J. Plasma Fusion Res. SERIES, Vol. 2 (1999) 376–380

Anomalous Relaxation and Stopping Power for Electron Beam in a Beam-Plasma System

HARA Kazumasa, MORI Ichiro*, KAWAKAMI Retsuo and TOMINAGA Kikuo
Faculty of Engineering The University of Tokushima
Tokushima, 770-8506 Japan

(Received: 9 December 1998 / Accepted: 16 June 1999)

Abstract

Anomalous relaxation and stopping power, i.e., deformation of electron beam's distribution function as a reaction of an emission of an elementary excitation (soliton) by the beam itself, in a beam-plasma system are analyzed theoretically. The results are compared with that of our experiments. It is shown that the beam electron emits the soliton by coherent interaction, that is to say that modulated or bunched wave numbe of the beam, $\boldsymbol{k}$, is equal to the wave number of surrounded plasma oscillation (emitted elementary excitation or the soliton), $\boldsymbol{k}_1$, and it is also shown that the beam slows down and its distribution function is deformed completely when the beam collides to the steep negative front of the ion wave by the '*Bremsstrahlung*' in the deceleration field.

Keywords:

distribution function, anomalous relaxation, stopping power, renormalization, soliton

1. Introduction

A relaxation of the beam electron can be described by the Fokker-Plank equation (FPE) for the beam distribution f_α:

$$\frac{\partial f_\alpha}{\partial t} = -\sum_{i=1}^{3} \frac{\partial}{\partial v_i}\{A_i f_\alpha\} + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} \frac{\partial^2}{\partial v_i \partial v_j}\{D_{ij} f_\alpha\}, \tag{1}$$

where the $\boldsymbol{A}_i$, $\boldsymbol{D}_{ij}$ are the friction coefficient (vector) and the diffusion coefficient (tensor) in the velocity space respectively. The first term in right hand side in eq. (1) is linear term since the $\boldsymbol{A}_i$ is related to the spontaneous emission or so called *Cherenkov emission* and does not include the f_α or the intensity of the elemantary excitation, while the second term is a nonlinear term which is related to the induced emission and the $\boldsymbol{D}_{ij}$ is functions of intensity of the elementary excitation and the f_α. We neglect the first term under the assumption that induced phenomenon overtakes the spontaneous one. At the following section, we discuss the quantity $\boldsymbol{D}_{ij}$.

2. The Nonlinear Theory

We start from the Vlasov and Poisson system and obtain the following set of self-consistent equations [1]:

$$D_{ij}(\boldsymbol{k},\boldsymbol{v}) = \left(\frac{e_\alpha}{m_\alpha}\right)^2 \sum_{k_1} \int \frac{d\omega_1}{2\pi} \frac{\boldsymbol{k}_{1i}\boldsymbol{k}_{1j}}{\boldsymbol{k}_1^2} |\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)|^2 G_\alpha(\boldsymbol{k}-\boldsymbol{k}_1,\boldsymbol{v},\boldsymbol{k}\cdot\boldsymbol{v}-\omega_1;\boldsymbol{k}',\boldsymbol{v}',\omega'), \tag{2}$$

$$\boldsymbol{E}(\boldsymbol{k},\omega) = -\sum_\alpha \frac{e_\alpha}{\varepsilon_0}\cdot\frac{i\boldsymbol{k}}{\boldsymbol{k}^2}\int d\boldsymbol{v}\int d\boldsymbol{v}'\int\frac{d\boldsymbol{k}'}{(2\pi)^3} G_\alpha(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega')\cdot f_\alpha^{(0)}(\boldsymbol{k}',\boldsymbol{v}',\omega'), \tag{3}$$

