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Hysteresis Effect in the Resonant Response of the Nonneutral Plasma Wave

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Abstract

Nonlinear resonant responses were studied by sweeping frequency of external oscillations for the fundamental axisymmetric plasma wave in a nonneutral plasma. Hysteresis phenomena were observed with increasing in the external amplitudes. The results were well explained by the forced nonlinear oscillation model.

Keywords:
nonneutral plasma, nonlinear, hysteresis, bifurcation

1. Introduction

Nonneutral plasma has excellent confinement properties, and this allows a detailed examination of the hysteresis effect. We present the experimental results on the hysteresis effect in nonneutral electron plasma waves. We swept slowly up and down the external driving frequency through the resonance of the modes. Sweeping the frequency upward enhanced the maximum response amplitude compared with that in the downward sweep. The jump like behavior of the response amplitude was also observed. Thus, the observed resonant response exhibited the feature of the hysteresis. The observed hysteresis was compared and discussed with the well known Duffing oscillator model. We focused on the nonlinear response for the n = 1 axisymmetric electrostatic wave, which corresponds to the center-of-mass motion of the plasma in the axial direction. The wave is relatively stable even with large amplitudes. In contrast with the fundamental mode, waves with higher mode numbers (n < 1,) easily decay to other waves via three and/or four wave interactions [1]. These effects are significant obstruct of the nonlinear resonant response since they cause drastic changes of the wave amplitudes in time. This is the reason why we focused on the fundamental mode.

2. Experiments

The nonneutral plasmas were produced by the pulsed injection of electron beams with the hot cathode and were confined with the modified Penning type [2], as is shown in Fig.1. The trapping system consisted of multi-ring electrodes and end grids. The ring electrodes were axially aligned with the same space of 1.2cm. Each ring was of 3cm-*i.d.* and 1.2cm-width. They served the axial trapping of electrons by appropriate electrostatic potential well. The well depth was 23V. The axially uniform magnetic field confined the electrons in the radial direction. The field intensity was $B = 0.07$T. The electron plasmas were typically of the total number, $N = 1.4 \times 10^8$, and the particle confinement time, $\tau_p \simeq$ 10sec. The cylindrical column of the electrons was of 1.6cm-diameter and 24cm-length. The resultant electron density was $n \simeq 6.0 \times 10^{12}\text{m}^{-3}$. The shot-to-shot reproducibility of the total electron number was within error of less than a few of percents.

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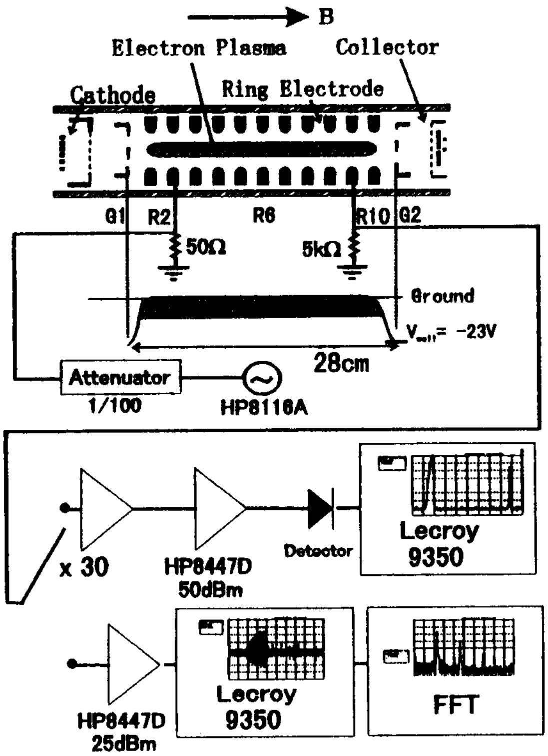

Fig. 1 A schematic of the experimental apparatus.

