech}^2(s')\tanh(s'). \quad (11)$$

The argument s' is

$$s' = [x_0 - (1 + B_n/2)v_A t]/D, \quad (12)$$

where x_0 is the initial particle position. We assume that the particles are in the far upstream region at $t = 0$; i.e., $x_0/D \gg 1$. We apply the Laplace transform to Eq. (10), and using the inversion formula, we find the velocity v_{by} for $t > 0$ as

$$v_{by} = \frac{1}{\Omega_b}\int_0^t \sin[\Omega_b(t-u)]F(u)du. \quad (13)$$

In order to obtain the velocity v_{by} in the downstream region at large t, we assume that

$$(x_0 - Mv_A t)/D \to -\infty. \quad (14)$$

Then, v_{by} can be found as

$$v_{by} = v_{bm}\cos\left[\Omega_b\left(t - \frac{x_0}{Mv_A}\right)\right], \quad (15)$$

where, v_{bm} is defined as

$$\frac{v_{bm}}{v_A} = 4\pi\frac{d_l^2\omega_{pa}^2}{c^2}\frac{\Omega_b^2}{\Omega_a^2}\left(1 - \frac{\Omega_b}{\Omega_a}\right)\operatorname{cosech}\left(\frac{\pi\Omega_b d_l}{v_A B_n^{1/2}(1 + B_n/2)}\right). \quad (16)$$

This equation indicates that heavy ions gyrate with the speed v_{bm} in the downstream region. In passing through the pulse, a heavy ion gains energy $m_b v_{bm}^2/2$.

If we apply the above discussion to the high-frequency mode, we can also obtain a finite velocity behind the pulse. Its magnitude is equal to the value which we showed in the previous paper [Eq. (A.14) in ref. 3]. In Fig. 1 we show v_{bm} as a function of the wave amplitude B_n. The dotted line is for the high-frequency mode, and the solid line is for the low-frequency mode. The v_{bm} for the low-frequency mode rapidly increases with the wave amplitude.

3. Numerical Simulation

We study the propagation of the low-frequency mode using a one-dimensional, fully electromagnetic code based on a three-fluid model:

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \boldsymbol{v}_j) = 0, \tag{17}$$

$$m_j \left[\frac{\partial}{\partial t} + (\boldsymbol{v}_j \cdot \nabla) \right] \boldsymbol{v}_j = q_j \boldsymbol{E} + \frac{q_j}{c} \boldsymbol{v}_j \times \boldsymbol{B}, \tag{18}$$

$$\frac{1}{c} \frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}, \tag{19}$$

$$\frac{1}{c} \frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \frac{4\pi}{c} \sum_j q_j n_j \boldsymbol{v}_j. \tag{20}$$

We assume periodic boundary conditions. As initial wave profiles, we use the solitary wave solution obtained from the KdV equation for the low-frequency mode and observe their evolution.

We simulate a hydrogen-helium plasma. Thus, we have chosen the mass and charge ratios as m_a/m_e = 1000, m_b/m_a = 4, q_b/q_a = 2. The density ratio is n_b/n_a = 0.1, and the magnetic field strength is $|\Omega_e|/\omega_{pe}$ = 0.5 in the upstream region, so that c/v_A = 68.3.

In Fig. 2 we show profiles of the magnetic field, light-ion velocity v_{ay} and heavy ion velocity v_{by} at time $\Omega_H t$ = 50. The initial amplitude is $B_n(0)$ = 0.03. The heavy ion has the finite velocity behind the pulse, as Eq. (16) predicts. This heavy ion motion produces perturbations behind the pulse, which can be seen in the profiles of v_{ay} and B_z.

The observed heavy ion speed v_{bm} is 2.2 × 10^{-3} v_A is smaller than the theoretical value given by Eq. (16), v_{bm} = 7.7 × 10^{-3} v_A. In deriving Eq. (16), we did not include the effect that heavy ion motion generates perturbations behind the pulse. Therefore, Eq. (16) is interpreted to give the upper limit of the velocity of accelerated heavy ions.

Figure 3 shows time variation of the wave energy of the main pulse; its denoted by $E_w(t)$. Even though we observe small-amplitude fluctuation, the main pulse certainly loses energy gradually.

