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Model of Silent Discharge Current and Its Chaotic Oscillation

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Abstract

A model of silent discharge has been proposed. This model can produce a current waveform similar to the experimental one. The current waveforms in the experiments showed sometimes chaotic fluctuations. The Lyapunov exponents, the Lyapunov dimension and correlation dimension of the experimental data have been analyzed. Some of them show a positive Lyapunov exponent and non-integer Lyapunov dimension and non-integer correlation dimension. Therefore they are deterministic chaos oscillations.

Keywords:

silent discharge, chaotic oscillation, the Lyapunov exponent, the Lyapunov dimension, the correlation dimension

1. Introduction

Recently, silent discharges have been used for gaseous waste disposal (e.g. NO_X, SO_X). To enhance the disposal efficiency, it is important to understand the phenomena happened in silent discharge.

First, we propose the simple model of silent discharges to describe the V-I characteristic .

Second, the experimental signals of the discharge current are analyzed in terms of the Lyapunov exponents, the Lyapunov dimension and the correlation dimension.

To our knowledge, the chaos analysis of the oscillations on silent discharge currents has not been reported.

Some of the discharge currents are shown to have a positive Lyapunov exponent, a non-integer Lyapunov dimension and a non-integer correlation dimension.

2. Experimental Setup

A discharge apparatus consists of a quartz glass tube (length 250mm, inner diameter 22mm, thickness 1 mm), an inner electrode (a tungsten wire, 0.15mm ϕ), an outer electrode (a copper sheet which cover the quartz glass tube) and an AC high voltage source (50Hz). A gas was air and pressure was 1atm. The AC voltage was changed from 4kV to 12kV. The discharge currents were recorded with a personal computer through a digitizer. The sampling frequency was 20kHz and the number of points was 16384.

The schematic diagram of the discharge apparatus is shown in Fig.1(a), and the cross section of the discharge tube is shown in Fig.1(b).

3. Results

3.1 Model of silent discharge

Since the discharge of ac-PDP (alternative current-Plasma Display Panel) is a kind of silent discharges, we think that the equivalent circuit for ac-PDP might be adapted to the discharge system in this experiment. The V-I characteristic of ac-PDP gaps resembles to that of trigger diodes for AC voltages [1]. Therefore we

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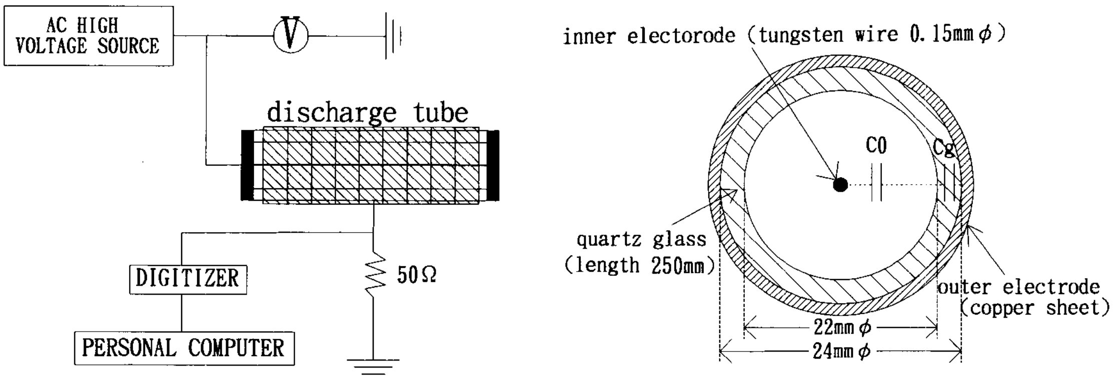
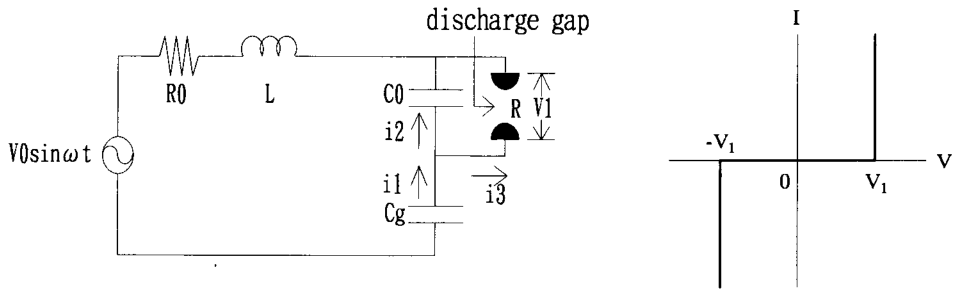


(a) (b)

Fig. 1 Schematic diagram of the experimental set up (a), the cross section of the discharge tube (b).



