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Langmuir Wave Excitation by Beating Gaussian Laser Beams in a Parabolic Plasma Channel

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Abstract

Two dimensional effect on the beat wave excitation of a plasma wave in a laser beat wave accelerator are examined by beating two gaussian laser beams of frequencies ω_1 and ω_2. Two gaussian laser beams of frequencies ω_1 and ω_2, propagating through a preformed plasma channel, exert a longitudinal ponderomotive force on the electrons at $\omega_1 - \omega_2$. The latter drives a large amplitude plasma wave when $\omega_1 - \omega_2$ is within a few percent of ω_{p0}, the plasma frequency on the axis of the plasma channel. In the cold plasma approximation an analytical solution for φ, the plasma wave potential is possible in the paraxial region when frequency mismatch $(\omega_1 - \omega_2) - \omega_{p0} \geq a/c$ where a is the plasma channel radius. In this case φ has a minimum on the axis. In a hot plasma the fundamental Langmuir eigen mode has a maximum on the axis and is strongly localized in the axial region. Its amplitude saturates due to the plasma resonance detuning caused by the relativistic mass effects.

Keywords:

laser accelerator, beat wave accelerator, plasma accelerator, accelerator, laser plasma interaction

1. Introduction

Laser beat wave accelerator has attracted considerable interest over the last two decades. It involves the excitation of a large amplitude plasma wave by the beat frequency ponderomative force due to the two lasers [1-8]. UCLA group has reported accelerating field gradients ~ 2.8GV/m using CO_2 lasers at 10.6μm and μ wavelengths with power density 10^{14}W/cm^2 and pulse duration 300ps, in a plasma density of 10^{16}cm^{-3}. They have achieved acceleration of 28MeV electrons to 2.8GeV [2,3]. Modena *et al.* [4], have observed electron acceleration upto 44MeV by the high amplitude plasma wave produced by Raman forward scattering of Nd: glass laser of intensity > 10^{18}W/cm^2, pulse length ≤ 1ps in an underdense plasma, n_e ~ 10^{19}cm^{-3}. Amiranoff *et al.* [5], have reviewed the beat-wave experiments at Ecole Polytechnique using Nd: glass lasers at $\lambda_1 = 1.0530\mu m$ and $\lambda_2 = 1.0642\mu m$, pulse duration ~100ps and laser energies ≤ 10J they obtained an energy gain of 1.3MeV, compatible with an accelerating gradients of 0.7GeV/m. Ghizzo *et al.* [6], have developed a Hilbert-Vlasov code for the study of the plasma beat wave accelerator with high radius of driver frequency to plasma frequency. It is seen that, with effects like beat frequency chirping (i.e. pump frequency linearly decreasing with time), amplitude limit to relativistic detuning can be enhanced, giving accelerating gradients ≥ 25GeV/m. Decker *et al.* [7], have investigated electron acceleration from space charge waves using particle-in-cell model. They observed large number of self-trapped electrons and multiple Raman forward scattering satellites. Esarey et al. [8], have given an elegant review of plasma based

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accelerator concepts.

The success of laser based accelerator scheme demands guiding of the pump lasers to long distances. The guiding could be achieved in a preformed plasma channel with a density minimum on axis or due to self focusing via nonlinear refraction. In either case the plasma frequency is nonuniform across the cross section of the lasers, hence the generated plasma wave would have nonuniform amplitude profile.

In this paper we study the excitation of a plasma wave by beating two lasers in a plasma channel with parabolic density profile. In section II we study the problem in cold plasma approximation. The thermal effects are considered in section III. The electron acceleration energy and length are given in section VI. A brief discussion of the results is given in section V.

2. Beat Wave Excitation

Consider a nonuniform plasma channel equilibrium density profile,

$$n(x)=n_0\,(1+\frac{x^2}{a^2}),\quad \omega_p^2(x)=\omega_{p0}^2(1+\frac{x^2}{a^2}). \tag{1}$$