Corresponding author's e-mail: mori@ee.tokushima-u.ac.jp

$$f_\alpha(\boldsymbol{k},\boldsymbol{v},\omega)=\int \mathrm{d}\boldsymbol{v}'\, G_\alpha(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega')\cdot f_\alpha^{(0)}(\boldsymbol{k}',\boldsymbol{v}',\omega'), \tag{4}$$

$$\Sigma_\alpha(\boldsymbol{k},\boldsymbol{v},\omega)=\left(\frac{e_\alpha}{m_\alpha}\right)^2\sum_{k_1}\int\frac{\mathrm{d}\omega_1}{2\pi}\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)\cdot\frac{\partial}{\partial\boldsymbol{v}}\{G_\alpha(\boldsymbol{k}-\boldsymbol{k}_1,\boldsymbol{v},\boldsymbol{k}\cdot\boldsymbol{v}-\omega_1)\cdot\boldsymbol{E}(-\boldsymbol{k}_1,-\omega_1)\cdot\frac{\partial}{\partial\boldsymbol{v}}\}, \tag{5}$$

$$G_\alpha^{(m=0)}(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega')=-\frac{\mathrm{i}}{4\pi D_{ij}}\cdot\frac{1}{|\boldsymbol{v}-\boldsymbol{v}'|}\cdot\exp\left(-\frac{|\boldsymbol{v}-\boldsymbol{v}'|}{\sqrt{D_{ij}}}\cdot\sqrt{\mathrm{i}(-\omega+c)}\right) \tag{6}$$

where the $f_\alpha^{(0)}(\boldsymbol{k}',\boldsymbol{v}',\omega')$, $\Sigma_\alpha(\boldsymbol{k},\boldsymbol{v},\omega)$, $G_\alpha(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega')$ are the Fourier transformation of the initial beam distribution, the collision frequency between electrons and the elementary excitations or self-energy term, and the Green's function respectively. In the original Green's function [1], m = 0-mode is the most important and we use only the m = 0-mode which is given in eq. (6). The terms $m \neq 0$ in the Green's function are proportional to $\boldsymbol{k}^{2m}$, then they vanish when we consider the coherent interaction, $\boldsymbol{k}=\boldsymbol{k}_1$, in eqs. (2), (5) and take the limit $\boldsymbol{k}\to 0$ in the $G_\alpha(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega')$. The meaning of the Green's function appears in the eq. (4) since the eq. (4) is transformation from the differential equation, eq. (1), to integral equation, so that the Green's function is a 'Kernel' or 'transition probability' which connects the initial function state $f_\alpha(t=0)$ to a final state $f_\alpha(t)$.

To solve the above set of equations we assumed firstly that $|\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)|^2=|E|^2$ = const. in the expressions of $D_{ij}(\boldsymbol{k},\boldsymbol{v})$, and $\Sigma_\alpha(\boldsymbol{k},\boldsymbol{v},\omega)$, then the $\boldsymbol{E}(\boldsymbol{k},t)$, $f^\alpha(\boldsymbol{k},\boldsymbol{v},t)$ are solved as initial value problems through inverse transformation. We have a relation between D_{ij} and Σ_α:

$$D_{ij}=\frac{C_1|\boldsymbol{E}(\boldsymbol{k},\boldsymbol{k}\cdot\boldsymbol{v})|^2}{2|\boldsymbol{v}-\boldsymbol{v}'|^3}(1-y)\exp(y),\quad C_1=\frac{1}{2\pi^2}\left(\frac{e_\alpha}{m_\alpha}\right)^2\left(\frac{k_i k_j}{\boldsymbol{k}^2}\right), \tag{7}$$

$$\Sigma_\alpha=y^2\cdot\frac{D_{ij}}{|\boldsymbol{v}-\boldsymbol{v}'|^2}, \tag{8}$$

where the y has a value of $0\sim 1$. The eq. (8) is anomalous collision frequency obtained by Tsytovich in quasilinear theory [2], if we exchange our correlational length in velocity space, $|\boldsymbol{v}-\boldsymbol{v}'|$, with $\boldsymbol{v}$.