(a) (b)

Fig. 2 Equivalent circuit of the experimental system (a) , and the V-I characteristic of the discharge gap (b).

propose the equivalent circuit as shown in Fig.2(a). In Fig.2(a), C_0 is the capacitance of the discharge region, C_g is the capacitance of the glass tube and R is the nonlinear resistance of the discharge region. Figure 2(b) shows the V-I characteristic of the discharge gap. The equations for the equivalent circuit are as follows.

$$\frac{\mathrm{d}i_1}{\mathrm{d}t} = \frac{1}{L}\left\{ V_0 \sin \omega t - \frac{1}{C_g}\int i_1 \,\mathrm{d}t - R i_3 - R_0 i_1 \right\}, \quad (1)$$

$$\frac{\mathrm{d}i_3}{\mathrm{d}t} = \frac{1}{C_0 R}\left(i_1 - i_3\right), \quad (2)$$

$$i_2 = i_1 - i_3 . \quad (3)$$

When we choose proper parameters of unknown elements, this model can produce a current waveform similar to experimental one. (see Fig.3)

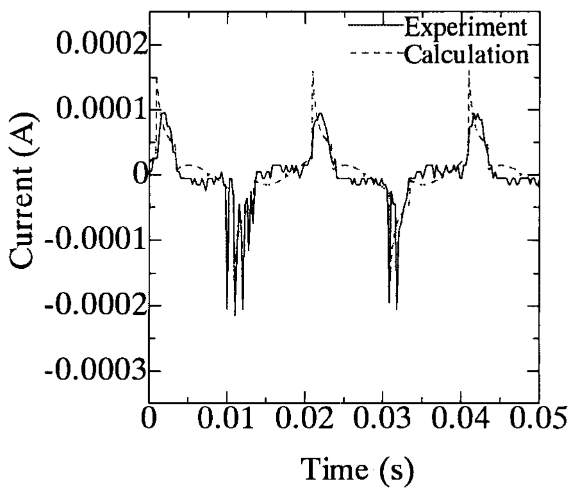


Fig. 3 Comparison between the current obtained experimentally and that obtained from the equivalent circuit (V = 6.0kV).

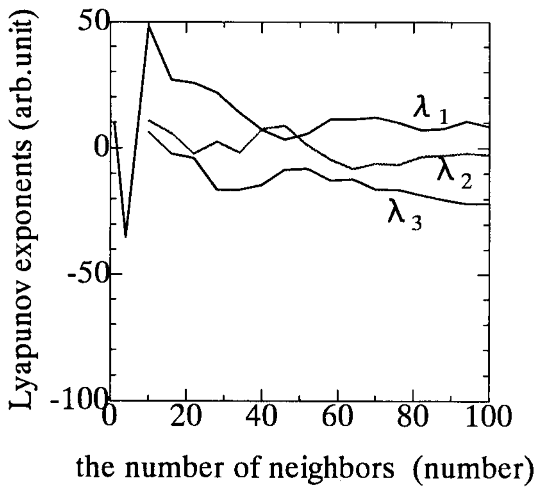


Fig. 4 Lyapunov exponents of the oscillation on a discharge current ($V = 6.0$kV).

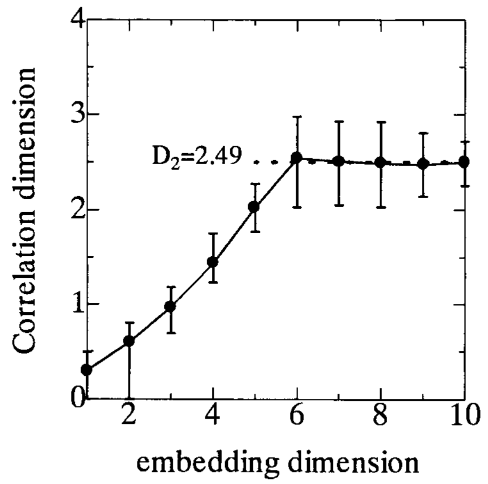


Fig. 6 Correlation dimension of the oscillation on a discharge current ($V = 6.0$kV).

