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Kinetic Simulation on Ion Acoustic Wave in Gas Discharge Plasma with Convective Scheme

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Abstract

Collective motion of ions in one dimensional system described by the velocity-coordinate phase space is simulated by the convective scheme, combining the kinetic equation with the Poisson equation. The system has an asymmetric concave electric potential, *i.e.* anode glow mode, in a stationary state, and in which ion current leaks through cathode. Also, spatially uniform ion source is added to maintain the potential. Equidistant spectral lines with respect to harmonics of fundamental mode of ion acoustic wave are observed and they have two local peaks at low and high frequency components. If dissipation is introduced, the lowest frequency mode becomes dominant. From theoretical analysis examined by the dispersion relation derived from a characteristic equation, the instability is found to be caused by asymmetry of the system such as flow velocity and sheath width. The linear theory is satisfactorily consistent with the simulation results.

Keywords:

ion acoustic wave, convective scheme, sheath, asymmetric boundary condition, nonlinear oscillation, gas discharge plasma

1. Introduction

Our motivation for present research is to discuss the physics of plasma and sheath system consistently. It has been reported that the system of DC discharged Ar plasma with thermionic cathode exhibits forced or self-excited chaotic oscillation [1-3]. We showed the result of analysis of chaotic oscillation on forced-oscillation system by assuming charge distributions for each species and solving the Poisson equation with linear approximation [1]. To improve the approximation for describing electric current composed of many ions motion, the self-consistent method is necessary. The convective scheme has an advantage that statistical errors arising from the limited particle number are small, because the distribution function can be treated as continuous function [4]. In the present paper, we review the simulation results of ion acoustic wave in the plasma-sheath system. The growth of the wave can be interpreted as the linear phenomena.The necessary condition to simulate the motion of the ions is that the time scale of spatial diffusion due to the scheme is much grater than that of the wave oscillation ($(D_x k^2)^{-1} >> 2\omega$) and the time scale of heating due to the scheme is much grater than that of the wave ($T_i / \Delta \varepsilon >> 2\omega$). The setting in our simulation is chosen to satisfy the above condition. The influence of the diffusion due to the scheme in phase space is not serious because the wave length and the time scale of ion acoustic wave are large.

2. Simulation Model and Results

Simulation setting is one dimensional oscillatory

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system described by the velocity-coordinate phase space, and we investigate only ion motion. The force acting on the ion is considered only electric field from the Poisson equation. The kinetic equation of ion distribution function and the Poisson equation are coupled together.

$$\frac{\partial f}{\partial t} + \upsilon \frac{\partial f}{\partial x} + \frac{e}{M} E \frac{\partial f}{\partial \upsilon} = C(f), \qquad (1)$$

$$\nabla \cdot E = \frac{e}{\varepsilon_0}(n - n_e). \qquad (2)$$

Then f denotes ion distribution function, $C(f)$ is the time variation due to collision, n and n_e denote ion and electron density, respectively. We arrange numerical mesh in two dimensional phase spaces and move the particles associated with individual cells to new locations determined by the equation of motion under the electric field. Particles are distributed into new four cells basing on the portion of phase space overlap. The equation of motion associated with individual cell is solved by using the Runge-Kutta method according to the equation of motion of an ion, $M\ddot{x} = eE$, if collision does not exist.

We make assumptions and conditions of system as follows. 1. Electron distribution is given by the Boltzmann distribution. 2. Boundary condition is that two electrodes are set as constant potentials, i.e., anode is zero and cathode is kept constant ϕ_0. 3. After solving the Poisson equation by iteration method, calculating norm of approximate solution and verifying convergence of solution on the electric potential, we proceed to the next time step. 4. The system is open for ions and most of ions escape from cathode. Then, ions must be supplied by a uniform source in space as a bias to sustain discharge and put as minimum cells of numerical meshes in velocity space. 5. The electric field between mesh points is calculated by using linear interpolation method in each time step. 6. We use the Debye length as a unit of length, and set the system size L as 100 and divide L into 2000 cells. The velocity space is divided into 200 cells in the unit of ion acoustic speed. The maximum velocity $| \pm \upsilon_{max} |$ are limited to 15 and the particles faster than the maximum speed are deleted.