As for the initial distribution function, $f_\alpha^{(0)}(\boldsymbol{k}',\boldsymbol{v}',\omega')$, we assume a delta function $\delta(\boldsymbol{v}'-\boldsymbol{v}_0)$, with initial beam velocity, $\boldsymbol{v}_0$ and with spatial form factor $f_1^{(0)}(\boldsymbol{k}')$, then we can obtain the beam distribution $f_\alpha(\boldsymbol{k},\boldsymbol{v},t)$:

$$f_\alpha(\boldsymbol{k},\boldsymbol{v},t)=-\frac{\mathrm{i}}{8\pi}f_1^{(0)}(\boldsymbol{k})\frac{n_b}{\{\frac{1}{2}C_1|E|^2(1-y)e^y\}^{\frac{3}{2}}}\cdot\frac{1}{\pi^{\frac{1}{2}}t^{\frac{3}{2}}}\cdot\left|\boldsymbol{v}-\boldsymbol{v}_0\pm\frac{\mathrm{Im}\Sigma_\alpha}{2\boldsymbol{k}}\right|^{\frac{9}{2}}\cdot\exp\left\{-\frac{|\boldsymbol{v}-\boldsymbol{v}_0\pm\frac{\mathrm{Im}\Sigma_\alpha}{2\boldsymbol{k}}|^5}{2C_1|E|^2(1-y)e^y\cdot t}\right\}\cdot\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}_0t)\cdot\exp(-\mathrm{Re}\Sigma_\alpha t), \tag{9}$$

where the n_b, $\mathrm{Im}\Sigma_\alpha$, $\mathrm{Re}\Sigma_\alpha$ are the beam electron density, the width of frequency broadening, the collision frequency between beam electron and a cloud of elementary excitation, i.e., phonon cloud under the presence of ions when we consider the collision with ion waves. The Σ_α is a function of frequency, and if beam electron collide to plasmon cloud, its angular frequency, ω, is given as $\omega\simeq\omega_{pe}\simeq\boldsymbol{k}\cdot\boldsymbol{v}_0$ (ω_{pe}: plasma frequency), then the elementary excitation (soliton) is not emitted since sum of the kinetic energy and the plasmon energy is conserved and fast energy exchange between them is possible. However if beam electron collide to ion wave, the total energy of electron and phonon cloud is not conserved and the reaction is progressive therefore the elementary excitation appears. In this case, Σ_α = *Function* ($\gamma\boldsymbol{k}\cdot\boldsymbol{v}_0$), where the $\gamma\equiv\sqrt{m_e/m_i}\simeq(1/500)$.

Equation (9) includes an intensity of electric field $|\boldsymbol{E}(t)|^2$, which can be obtained as:

$$\boldsymbol{E}(\boldsymbol{k},t)=\sum_\alpha\left(\frac{e_\alpha}{\varepsilon_0}\right)\left(\frac{\mathrm{i}\boldsymbol{k}}{\boldsymbol{k}^2}\right)f_1^{(0)}\left(\frac{4^q n_b v_0}{\pi^{\frac{1}{2}}\{C_1|E|^2t\}^{(\frac{3}{2}-q)}}\right)\cdot\left(\frac{1}{\boldsymbol{k}\cdot\boldsymbol{v}_0t}\right)\cdot\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}_0t)\sinh\left(\frac{\mathrm{Im}\Sigma_\alpha}{2}t\right)\cdot\Gamma\left(q,a_1\left|\frac{\mathrm{Im}\Sigma_\alpha}{2\boldsymbol{k}}\right|^\nu\right), \tag{10}$$

where $\Gamma(q,x)$ is an incomplete Gamma function and variables q, x are given as $q=1.3$, $x=a_1|\mathrm{Im}\Sigma_\alpha/(2k)|^\nu$, $a_1=1/\{4C_1\}|E|^2t\}$, $\nu=5$.

At t = 0 $\boldsymbol{E}(\boldsymbol{k},t)$ has not value, however at the limit $t\to$

0, $\boldsymbol{E}(\boldsymbol{k}, t) \to 0$ owing to $\sinh\{(\mathrm{Im}\Sigma_\alpha/2)t\}$ and $\Gamma(q, x \to \infty) \to 0$, while from the character $\mathrm{Im}\Sigma_\alpha \propto \exp\{-\mathrm{Re}\Sigma_\alpha t\}$ and $\Gamma(q, x \to 0) \to \Gamma(q) \simeq 0.8975 = \text{const.}$, we get $\boldsymbol{E}(\boldsymbol{k}, t \to \infty) \to 0$ then the elementary excitation has finite life time.