Let two collinear gaussian laser beams of equal amplitude propagate through the plasma channel. The electric fields associated with these beams can be written as

$$\vec{E}_1=\hat{y}A_0\,e^{-x^2/r_0^2}\,e^{-i(\omega_1 t-k_1 z)},$$

$$\vec{E}_2=\hat{y}A_0\,e^{-x^2/r_0^2}\,e^{-i(\omega_2 t-k_2 z)}, \tag{2}$$

where $\omega\equiv\omega_1-\omega_2<<\omega_1,\omega_2$. They produce oscillatory electron velocities

$$\vec{v}_1=\frac{e\vec{E}_1}{mi\omega_1},\quad \vec{v}_2=\frac{e\vec{E}_2}{mi\omega_2}, \tag{3}$$

and exert a longitudinal ponderomotive force $\vec{F}_p=e\nabla\varphi_p$ on the electrons, with ponderomotive potential

$$\varphi_p=-\frac{m}{2e}\,\vec{v}_1\cdot\vec{v}_2. \tag{4}$$

The ponderomotive force imparts oscillatory velocity $\boldsymbol{v}$ to the electrons. Solving the equation of motion

$$m\frac{\partial v}{\partial t}=e\nabla\varphi_p$$

One obtains $\vec{v}=-\frac{e\nabla\varphi_p}{mi\omega}$. Using in the equation of continuity one obtains

$$n^{NL}=\frac{n_0^0\,e\nabla^2\varphi_p}{m\omega^2}\cong\frac{e\nabla.(n_0^0\,\nabla\varphi_p)}{m\omega^2}. \tag{5}$$

The self-consistent potential φ at ω also produces a linear density perturbation, $n^L=\frac{n_0^0 e\nabla^2\varphi}{m\omega^2}$. Using the linear and nonlinear density perturbations in the Poisson's equation we get

$$\nabla.(\varepsilon\nabla\varphi)=4\pi e n^{NL}\cong -k_z^2\,\varphi_p, \tag{6}$$

where

$$\varepsilon=1-\frac{\omega_p^2(x)}{\omega^2}=\Delta-\frac{x^2}{a^2},\quad \Delta=1-\frac{\omega_{p0}^2}{\omega^2}, \tag{7}$$

Here we have assumed

$$\frac{\partial\phi_p}{\partial x}<<k_z\phi_p.$$

If the collisions are taken into account then Δ is changed to $\Delta'=\Delta+\frac{i\nu}{\omega}$. Using z variation of φ as $e^{ik_z z}$ where $k_z=k_1-k_2$, Eq.(6) could be written as

$$\frac{\partial^2\varphi}{\partial x^2}-\frac{2x}{a^2(\Delta'-x^2/a^2)}\frac{\partial\varphi}{\partial x}-k_z^2\,\varphi=-\frac{1}{(\Delta'-x^2/a^2)}k_z^2\,\varphi_p. \tag{8}$$

Defining $\xi=x/a\sqrt{\Delta}$, Eq.(8) can be written as

$$\frac{\partial^2\varphi}{\partial\xi^2}-\frac{2\xi}{(1+i\nu/\omega\Delta-\xi^2)}\frac{\partial\varphi}{\partial\xi}-k_z^2\,a^2\Delta\varphi=-\frac{a^2}{(1+i\nu/\omega-\xi^2)}k_z^2\,\varphi_p. \tag{9}$$

We may deduce the following inferences here.

In the paraxial region $x<<a\sqrt{\Delta}$, $\frac{\partial^2\varphi}{\partial x^2}<k_z^2\varphi$, the plasma wave potential is $\varphi\cong\frac{\varphi_p}{\varepsilon}$.

This is valid when $\frac{\partial^2\varphi}{\partial x^2}<<k_z^2\varphi$ or $k_z^2a^2>2/\Delta$ and for $x>>a\sqrt{\Delta}$. One may notice that φ changes sign when one goes from the inner region to the outer one. Around $x\cong a\sqrt{\Delta}$, $\varepsilon\cong 0$ and Eq.(8) takes the form

$$\frac{\partial\varphi}{\partial x}=\frac{k_z^2\,a^2}{2x}\,\varphi_p. \tag{10}$$

Integrating Eq.(10) with respect to x we get,

$$\varphi=\frac{k_z^2\,a^2}{2}\int_{x_1}^{x}\frac{\varphi_p}{x}\,dx+\varphi(x), \tag{11}$$

where x_1 is some arbitrary point between the axis and the critical density layer. For $x\sim 0.5a\sqrt{\Delta}$ one may take $\varphi_p\cong(\varphi_p/\varepsilon)_{x_1}\cong 4\varphi_p/3\Delta$ and Eq.(11) can be written as

$$\varphi=\frac{4\varphi_p(x)}{3\Delta}+\frac{k_z^2\,a^2}{2}\int_{0.5a\sqrt{\Delta}}^{1.5a\sqrt{\Delta}}\frac{\varphi_p}{x}\,dx,\quad \text{for } x<1.5a\sqrt{\Delta}. \tag{12}$$

We have solved Eq.(8) numerically and the variation of φ as a function of x is plotted in Fig. 1 for the following parameters: $\nu/\omega\cong 10^{-3}$, $\Delta\cong 0.05$, $k_z a=10$.