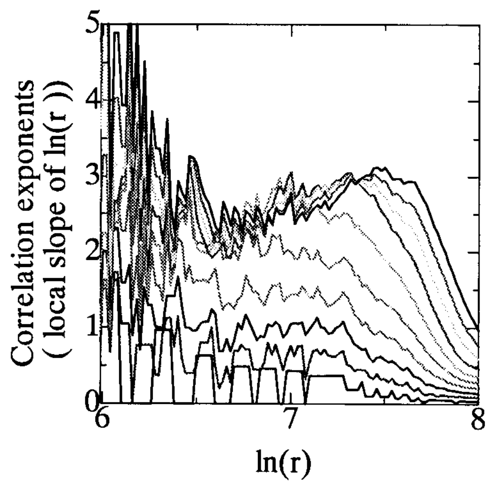


Fig. 5 Local slope of lnC(r) ($V = 6.0$kV).

3.2 Chaos analysis on experimental data

The Lyapunov exponents were estimated for the time series data of the silent discharge currents ($V = 6.0$kV) according to the method proposed by W.Huang et al. [2].

The typical result is shown in Fig.4. One of the Lyapunov exponents is clearly positive. The Lyapunov dimension is calculated as follows

$$D_{KY} = j + \left(\sum_{i=1}^{j} \lambda_i \right) / \left| \lambda_{i+1} \right| \quad \left(\sum_{i=1}^{j} \lambda_i \geq 0, \sum_{i=1}^{j+1} \lambda_i < 0 \right) \tag{4}$$

where λ_i is the Lyapunov exponents. The Lyapunov dimension of the signal D_{KY} is 2.38 and non-integer. The correlation dimension D_2 is calculated from the method proposed by P.Grassberger and I.Procaccia [3].

The local slope of $\ln C(r)$ ($C(r)$:the correlation integral) is plotted as a function of ln(r) in Fig.5. For the scaling of $\ln(r) = 6.6 \sim 7.2$ in Fig.5, the correlation dimension is approaching to 2.49 as the embedding dimension is increasing (see Fig.6). Therefore the correlation dimension of the oscillation on the discharge current is $D_2 = 2.49$.

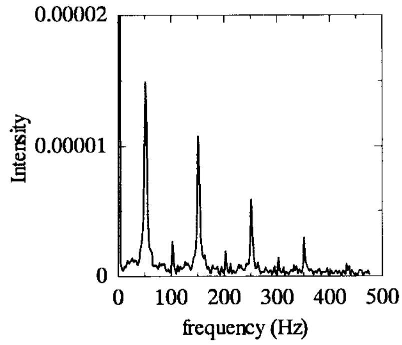
Since the set of the sign of the Lyapunov exponents is (+,0,–) and the Lyapunov dimension is non-integer ($D_{KY} = 2.38$) and the correlation dimension is also non-integer ($D_2 = 2.49$), the oscillation on the discharge current at $V = 6.0$kV is a deterministic chaos .

Figure7 shows the frequency component of the oscillation of the silent discharge current at (a) $V = 6.0$kV (b) $V = 9.4$kV. The frequencies of the oscillations of the discharge currents are normally composed of $50 + 100 \times n$Hz ($n = 0,1,2\cdots$). However, at low voltage they are $50 \times m$Hz ($m = 1,2,3\cdots$). The above mentioned equivalent circuit can also produce the higher harmonics, however, not chaotic oscillations so far.

4. Conclusion

We have shown that some fluctuations on the current of silent discharges are deterministic chaos in terms of the Lyapunov exponents, the Lyapunov dimension and the correlation dimension.

We have also proposed a model of silent discharges. This silent discharge model can produce a



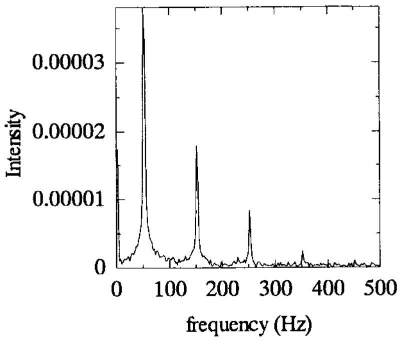


Fig. 7 Fourier spectra of a silent discharge current, (a) $V = 6.0$kV (b) $V = 9.4$kV.

current waveform similar to the experimental one. But the model can not produce chaotic oscillations so far. The improvement to show such oscillations is under way.

The chaos analysis programs that were used in this study, were made by Mr. Terada.

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