From the eq. (9), we can derive a stopping power analytically when the beam passing through the plasma, $\partial W/\partial t$ or $\partial W/\partial x \simeq (1/v)\cdot\partial W/\partial t$, where the W is energy per unit volume.

$$\frac{\partial W}{\partial t}=\frac{\partial}{\partial t}\left\{\int \frac{m v^2}{2} f(\boldsymbol{v})\,\mathrm{d}^3\boldsymbol{v}\right\}$$

$$=f_1^{(0)}\frac{1}{(\pi t)^{\frac{3}{2}}}\left(\frac{e_\alpha}{m_\alpha}\right)^2\frac{m}{v_0}\left(\frac{\boldsymbol{k}_i\boldsymbol{k}_j}{\boldsymbol{k}^2}\right)\frac{n_b}{\{\frac{1}{2}C_1|E|^2(1-y)e^y\}^{\frac{3}{2}}}$$

$$\cdot\frac{|\boldsymbol{E}(\boldsymbol{k},\boldsymbol{k}\cdot\boldsymbol{v})|^2}{|\boldsymbol{v}-\boldsymbol{v}'|^3}\cdot(1-y)e^y\{2C_1|E|^2(1-y)e^y t\}^{\frac{11}{10}}$$

$$\cdot\left[\frac{2}{3}\left(\frac{\mathrm{Im}\Sigma_\alpha}{2k}\right)^3+2v_0^2\left(\frac{\mathrm{Im}\Sigma_\alpha}{2k}\right)\right]$$

$$\cdot\left[\frac{\mathrm{i}}{16\pi}\Gamma\left(\frac{21}{10},\alpha\left|\frac{\mathrm{Im}\Sigma_\alpha}{2k}\right|^5\right)\right.$$

$$\left.\frac{9\mathrm{i}}{160\pi}\Gamma\left(\frac{11}{10},\alpha\left|\frac{\mathrm{Im}\Sigma_\alpha}{2k}\right|^5\right)\right]\cdot\exp(-\mathrm{Re}\Sigma_\alpha t),$$

$$\alpha=1/\{2C_1|E|^2(1-y)e^y t\}. \tag{11}$$

The stopping power in the eq. (11) is related to $\mathrm{Im}\Sigma_\alpha$ which is a loss function as in the theory of dielectric materials and the imaginary unit 'i' represents that an imaginary part of permittivity of electron gas is contributed. Since the $\mathrm{Im}\Sigma_\alpha$ is width of frequency, $\mathrm{Im}\Sigma_\alpha/(2k)$ means phase velocity and we limit the interaction in following velocities region: $\boldsymbol{v}_0 - \mathrm{Im}\Sigma_\alpha/(2\boldsymbol{k}) < \boldsymbol{v} < \boldsymbol{v}_0 + \mathrm{Im}\Sigma_\alpha/(2\boldsymbol{k})$. The $\boldsymbol{v}_0$ is initial beam velocity and we assume also the inequality $\mathrm{Im}\Sigma_\alpha/(2\boldsymbol{k}) \ll \boldsymbol{v}_0$. In the stopping power, $\partial W/\partial x \simeq (1/v_0)\partial W/\partial t$ depends on the quantity $[2\{\mathrm{Im}\Sigma_\alpha/(2k) + \{2/(3v_0^2)\}\cdot\mathrm{Im}\Sigma_\alpha/(2k)\}^3]$. The first term is independent on $\boldsymbol{v}_0$, while the second term shows a character of $(1/\boldsymbol{v}_0^2)$. The character is the same as in the linear theory by Bethe [3]. In linear theoretical results of low velocity ion incidence into solid [4,5], the stopping power is proportional to incident velocity, $\boldsymbol{v}_0$. These results are related to the quantity, $\boldsymbol{A}_i(\boldsymbol{v}, t)$, the first term in the Fokker-Planck eq. (1).