4. Summary

We have demonstrated the collisionless damping of the low-frequency magnetosonic pulses in a two-ion-species plasma. Firstly, we have theoretically predicted that heavy ions gain some energy from a solitary pulse

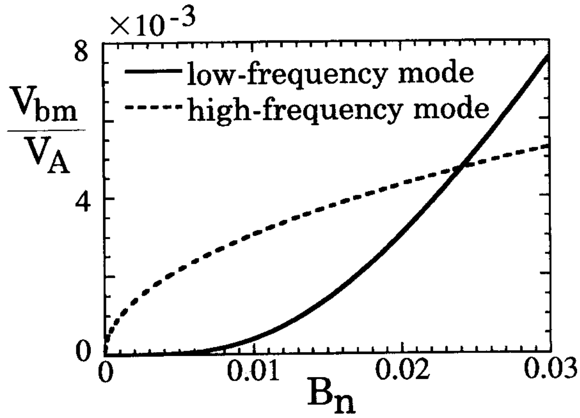

Fig. 1 Speed of accelerated heavy ions behind the pulse, v_{bm}, as a function of the normalized wave amplitude B_n. The solid line is for the low-frequency mode and the dotted line is for the high-frequency mode.

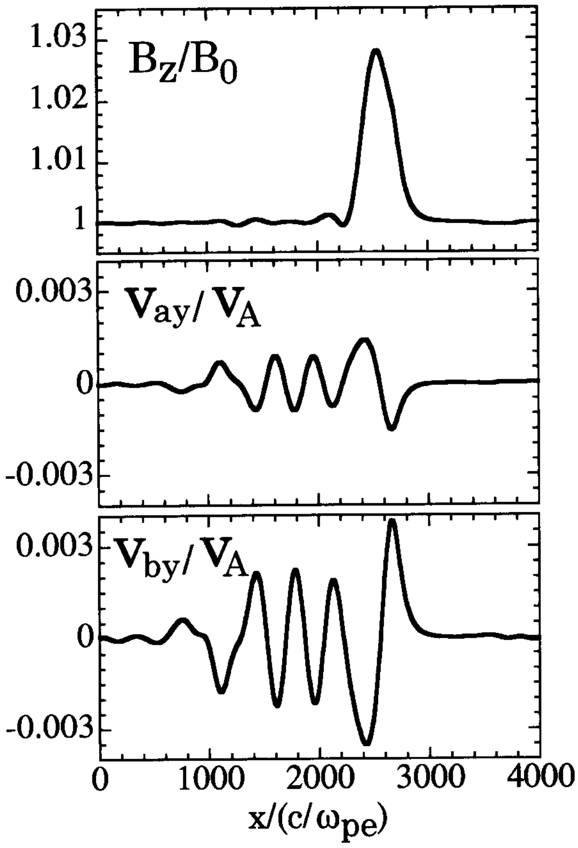

Fig. 2 Profiles of the magnetic field, light ion velocity v_{ay}, and heavy ion velocity v_{by} at $\Omega_H t$ = 50. The initial amplitude is B_n = 0.03.

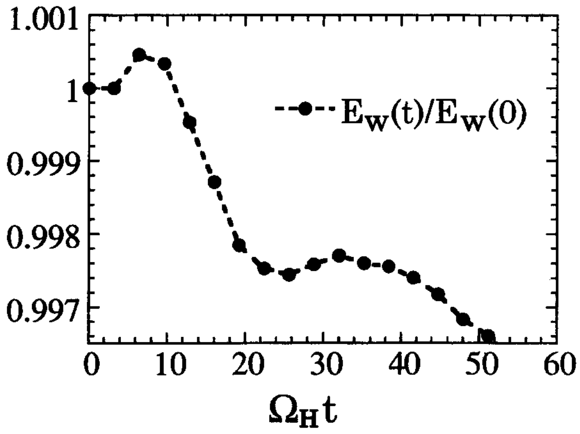

Fig. 3 Time variation of wave energy $E_w(t)$ of the original pulse.

of the low-frequency mode. Next, we carried out simulations using a one-dimensional, fully electromagnetic code based on a three-fluid model. We showed that the solitary pulse of the low-frequency mode is damped.

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