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Nonlocal Effects in an Excitation of Ion Acoustic Waves by an Ion Beam in a Plasma Cylinder

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Abstract

An ion beam propagating through a plasma cylinder drives electrostatic ion acoustic waves to instability via Cerenkov interaction. The growth rate of the ion acoustic wave instability increases with the beam density and scales as the one-third power of the beam density. The real frequency of the unstable wave increases as almost the square root of the beam energy. However, the growth rate of the instability decreases with the plasma density.

Keywords:
ion acoustic waves, ion beam, plasma cylinder, growth rate

1. Introduction

Electrostatic ion acoustic waves are studied in a wide variety of situations, ranging from small-scale laboratory experiments [1-5] to space plasmas [6-8]. Liu and Tripathi [9] have studied ion acoustic waves in an infinite medium. However, nonlocal effects for these waves are not considered so far. In this paper, we develop a non-local theory of ion acoustic waves driven to instability by an ion beam in a plasma cylinder. In Sec. 2, we carry out the instability analysis. The plasma and beam responses are obtained using fluid theory. We obtain the growth rate of the instability using first order perturbation theory. Results and discussions are given in Sec. 3.

2. Instability Analysis

Consider a cylindrical plasma column of radius a, equilibrium density n^{o}_{po}, temperature $T(= T_e$ for electrons and $= T_i$ for ions), immersed in a static magnetic field $\vec{B}_s \parallel \hat{z}$. The plasma is collisionless. An ion beam with velocity $v_{ob}\hat{z}$, mass m_b, density n^{o}_{ob} and radius r_o $(= a)$ propagates through the plasma along the magnetic field. The beam plasma system prior to the perturbation is quasineutral, since we have taken $n^{o}_{op} \gg n^{o}_{ob}$. We assume that temperature is not modified by the perturbation (isothermal approximation). The equilibrium is perturbed by an electrostatic perturbation

$$\phi = \phi_0 \, e^{-i(\omega t - k_z z)}. \tag{1}$$

The response of plasma electrons to the perturbation is governed by the equation of motion which on linearization yields the perturbed velocity

$$\vec{v}_{1\perp} = \frac{e}{m}\frac{[i\omega\nabla_\perp\phi + \nabla_\perp\phi \times \vec{\omega}_c]}{\omega^2 - \omega_c^2} - \frac{T_e}{mn^{o}_{op}}\frac{[i\omega\nabla_\perp n_{1p} + \nabla_\perp n_{1p} \times \vec{\omega}_c]}{\omega^2 - \omega_c^2}, \tag{2}$$

$$v_{1z} = \frac{ek_z\phi}{m\omega} + \frac{T_e}{m}\frac{k_z n_{1p}}{\omega n^{o}_{op}} \tag{3}$$

where $-e$, m, and ω_c are the electron charge, mass and cyclotron frequency and subscript 1 refers to perturbed quantities. Using Eqs. (2) and (3) in the continuity equation, we obtain

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$$(1 - \frac{v_{te}^2 k_z^2}{\omega^2}) n_{1p} + \frac{v_{te}^2 \nabla_\perp^2 n_{1p}}{\omega^2 - \omega_c^2} = \frac{n_{op}^o e}{m} (\frac{\nabla_\perp^2 \phi}{\omega^2 - \omega_c^2} - \frac{k_z^2 \phi}{\omega^2}), \quad (4)$$

where $v_{te} = (T_e/m)^{1/2}$ is the thermal velocity of electrons and $\omega_c = eB_s/mc$. In the limit of short parallel wavelength ($\omega \gg k_z v_{te}$) and $\omega \gg \omega_c$, Eq. (4) is modified as

$$n_{1p} = \frac{n_{op}^o e\phi}{m v_{te}^2} = \frac{n_{op}^o e\phi}{T_e}. \quad (5)$$