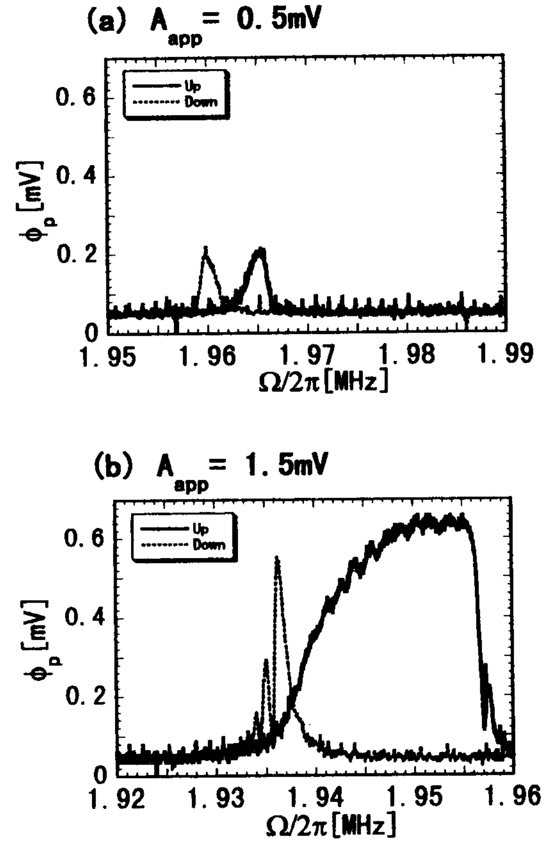

Fig. 2 The hysteresis curves of the n = 1 axisymmetric electrostatic wave. (a) A_{app} = 0.5mV, (b) A_{app} = 1.5mV. The solid and dotted curves represent the amplitude observed in the upward and the downward sweeps respectively.

We observed the hysteresis phenomena for the excited waves by linearly sweeping frequency of the externally applied sinusoidal oscillations through the linear resonance one. As is well known, the frequency sweep speed brings about of large modifications for the nonlinear resonant response, such as lowering of the maximum peak. Nonlinear effect reveals its feature in the case of sufficiently slow frequency sweep. Even small error of the plasma parameters by shot-to-shot made was unable to observe the hysteresis effect. Success for the measurement, therefore, required slow sweep enough to observe the nonlinear resonant response within a single shot. The typical sweep period was as long as 50ms. It was two orders as small as the varying period of the plasma parameters. In the experiment we exited the axisymmetric electrostatic wave applying the external oscillations to the electrode R2. The excited waves were detected with R10 via the amplifier. The power spectra at any periods were obtained by the usual fast-Fourier transformation (FFT). The external oscillation applied was of the form of A_{app} sin (Ωt). Here A_{app} was the amplitude; the frequency Ω was linearly swept through the linear resonance one ω_0, namely $\Omega = \omega_0 + \beta t$. The external amplitude was of 0.1 ~ 100mV. The range of the sweep speed β was typically 3 ~ 1,000MHz/sec. The resonant response strongly depended on the amplitude A_{app} and the sweep speed β. Especially the amplitude played a major role of the nonlinear resonant response; the sweep speed brought about only a modification in the response for any amplitudes.

3. Results and Discussion

The observed $n = 1$ wave amplitude ϕ_p is shown in Fig.2 for several values of external amplitude A_{app}. In the case the sweep speed was β = 3MHz/s. The frequency sweep rate $d\Omega/dt/\Omega^2$ was less than $1s^{-6}$. The condition might assure adiabaticity.

For sufficiently small A_{app}, the amplitude ϕ_p exhibited symmetric response around the linear resonant frequency as shown in Fig.2(a). Both the frequencies corresponding to the maxima shifted from the linear resonant one to the higher and lower ones depending on the direction of the frequency sweep. They decreased with β. The response also accompanied the maxima with severals of small peaks. This resulted from modulation among the frequencies due to the finite sweep speed.

For relatively large A_{app}, in contrast to the above case, the amplitude ϕ_p increased monotonically with the frequency in the upward sweep. Then it suddenly decreased as shown in Fig.2(b); in the downward sweep, it never gave rise to the similar response. The maximum amplitude and the width of the maximum peaks in the upward sweeps are larger than those in the downward sweeps. Thus the observed resonant response exhibited the feature of the hysteresis in the transit nonlinear response.

Another response experiment without the frequency sweep showed that the amplitude caused the displacement of the resonant frequency (Fig.3). The frequency shift was approximately in proportion to the square of the excited wave amplitude ϕ_p; $\Delta\omega/\omega_0 \approx 0.12 \times \phi_p^2$. The Q-value for the wave, on the other hand, was of the order of 10^3 with the damping coefficient of $\lambda \sim 3 \times 10^3 s^{-1}$. The hysteresis phenomenon was expected to be caused when $\Delta\omega/\omega_0 > Q^{-1}$ and the frequency sweep speed through the resonance $\omega_0/\beta > \lambda^{-1}$. The ϕ_p more than 0.1mV assured the former condition. The latter was satisfied for the case of β = 3MHz/s. The finiteness for the speed brought about only continuously rapid change in the response instead of the jump in the stationary resonant response. Taking into account of the fact, the present resonant response ϕ_p with the external field can be described by the following equation;

$$\frac{d^2\phi_p}{dt^2} + 2\lambda\frac{d\phi_p}{dt} + \omega_0^2\,\phi_p\,(1+\gamma\phi_p^2) = A_{ext}\cos(\Omega t). \qquad (1)$$

where λ is the damping rate for the wave amplitude and γ is the increment coefficient for the nonlinear frequency shift.