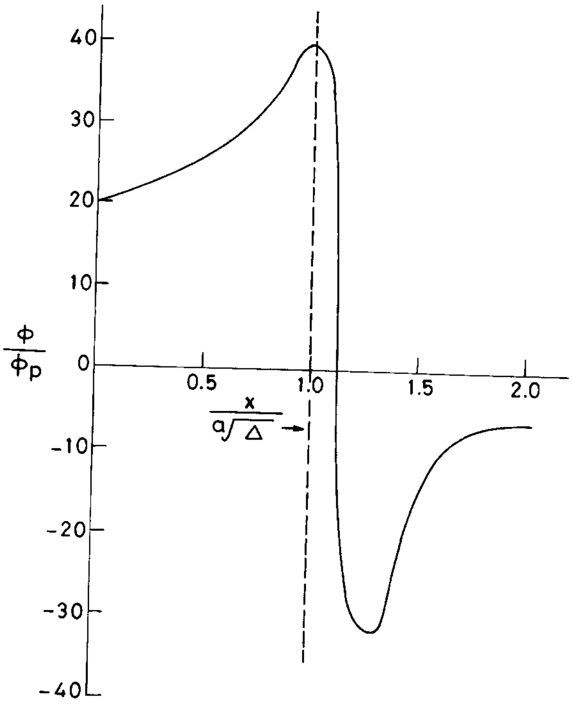

Fig. 1 Variation of φ as a function of x for the following parameters: $v/\omega \cong 10^{-3}$, $\Delta \cong 0.05$, $k_z a = 10$.

3. Thermal Effects

When thermal effects are included the potential of the plasma wave can be expressible in terms of φ_p as

$$\varepsilon\varphi = -\chi_e\varphi_p\,, \tag{13}$$

where $\varepsilon = 1 + \chi_e$, and $\chi_e = -(\omega_p^2 + k_z^2 v_{th}^2)/\omega^2$. Equation (13) can be written as $(-k^2 + \frac{\omega^2-\omega_p^2}{v_{th}^2})\varphi = \varphi_p$. Replacing k_z by $k_z - i\frac{\partial}{\partial x}$ and $k_\perp^2$ by $-\frac{\partial^2}{\partial x^2}$ we obtain,

$$-2ik_z\frac{\partial\varphi}{\partial z} + \frac{\partial^2\varphi}{\partial x^2} + \left(\frac{\omega^2-\omega_{p0}^2-k_z^2 v_{th}^2}{v_{th}^2} - \frac{\omega_{p0}^2 x^2}{v_{th}^2 a^2}\right)\varphi = \frac{\omega^2}{v_{th}^2}\varphi_p\,. \tag{14}$$

Defining $\lambda_{es} = (av_{th}/\omega_{p0})^{1/2}$, $\eta = x/\lambda_{es}$, $\lambda = (\frac{\omega^2-\omega_{p0}^2-k_z^2 v_{th}^2}{v_{th}^2})\frac{av_{th}}{\omega_{p0}}$, Eq.(14) can be written as

$$-2ik_z\lambda_{es}^2\frac{\partial\varphi}{\partial z} + \frac{\partial^2\varphi}{\partial\eta^2} + (\lambda-\eta^2)\varphi = \frac{\omega^2 a v_{th}}{v_{th}^2\omega_{p0}}\varphi_p\,. \tag{15}$$

When the first term on the LHS and RHS are ignored Eq.(15) has Hermite polynomial solution, and the eigen values are $\lambda = (l + 1/2)$. When all the terms in Eq.(15) are finite we write

$$\varphi = A(z)\psi_l \tag{16}$$

Substituting this expression for φ in Eq.(15), multiplying the resulting equation by ψ_l^* and integrating over x we get

$$2ik_z\lambda_{es}^2\frac{\partial A}{\partial z} = \frac{\omega_{p0}^2 a v_{th}}{v_{th}^2\omega_{p0}}\varphi_p I, \quad A = P_z\,. \tag{17}$$

where

$$P = \frac{\omega_{p0}^2}{v_{th}^2}\frac{a v_{th} I}{\omega_{p0} 2ik_z\lambda_{es}^2}, \quad I = \frac{\int\varphi_p\psi^* d\eta}{\int\varphi\varphi^* d\eta}\,. \tag{18}$$