Two solutions for electric potential expressed by a concave and a convex curve, respectively, are obtained. We take notice of the concave electric potential of Fig. 1, which is confirmed by our previous theory and experiments [1]. Here, we summarize simulation results. 1. Equidistant line spectra are observed and they peak at a low frequency component and at a specific mode, *i.e.* a high frequency component, as shown in Fig. 2. 2. Standing waves exist in the flat part of the electric potential if scaling up the electric potential curve. 3. The high-frequency component is dominant with non-dissipative system after the simulation time develops. 4. Selecting the maximum value of power spectra on high frequency component, we obtain the dispersion relation such as linear line with respect to wave number. These modes are found to be the ion acoustic wave with mode number 1, 2 and so on. 5. If we examine the relation between remained ion density in the system and mode number, it is found that the product of wave number and the Debye length, *i.e.*, characteristic length, is kept constant. 6. The spectrum of low-frequency component becomes clearer than the high frequency component, if a

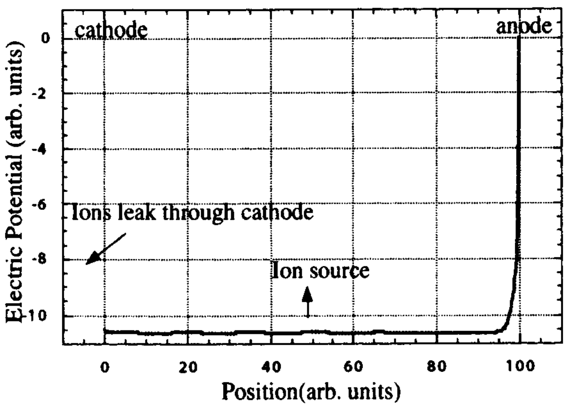

Fig. 1 A typical electric potential curve. Ions leak though cathode and the system is supplied by uniform source S.

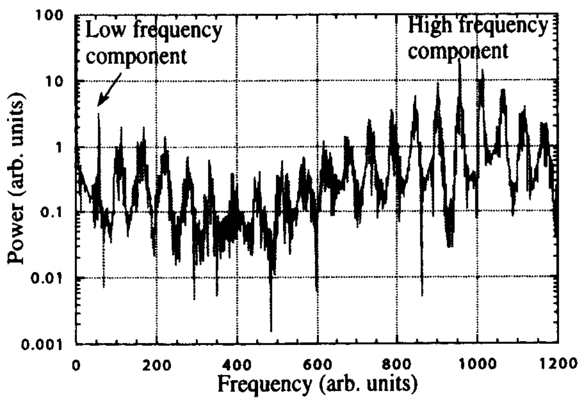

Fig. 2 Equidistant line spectra with respect to higher harmonics of fundamental mode. Two local peaks at low and high components are observed.

dissipative term proportional to velocity ($-\nu\dot{x}$) is added to the equation of motion, which is a model of ion-neutral collision, as is shown in Fig. 3.

3. Theoretical Analysis and Discussion

By linear fluid analysis, we introduce the characteristic equation, determine the dispersion relation and examine growth rate. We can specify the collision source by the simplified Fokker-Planck collision term as $C(f) = -\frac{\partial}{\partial_v}(-\nu v f) + S\delta(v)$. Then S denotes the intensity of uniform source in space and ν denotes the friction coefficient. Fluid equations are obtained as follows.