To compare the Duffing equation and the experimentally obtained Hysteresis Curves, we investigated the dependence of the frequency width $\delta\varpi$, as the representing parameters which characterize the behavior of the hysteresis curves. Here the $\delta\varpi$ is defined as the width between the frequencies at 1/e of the maximum value of ϕ_p in the upward response normalized by ω_0. Figure 4 shows the relation between the frequency width $\delta\varpi$ and the external amplitude. The hysteresis curves (a),(b) in fig.2 are corresponding to the points labeled as (a),(b) in fig.3. At small external amplitude A_{ext}, the frequency width $\delta\varpi$ in the upward

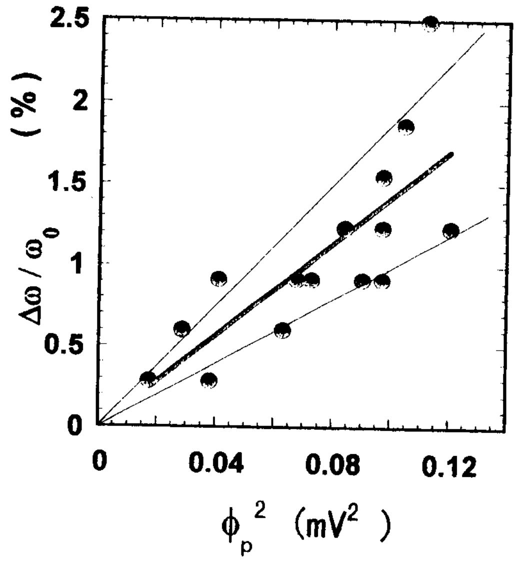

Fig. 3 The nonlinear frequency shift caused by the amplitude of the wave. The lines represent the best fitting ones with the form of $\Delta\omega/\omega_0$ = $(14.2 \pm 4.5) \times 10^{-2} \times \phi_p^2$.

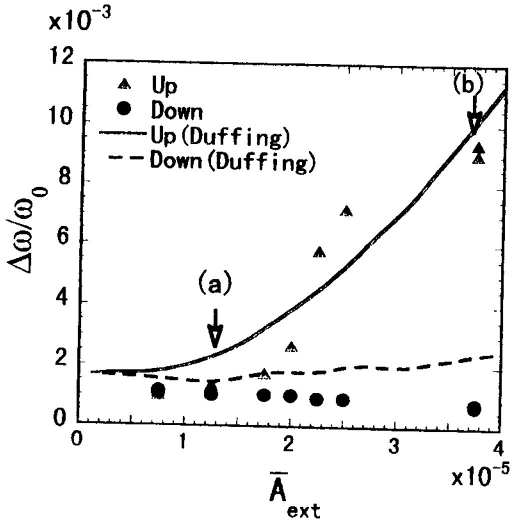

Fig. 4 The relation between the frequency width $\delta\varpi$ and the external amplitude. The external amplitude was normalized as $\bar{A}_{ext} = A_{ext}\sqrt{\gamma}/\omega_0^2$. The triangle and the circle represent the experimentally obtained frequency width in the upward and the downward sweeps respectively. The curves indicate the frequency width calculated from the eq.1. The solid and dotted curves represent that in the upward and the downward sweeps respectively.

and the downward sweeps had same value. The frequency width $\delta\varpi$ increased with the raise of the A_{ext} in the upward sweep. On the other hand, in the downward sweep, the frequency width $\delta\varpi$ remained constant for all range of A_{ext}. The curves indicate the frequency width $\delta\varpi$ calculated from eq.1 using the asymptotic methode [3]. In the calculation, the coefficients in eq.1 were experimentally decided. The curves qualitatively agree with the experimental results.

The mechanism which causes the hysteresis effect in nonneutral plasma was investigated by Lamb and Morales [4]. They predicted that the ponderomotive effect of the axisymmetric standing wave can cause the Duffing type hysteresis effect. Their analysis showed that the hysteresis was soft spring type ($\gamma < 0$), in which the frequency width $\delta\varpi$ in the downward sweep was larger than that in upward sweeps. Our experimental result of the $n = 1$ axisymmetric wave was just the opposite ($\gamma > 0$). The difference leads us to the further investigation.

4. Conclusion

In summary, we clearly observed the hysteresis effect of $n = 1$ axisymmetric electrostatic modes in a nonneutral electron plasma column. We studied the dependence of the frequency width $\delta\varpi$ to the external driving amplitude A_{ext}. With the raise of external driving amplitude A_{ext}, $\delta\varpi$ in the upward sweep increases but that in the downward sweep remain constant. The results were well explained by the Duffing equation model.

Acknowledgement

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