Initially $\lambda \cong 1$. However, as φ acquires large amplitude, the mass of the electrons increases due to relativistic effects.

$$\lambda \cong 1 + \frac{\partial\lambda}{\partial m}\Delta m,$$

$$\Delta m \cong \frac{v^2}{2c^2}m_0 \cong \frac{m_0 e^2 k_z^2 A^2}{2m_0\omega^2 c^2}\frac{\varphi\varphi^*}{}\,. \tag{19}$$

Using Eq.(19), and ignoring $\frac{\partial\varphi}{\partial z}$ term in Eq.(15), we obtain the saturation state

$$A\int_{\infty}^{\infty}\frac{\partial\lambda}{\partial m}\Delta m\varphi\varphi^* d\eta = \frac{\omega_p^2}{v_{th}^2}\frac{av_{th}}{\omega_{p0}}\int_{-\infty}^{\infty}\varphi_p\psi^* d\eta. \tag{20}$$

Which gives

$$A^3 = \frac{m_0^2\omega^2}{k_z^2 e^2}\frac{\omega_{p0}^2}{v_{th}^2}\frac{1}{m}\frac{\varphi_{p0}}{(0.5+\lambda_{es}^2/r_0^2)^{-1/2}}\,. \tag{21}$$

4. Electron Acceleration

The large amplitude plasma wave can accelerate an electron beam to high energy. Following Liu and Tripathi [9]. The acceleration energy can be written as

$$W_A = m_0 c^2\Delta\gamma, \quad = \frac{2ecA}{\omega}\frac{\beta}{(1-\beta)},$$

$$= \left(\frac{m_0^2\omega^2}{k_z^2 e^2}\frac{\omega_{p0}^2}{v_{th}^2}\frac{1}{m}\frac{\varphi_{p0}}{(0.5+\lambda_{es}^2/r_0^2)^{-1/2}}\right)^{1/3}\frac{2ec}{\omega}\frac{\beta}{(1-\beta)}, \tag{22}$$

where $\beta = \frac{\omega}{kc}$. The acceleration length can be written as

$$W_A \cong eAL_A\,. \tag{23}$$

5. Discussion

Two gaussian laser beams propagating through a preformed plasma channel produce a large amplitude plasma wave when beat frequency is closer to, within a few percent, the plasma frequency at the center of the channel. In the cold plasma approximation the potential has minimum on the axis and increases with $x/a\sqrt{\Delta}$. In the hot plasma approximation the plasma eigen mode has a maximum on the axis and is localized strongly in the axial region. The relativistic mass effects causes plasma resonance detuning and saturates the amplitude of the plasma wave.

In the cold plasma approximation the acceleration energy follows the behavior of potential function. In a hot plasma the thermal motion deteriorates the acceleration energy.

References

[1] M.N. Rosenbluth and C.S. Liu, Phys. Rev. Lett. **29**, 701 (1972).

[2] C.E. Clayton, M.J. Everett, A. Lal, D. Gordon, K.A. Marsh and C. Joshi, Phys. Plasmas **1**, 1753 (1994).

[3] M. Everett, A. Lal, D. Gordon, C.E. Clayton, K.A. Marsh and C. Joshi, Nature **368**, 527 (1994).

[4] A. Modena *et al.*, Nature **337**, 606 (1995).

[5] F. Amiranoff *et al.*, in Advanced Accelerator Concepts, AIP Conf. Proc. **335**, P. Schoessow, Ed. New York: Amer. Inst. Phys., p 612 (1995).

[6] Alain Ghizzo *et al.*, IEEE Trans. Plasma Sci. **24**, 370 (1996).

[7] C.D. Decker *et al.*, IEEE Trans. Plasma Sci. **24**, 379 (1996).

[8] E. Esarey, P. Sprangle, J. Krall and T. Ting, IEEE Trans. Plasma Sci. 252 (1996).

[9] C.S. Liu and V.K. Tripathi, *Interaction of Electromagnetic Waves with Electron Beams and Plasmas* (World Scientific 1994).