$$\frac{\partial n}{\partial t} + \frac{\partial n v}{\partial x} = S, \qquad (3)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial}{\partial x}v = \frac{e}{M}\left(-\frac{\partial \phi}{\partial x}\right) - \frac{1}{nM}\frac{\partial P}{\partial x} - \nu v - S\,\frac{v}{n}, \qquad (4)$$

where P denotes the ion pressure. We neglect the second term and the last term of eq. (4). Assuming electron distribution as the Boltzmann distribution, we can write the Poisson equation as

$$-\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\varepsilon_0}\left[n - n_{e0}\exp\left(\frac{e\phi}{T_e}\right)\right]. \qquad (5)$$

From eq. (3), the equation of flux as $\Gamma = \Gamma_L + S(x-L) - \frac{\partial}{\partial_t}\int_L^x n\mathrm{d}x$ is obtained. Here, Γ_L denotes the flux from anode. Because we validate the existence of stationary solution under charge neutrality, we express the stationary solution with ^{0} and the first order perturbation with ^{1}, *i.e.*, $n = n^0 + n^1$ and $\Gamma = \Gamma^0 + \Gamma^1$. Combining eq. (3) and eq. (4) and using condition of charge neutrality, we obtain next equation.

$$\frac{\partial^2 n^1}{\partial t^2} + \nu\frac{\partial n^1}{\partial t} = C_s^2\frac{\partial n^1}{\partial x^2} + \frac{\partial^2}{\partial x^2}\left[2\frac{\Gamma^0}{n_0}\Gamma^1 - \left(\frac{\Gamma^0}{n_0}\right)^2 n^1\right], \qquad (6)$$

where C_s is the ion acoustic speed $\sqrt{\frac{T_e}{M}}$ and the relation $\frac{\partial \Gamma^1}{\partial x} = -\frac{\partial n^1}{\partial t}$ is used. We assume the distribution of ion flow velocity and solve eq. (6) under the condition of continuation of the solution between sheath regions and bulk plasma. In sheaths the flow is approximated by a linear function of x and in the bulk plasma the flow is settled zero. Because the number of ion entering from the anode sheath to bulk plasma is much smaller than the number of ion in bulk plasma, the average velocity of ion in the bulk plasma, *i.e.* the flow velocity, shows small value compared to one particle velocity and can be negligible. Actually, we confirm that the flow distribution in simulation is similar to our model. To separate the variables in eq. (6), we introduce a function n^1 as $\tilde{n}$ ($n^1 = \tilde{n}(x)\exp(-\mathrm{i}\omega t)$) and set $v^0 = \Gamma^0/n^0$. First, let's consider the non-dissipative case of $\nu = 0$. At the cathode sheath or the anode sheath, we assume the ion flow velocity as linear function and expand the first order density as the Taylor series.

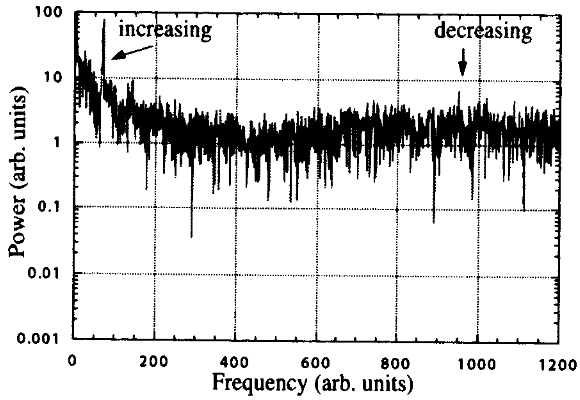

Fig. 3 Power spectra when the dissipation is introduced $\nu = 0.01$. Other parameters are the same as Fig. 2. The low frequency component becomes dominant, compared to Fig. 2.

$$v^0(x) = u_0 - \frac{u_0}{d_0}x, \quad \tilde{n} = \sum a_n x^n, \qquad (7)$$

where $\tilde{n}(0) = 0$ is postulated. At the anode sheath,

$$v^0(x) = \frac{u_L}{d_L}(x-L) + u_L, \quad \tilde{n} = \sum b_n (x-L)^n, \qquad (8)$$

where $\tilde{n}(L) = 0$ is postulated. In the bulk plasma region, we set the ion flow velocity as zero and set the density change as a wave form sinusoidally,

$$v^0(x) = 0, \quad \tilde{n} = Ae^{-\mathrm{i}k_r x}e^{kx} + Be^{\mathrm{i}k_r x}e^{-kx}. \qquad (9)$$