By replacing $-e$, m, ω_c and ω_{ci} by e, m_i, ω_{ci} and v_{ti}, respectively, in Eq. (4), we obtain

$$(1 - \frac{v_{ti}^2 k_z^2}{\omega^2}) n_{1p} + \frac{v_{ti}^2 \nabla_\perp^2 n_{1p}}{\omega^2 - \omega_{ci}^2} = \frac{n_{op}^o e}{m} (\frac{\nabla_\perp^2 \phi}{\omega^2 - \omega_{ci}^2} - \frac{k_z^2 \phi}{\omega^2}), \quad (6)$$

where $v_{ti} = (T_i/m_i)^{1/2}$ is the thermal velocity of ions and $\omega_{ci} = eB_s/m_i c$. In the limit $\omega \ll \omega_{ci}$, $k_\perp v_{ti} < \omega_{ci}$, i.e., $k_\perp v_{ti}/(\omega - \omega_{ci}) \ll 1$ or $k_\perp \rho_i \ll 1$, Eq. (6) is modified as

$$n_{1i} = \frac{1}{4\pi e} [\frac{\omega_{pi}^2}{\omega_{ci}^2} \nabla_\perp^2 \phi + \frac{\omega_{pi}^2}{\omega^2} k_z^2 \phi], \quad (7)$$

where $\omega_{pi}^2 = 4\pi n_{op}^o e^2/m_i$. The response of cold ion beam can be obtained by solving the fluid equations of motion and continuity which on linearization yields the perturbed beam velocity and beam density

$$v_{1b} = \frac{e}{m_b} \frac{k_z \phi}{(\omega - k_z v_{ob})^2}, \quad (8)$$

$$n_{1b} = \frac{n_{ob}^o e k_z^2 \phi}{m_b (\omega - k_z v_{ob})^2}. \quad (9)$$

Using Eqs. (5), (7) and (9) in the Poisson's equation $\nabla^2 \phi = 4\pi e (n_{1e} - n_{1i} - n_{1b})$, we obtain

$$\nabla_\perp^2 \phi + p^2 = \frac{\omega_{pb}^2 k_z^2 \phi}{(\omega - k_z v_{ob})^2 (1 + \frac{\omega_{pi}^2}{\omega_{ci}^2})}, \quad (10)$$

where

$$p^2 = \frac{\frac{\omega_{pi}^2}{\omega^2} k_z^2 - k_z^2 - \frac{\omega_p^2}{v_{te}^2}}{(1 + \frac{\omega_{pi}^2}{\omega_{ci}^2})}, \quad (11)$$

and $\omega_p^2 = 4\pi n_{op}^2 e^2/m$, $\omega_{pb}^2 = 4\pi n_{ob}^o e^2/m_i$. Equation (10) can be rewritten for axially symmetric case as

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} + \frac{\partial \phi}{\partial r} + p^2 \phi = - \frac{\omega_{pb}^2 k_z^2 \phi}{(\omega - k_z v_{ob})^2 (1 + \frac{\omega_{pi}^2}{\omega_{ci}^2})}, \quad (12)$$

Now we attempt a solution of Eq. (12) and evaluate the growth rate of unstable mode in the beam plasma system using perturbation theory. In the absence of the beam, i.e., when the right-hand side is zero , Eq. (12) reduces to the Bessel equation and gives a well known solution

$$\phi = A J_0(p_n r), \ p = p_n. \quad (13)$$

ϕ must vanish at $r = a$; hence $J_0(p_n a) = 0$, i.e., $p_n = x_n/a$ ($n = 1, 2, 3......$), x_n are the zeros of the Bessel function $J_0(x)$. In the presence of the beam, the wave function can be expressed in a series of orthogonal sets of wave functions:

$$\phi = \sum_m A_m J_0(p_m r). \quad (14)$$

Substituting value of Eq. (14) in Eq. (12), multiplying both sides by $r J_0(p_n r)$, integrating over r from 0 to a (where a is the plasma radius), and retaining only the dominant mode $m = n$, we obtain

$$p^2 - p_n^2 = - \frac{\omega_{pb}^2 k_z^2}{(\omega - k_z v_{ob})^2 (1 + \frac{\omega_{pi}^2}{\omega_{ci}^2})}, \quad (15)$$