$$\boldsymbol{A}(\boldsymbol{v},t)=-\frac{e^2}{m\varepsilon_0}\int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\frac{\boldsymbol{k}}{\boldsymbol{k}^2}\cdot\left\{\frac{1}{\varepsilon(\boldsymbol{k},\boldsymbol{k}\cdot\boldsymbol{v})}\right\}$$

$$=\frac{\pi e^2}{m\varepsilon_0}\int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\frac{\boldsymbol{k}}{\boldsymbol{k}^2}$$

$$\cdot\frac{\omega(\boldsymbol{k})\cdot\delta[\boldsymbol{k}\cdot\boldsymbol{v}-\omega(\boldsymbol{k})]}{|\omega\frac{\partial}{\partial\omega}\varepsilon_1(\boldsymbol{k},\omega)|_{\omega=\omega(\boldsymbol{k})}}, \tag{12}$$

where $\varepsilon_1(\boldsymbol{k}, \omega)$ is a real part of permittivity. We have neglect the term with respect to $\boldsymbol{A}(\boldsymbol{v}, t)$ because coherent interaction such an induced emission or absorption will exceed the incoherent spontaneous effect. However, if we consider a nonlinear theory by using $\boldsymbol{A}(\boldsymbol{v}, t)$, it is sufficient to replace their δ-function with our Green's function with finite width. The merit of our nonlinear theory is 'no cut off' in the $\boldsymbol{k}$-space even if at $\boldsymbol{k} = 0$, while in the above linear theories, there are cut off in the coherent region such as $\boldsymbol{k}_{\text{minimum}}$ with proper reason to avoid divergence.

Figure 1 shows the results of a soliton obtained by theory at beam energy of 1920 eV and beam density $n_b = 10^{14}\ \mathrm{m}^{-3}$ by using the $\gamma \equiv \sqrt{m_e/m_i} \simeq (1/500)$. The vertical axis shows the electric field in unit volume of $\boldsymbol{k}$-space, $\boldsymbol{E}(\boldsymbol{k}, t)$, i.e., if we represent $\boldsymbol{E}(\boldsymbol{r}, t)$ with (Volt/λ) where λ is wave length, then the $\boldsymbol{E}(\boldsymbol{k}, t) = \lambda^3\cdot\boldsymbol{E}(\boldsymbol{r}, t)$.

The collision frequency or the self energy term, Σ_α, is a function of time. The value of the Σ_α becomes

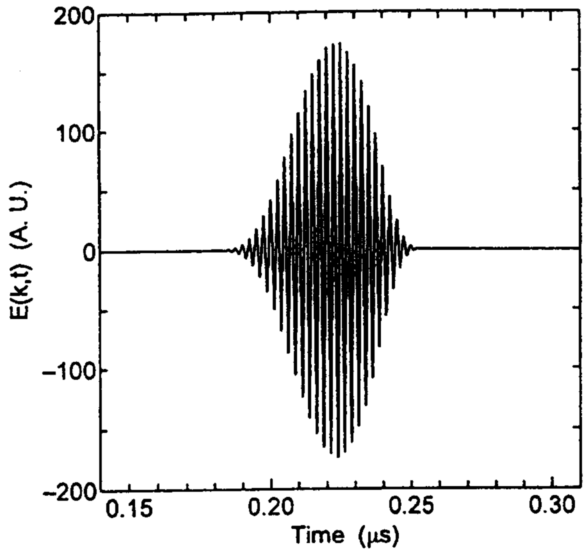

Fig. 1 An example of the calculated wave packet (soliton) is shown. The following constants are used. $|\boldsymbol{v} - \boldsymbol{v}'| \simeq 10^4$ (m/s), $y = 0.72$, $\gamma = \{m_i/m_e\}^{1/2} \simeq 500$.

suitable for soliton excitation, the soliton starts to increase. The starting time is at 1.72×10^{-7}s.