Considering that the mode is ion acoustic wave, we substitute these conditions of eqs. (7), (8) and (9) into eq. (6) and use next relations, $\omega = \Omega + \mathrm{i}\gamma$, $k = k_r + \mathrm{i}\kappa$, $\Omega = C_s k_r$, $\gamma = C_s\kappa$, where γ denotes the growth rate. We expand the elements setting $\kappa^2 \ll 1$ and examine the dependence of growth rate γ on wave number k_r. It is found that the product of mode number and sum of sheath width is constant as shown in Fig. 4. It agrees simulation results. Next, let's examine the growth rate in the dissipative case of $\nu \neq 0$. Then Ω and κ are replaced as follows, $\Omega = C_s k_r\left(1 - \frac{\nu^2}{8C_s^2 k_r^2}\right)$, $\kappa = \frac{1}{C_s}\left[\gamma\left(1 - \frac{\nu^2}{8C_s^2 k_r^2}\right) + \frac{\nu}{2}\right]$. We use these relations and calculate the dependence of γ on k_r in the same way. As a result, the condition of parameters, which the fundamental mode increases, is found. The lower curve of Fig. 4 shows that growth rate of the fundamental mode is positive. The increasing of intensity of low frequency component shown in Fig. 3 can be explained.

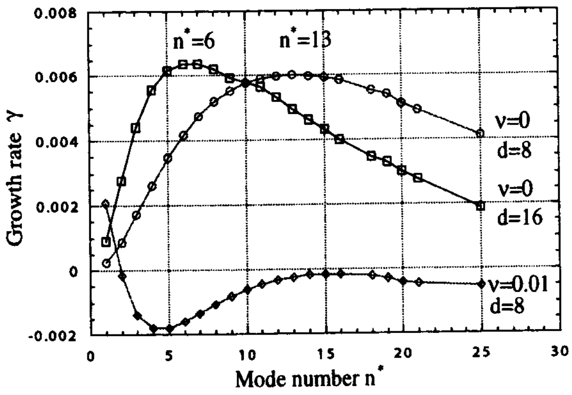

Fig. 4 Theoretical growth rate from fluid approximation. Upper two curves show the comparison when sheath width is settled as half width. It is found that peak mode number becomes two times. Therefore, the product of mode number n^* and sum of sheath width d is kept constant (n^*d = const.) with non-dissipation. The growing mode appears when the system is asymmetrical, does not appear when symmetrical. Lower curve shows that only growth rate of fundamental mode is positive and other modes are negative with dissipation.

4. Conclusions

Using the convective scheme and assuming the Boltzmann distribution for electrons, we calculate self-consistent motions of ions in the concave electric potential. From numerical and theoretical analysis, it is found that asymmetry of the boundary, whose factors are ion flow velocity and sheath width at cathode and anode, destabilizes the ion acoustic wave in the bulk plasma. The condition that the growth rate of the low frequency component can be positive in some region of parameters is obtained. Our result suggests that the oscillation of electric current observed in undriven experimental system can be interpreted by the fundamental mode of destabilized ion acoustic wave. The discussion on chaotic behaviors and period-doubling bifurcation of driven system will be given elsewhere.

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References

[1] Y. Matsunaga and T. Kato: J. Phys. Soc. Jpn. **63**, 4396 (1994).

[2] P.Y. Cheung and A.Y. Wong: Phys. Rev. Lett. **59**, 511 (1987).

[3] P.Y. Cheung, S. Donovan and A.Y. Wong: Phys. Rev. Lett. **61**, 1360 (1988).

[4] W.N.G. Hitchon, D.J. Koch and J.B. Adams: J. Compt. Phys. **83**, 79 (1989).