Substituting value of p^2 from Eq. (11), Eq. (15) can be rewritten as

$$\left(\omega^2 - \frac{\omega_{pi}^2}{\alpha} \frac{k_z^2}{(p_n^2 + k_z^2 + \frac{\omega_{pi}^2}{c_s^2})} \right) = \frac{\omega_{pb}^2 \omega^2 k_z^2}{\alpha (\omega - k_z v_{ob})^2 (p_n^2 + k_z^2 + \frac{\omega_{pi}^2}{c_s^2})}, \quad (16)$$

where

$$\alpha = 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{p_n^2}{(p_n^2 + k_z^2 + \frac{\omega_{pi}^2}{c_s^2})}. \quad (17)$$

Equation (16) can be rewritten as

$$(\omega^2 - \alpha_1^2)(\omega - k_z v_{ob})^2 = \frac{\omega_{pb}^2 \omega^2 k_z^2}{\alpha (p_n^2 + k_z^2 + \frac{\omega_{pi}^2}{c_s^2})}, \quad (18)$$

where

$$\alpha_1^2 = \frac{c_s^2 k_z^2}{\alpha (1 + \frac{(p_n^2 + k_z^2) c_s^2}{\omega_{pi}^2})}, \quad (19)$$

and $c_s = (T_e/m_i)^{1/2}$ is the ion acoustic speed. Here $\omega = \alpha_1$ corresponds to the dispersion relation of ion acoustic wave and $\omega \simeq k_z v_{ob}$ corresponds to the beam mode. We are looking for solutions when $\alpha_1 \simeq k_z v_{ob}$, i.e., when the beam is in Cerenkov resonance with the ion acoustic mode. In this case the two factors on the left-hand side of Eq. (18) are simultaneously zero in the limit as $n^o_{ob} \to 0$. When $n^o_{ob} \neq 0$, we expand as $\omega = \alpha_1 + \delta = k_z v_{ob} + \delta$ where δ is the modification in ω due to the right-hand side of Eq. (18). Then Eq. (18) gives the growth rate of the unstable mode

$$\gamma = \mathrm{Im}\delta_i = \frac{\sqrt{3}}{2}\left(\frac{\alpha_1\,\omega^2_{pb}k^2_z}{2\alpha(p^2_n + k^2_z + \frac{\omega^2_{pi}}{c^2_s})}\right)^{1/3}. \qquad (20)$$

The real frequency of the unstable mode in terms of beam energy is given by

$$\omega_r = k_z\left(\frac{2eV}{m_b}\right)^{1/2} - \frac{1}{2}\left(\frac{\alpha_1\,\omega^2_{pb}k^2_z}{2\alpha(p^2_n + k^2_z + \frac{\omega^2_{pi}}{c^2_s})}\right)^{1/3}. \qquad (21)$$

3. Results and Discussions

In Fig.1, we have plotted the dispersion curve of an ion acoustic wave and beam mode for typical parameters of an ion beam plasma experiment, e.g., plasma density $n^o_{op} \simeq 1 \times 10^{10}\mathrm{cm}^{-3}$, ion plasma frequency $\omega_{pi} = 2.03 \times 10^7$rad./sec (argon plasma), guide magnetic field $B_s = 3 \times 10^3$Gauss, ion cyclotron frequency $\omega_{ci} = 6.84 \times 10^5$rad/sec (argon), radius of plasma cylinder $a = 2$cm, mode number $n = 1$, i.e., first zero of the Bessel function, electron temperature $T_e \simeq 4$eV, and argon beam energy $E_b = 1.4$eV. The frequency and the corresponding wave number of the unstable wave are obtained by the point of intersection between the beam mode and the plasma mode (Cerenkov interaction) and are as follows: $\omega = 6.8 \times 10^6$rad./sec, $k_z = 27.2\mathrm{cm}^{-1}$. Using Eq. (20), we have plotted in Fig. 2 the normalized growth rate of the ion acoustic wave instability as a function of the normalized beam density for the same parameters as in Fig. 1 and for beam density $n^o_{ob} \simeq 10^8 - 9 \times 10^8\mathrm{cm}^{-3}$, unstable wave frequency $\omega = 6.8 \times 10^6$rad./sec and axial wave number $k_z = 27.2\mathrm{cm}^{-1}$. From Fig. 2, it can be seen that the growth rate increases with the beam density and scales as the one-third power of the beam density. When we evaluate Eq. (20) numerically for the same parameters as in Fig. 1 and for beam density $n^o_{ob} \simeq 1 \times 10^8\mathrm{cm}^{-3}$, the growth rate γ of the instability turns out to be $2.0 \times 10^6\mathrm{sec}^{-1}$. The real frequency of the unstable wave increases with the beam energy [cf. Eq. (21)] and scales as one-half power of the beam energy. The growth rate