Figure 2 shows the beam distribution function, $f(\boldsymbol{k}, \boldsymbol{v}, t)$, in the vertical axis which means an electron number in the cubic, λ^3, composed of the wave length, λ. As the eq.(9) is the function of $|\boldsymbol{E}(\boldsymbol{k}, t)|^2$, we must

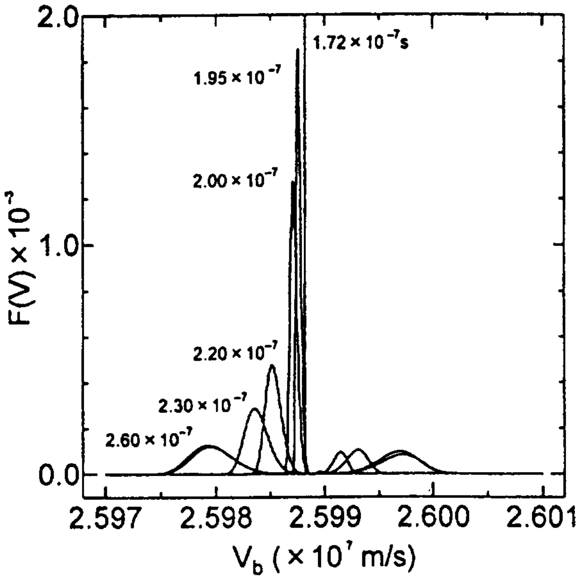

Fig. 2 Relaxation of a beam is shown. At $t = 1.72 \times 10^{-7}$(s), where the soliton starts to increase as in fig. 1, the beam function $\delta(t - 1.72 \times 10^{-7})$ also starts up.

Fig. 3 The electronic stopping power $\partial W/\partial t$ for beam electron is shown as a function of time.

solve eq. (10) simultaneously. We choose the time that the beam's distribution function is proportional to $\delta(t - 1.72 \times 10^{-7})$. The beam is relaxed out within 100ns after the starting.

Analytical results of $\partial W/\partial t$ are shown in fig. 3. Mechanism of the stopping power is the following: at least, 2nd-order nonlinear quantity in the eqs. (2) and (5) is $|\boldsymbol{E}(\boldsymbol{k}_1, \omega_1)|^2 \cdot G_\alpha(\boldsymbol{k} - \boldsymbol{k}_1, \boldsymbol{v}, \boldsymbol{k}\cdot\boldsymbol{v} - \mathrm{i}\Sigma_\alpha - \omega_1)$ which means that two electrons having $(\omega, \boldsymbol{k})$ and $(\omega_1, \boldsymbol{k}_1)$, interact trough exchanging the phonon under the presence of ion. The coupled electrons may behave as Bose particle in which the force is gravitation. The Σ_α includes the above quantity and is accompanied with a factor $(e_\alpha/m_\alpha)^2$, moreover, $\boldsymbol{E}(\boldsymbol{k}, \omega)$ is proportional to $(e_\alpha/\varepsilon_\alpha)$ and the Σ_α includes the factor $|\boldsymbol{E}(\boldsymbol{k}_1, \omega_1)|^2$, so that if charge accumulation occurs, then the stopping power increases by the 4th order of accumulated charge even in the 2nd order nonlinear while the soliton occurs by 3rd-order nonlinear. For the actual calculation of the Σ_α, one must obtain a eigen value since eq. (5) is a operator.

3. Experiment

A apparatus consists of a beam gun and a cylindrical container in which plasma is generated by an electron beam. The container is made of stainless steel, which is 45cm in length and 16cm in diameter, and is filled with Ar gas of $10^{-4} \sim 10^{-3}$Torr. A mirror magnetic field with 80Gauss at a central part and mirror ratio of 1.4 is used. The electron beam with 1920V and 18mA injected into plasma in the density of $10^8 \sim 10^9$cm^{-3}.

Figure 4 shows the solitons and the signal of 1 MHz component of the ion wave detected with 300kHz band width observed in our experiments. The solitons are indicated by arrows. which are detected by a mixer with passive device only, while the ion wave is obtained by the frequency analyzer, HP-8554L-8552B system. The signal of the ion wave delays from that of the soliton, the reason is the following: The soliton has large intensity, then the passive device can be used so that there is no delay, however the signal of ion wave pass through the frequency analyzer by amplification and some delay is occurred but the main delay comes from 'the uncertainty' and also a sampling rule that the band width $\Delta\omega$ = 300kHz corresponds to time uncertainty (time delay) Δt with the relation $\Delta\omega\Delta t \geq 0.5$ *(cycle)* so that delay time is to be 1.7μs.