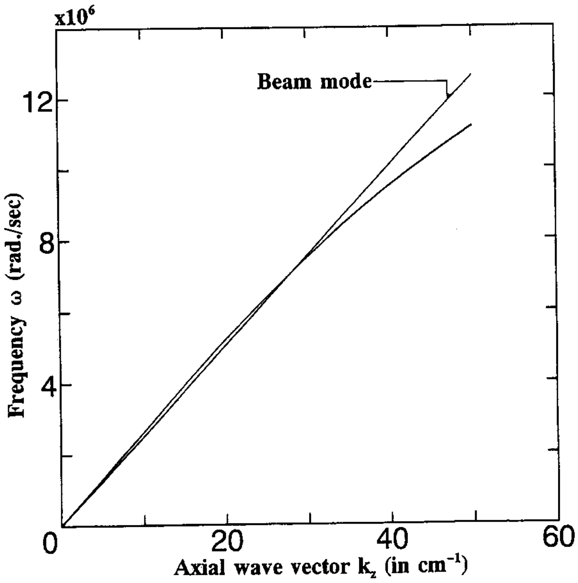

Fig. 1 Dispersion curve of ion acoustic wave and beam mode for plasma density $n^o_{op} \simeq 1 \times 10^{10}\mathrm{cm}^{-3}$, guide magnetic field $B_s = 3 \times 10^3$Gauss, plasma radius a = 2cm, mode number $n = 1$, electron temperature $T_e \simeq 4$eV and argon beam energy $E_b = 1.4$eV.

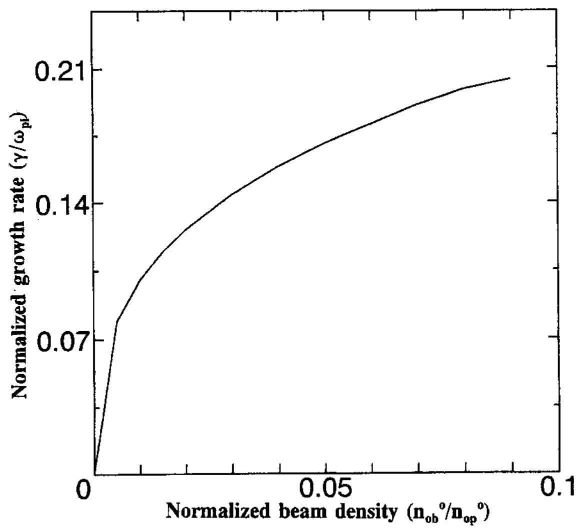

Fig. 2 Normalized growth rate of the ion acoustic wave instability as a function of the normalized beam density for the same parameters as Fig.1 and for beam density $n^o_{ob} = 10^8 - 9 \times 10^8\mathrm{cm}^{-3}$, unstable wave frequency $\omega = 6.8 \times 10^6$rad./sec and axial wave number $k_z = 27.2\mathrm{cm}^{-1}$.

of the instability decreases with the plasma density (in this case). By increasing mode number n, $k_2(= p_n^2 + k_z^2)$ increases in the denominator for the growth rate expression Eq. (20) which implies that the growth rate decreases slightly.

In conclusion, we may say that electrostatic ion acoustic waves are driven to instability by an ion beam in a plasma cylinder via Cerenkov interaction. The growth rate of the instability increases with the beam density and scales as the one-third power of the beam density. The real frequency of the unstable wave increases as almost the one-half power of the beam energy. However, the maximum beam plasma interaction occurs at the lower plasma density. The growth rate of the instability decreases slightly with the mode number.

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