Figure 5 shows a oscilloscope trace of the solitons and also the ion wave in rather high plasma density. The soliton is not hyperbolic-secant function, in contrast to the lower density case, but has many fine splits,

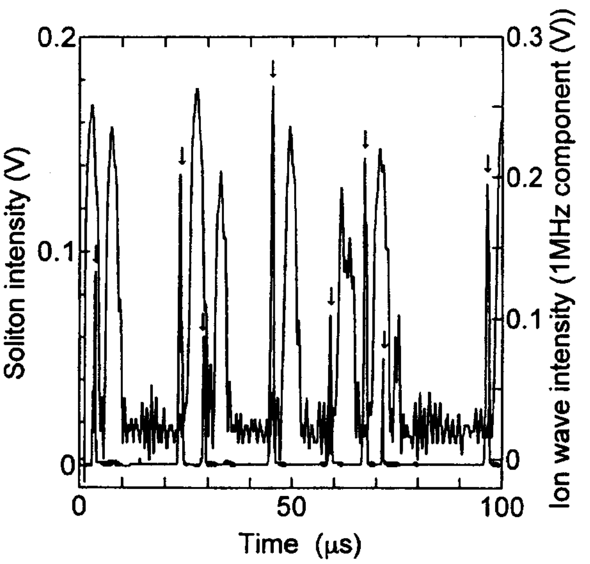

Fig. 4 The soliton and the signal of 1MHz-component of the ion wave in the experiment is shown. The soliton is indicated by arrow while the ion wave is detected by 300kHz-band width so that the signal is delayed.

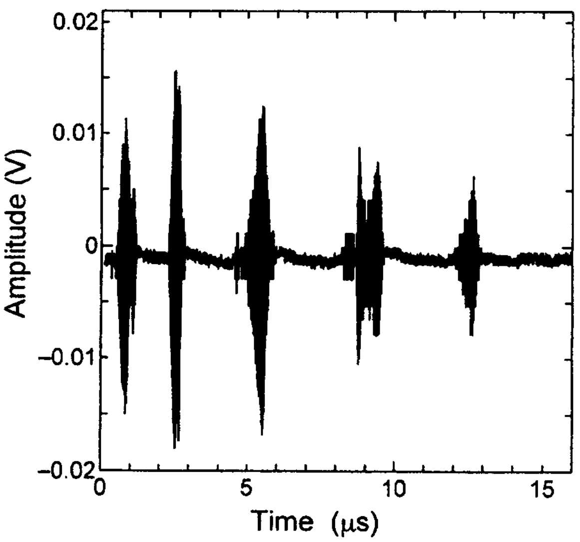

Fig. 5 The coupling between the soliton and the ion wave is shown. The coupling appears when the plasma density which is proportional to the gas pressure is higher in such the pressure of 4×10^{-4} Torr. The ripple of horizontal line shows the ion wave.

showing that the heavy load, such as the nonlinear Landau damping and the ion wave turbulence, make deform the shape. The ion wave appears as a triangle with steepening negative front. The appearance of soliton coincide actually with the ion wave. This means that the beam electron emits the soliton when the beam collide to the steep negative front of the ion wave by the *Bremsstrahlung* in the deceleration field.

References

[1] I. Mori, T. Morimoto, R. Kawakami and K. Tominaga, *to be published in* J. Plasma Fusion Res. **2** (1999).

[2] V.N. Tsytovitch, *Nonlinear Effects in Plasmas* (Trans. by M. Hamberger, New York-London, Plenum Press, 1970).

[3] H.A. Bethe, Ann. Phys. (Leipzig) **5**, 325 (1930).

[4] O.B. Firsov, Zh. Eksp. Teor. Fiz. **36**, 1517 (1959) [Sov. Phys. JETP **9**, 1076 (1959)].

[5] J. Lindhard, M. Scharff and H.E. Schiøtt, Kgl. Danske. Videnskab. Sel-skab, Mat-fys. Meddele., **33**, No.14 (1963).