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# Electron Heating at Sub-Harmonic Electron Cyclotron Frequency in an Electron Beam-Plasma System

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## Abstract

Electron heating occurs in an electron beam-plasma system at $\omega \simeq \frac{3}{2}\omega_{ce}$ by launching a space charge wave on $V_b$ = 1500V and 1000V, where $V_b$ denotes the acceleration voltage of beam electrons. Experimental results are qualitatively consistent with the nonlinear coupling coefficient of the sub-harmonic electron cyclotron (EC) resonance. This suggests that the electron heating is due to the sub-harmonic EC resonance.

## Keywords:

space charge wave, sub-harmonic electron cyclotron resonance

## 1. Introduction

Ion heating at sub-harmonic ion cyclotron frequencies ($\omega \simeq m\omega_{ci}/2$) was investigated [1-6]. Here, $m$ is an odd integer. Recently, electron heating at half the electron cyclotron frequency by extraordinary wave, which can be applicable for ITER, is investigated theoretically [7]. In these circumstances, experimental studies for the sub-harmonic electron cyclotron (EC) frequencies is useful. However, only a few studies were carried out [8]. In this paper, we study sub-harmonic EC resonance by employing a space charge wave in an electron beam-plasma system at $\omega \simeq \frac{3}{2}\omega_{ce}$ ($\omega_{ce} \equiv |e|B/m_e$).

Sub-harmonic heating is due to the nonlinear Landau (cyclotron) damping resulting from the self-interaction of waves. The resonance condition for nonlinear Landau damping in a magnetic field is given by $\omega_k - \omega_{k'} - (k_\parallel - k'_\parallel)\, v_\parallel = m\omega_{ce}$, where ($\omega_k$, $\mathbf{k}$) and ($\omega_{k'}$, $\mathbf{k}'$) are frequencies and wave vectors of two waves, and $m$ is an integer. In the case of self-interaction, ($\omega_k$, $\mathbf{k}$) = ($-\omega_{k'}$, $-\mathbf{k}'$) = ($\omega$, $\mathbf{k}$) and then ($\omega_{k''}$, $\mathbf{k}''$) = ($2\omega$, $2\mathbf{k}$). The resonance condition is presented by

$$\omega - k_\parallel v_\parallel = \frac{m}{2}\,\omega_{ce}.$$

The term $k_\parallel v_\parallel$ can be neglected since $k_\parallel \ll k_\perp$ in the space charge wave, and then the resonance condition is given by $\omega \simeq m\omega_{ce}/2$. Therefore, the resonance at $\omega \simeq \frac{3}{2}\omega_{ce}$ (i.e. $m$ = 3) can be occurred.

In this paper, the first experimental results are presented and are qualitatively compared with the nonlinear wave-particle coupling coefficient. In Section 2, we describe the experimental results. In Section 3, the kinetic wave equation and the transport equation are described. In Section 4, the numerically calculated coupling coefficients are compared with the experimental results. Finally, we present a summary in Section 5.

## 2. Experimental Results

### 2.1 Experimental procedures

Experiments are performed in a linear device of about 9 cm diameter and 50cm length. A mono-energetic electron beam from electron gun is

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continuously injected along a magnetic field of $B_0 \simeq 70G$ ($\omega_{ce}/2\pi$ = 198MHz). Acceleration voltage, current and diameter of electron beam are $V_b \simeq$ 500–1500V, $I_b \simeq$ 0.5–2mA and $D_b \simeq$ 2mm (diameter of the anode hole of the electron gun is 2mm), respectively. Length from the gun to target is $\simeq$ 40cm. Langmuir probe is placed 20cm away from the gun at the plasma center ($r$ = 0mm). Working gas is argon, and gas pressure is 1.5mTorr. In the beam-generated plasma, electron density is $n_e \simeq 5$–$10 \times 10^{10}$cm$^{-3}$. The space charge waves are excited by applying an external RF signal with the frequency $\omega/2\pi$ = 250–350MHz and the power $P$ = 1W to the electrode of the gun.

## 2.2 Electron heating

Figure 1 (a) shows $\Delta T_e$ versus $\omega/\omega_{ce}$ for $V_b$ = 1500V (solid circles) and $V_b$ = 1000V (open circles). Here, $\Delta T_e$ means the change of the electron temperature by applying the external RF signal ($P$ = 1W). Electron temperature before applying the external RF signal is $T_{e0} \simeq$ 7eV, 4eV for $V_b$ = 1500V and $V_b$ = 1000V, respectively. For $V_b$ = 1500V, $\Delta T_e$ has a peak at $\omega/\omega_{ce} \simeq 1.5$, and becomes small in $\omega/\omega_{ce} < 1.4$ or $\omega/\omega_{ce} > 1.6$. For $V_b$ = 1000V, $\Delta T_e$ has a smaller peak at $\omega/\omega_{ce} \simeq 1.5$, and $\Delta T_e$ remains positive at $\omega/\omega_{ce}$ = 1.4 or 1.6. This is different from the former case ($V_b$ = 1500V). The fractional change of electron temperature $\Delta T_e/T_{e0}$ is the same order for both cases. The fractional change of electron density $\Delta n_e/n_{e0}$ versus $\omega/\omega_{ce}$ are shown in fig. 1 (b). In both cases, electron density decreases with increasing in $\Delta T_e$. Electron density before applying the external RF signal is $n_{e0} \simeq$ 6, $10 \times 10^{10}$cm$^{-3}$ for $V_b$ = 1500V and $V_b$ = 1000V, respectively. Figure 1 (c) shows the fractional change of energy density $\Delta(n_eT_e)/(n_{e0}T_{e0})$ versus $\omega/\omega_{ce}$. In spite of decrease in electron density, energy density increases at $\omega/\omega_{ce} \simeq 1.5$ due to the increase in electron temperature for both cases.

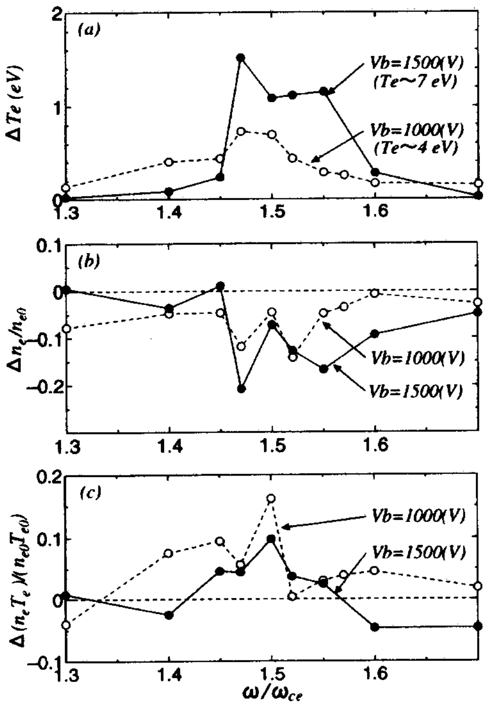


Fig. 1 (a) Change of the electron temperature ($\Delta T_e$), (b) fractional change of the electron density ($\Delta n_e/n_{e0}$), (c) fractional change of the energy density ($\Delta n_eT_e/n_{e0}T_{e0}$) versus $\omega/\omega_{ce}$ for $V_b$ = 1500V (solid circles) and $V_b$ = 1000V (open circles), respectively. For the case of $V_b$ = 1500V, $I_b$ = 1.7mA, $n_e = 6 \times 10^{10}$cm$^{-3}$, $T_{e0}$ = 7eV. For the case of $V_b$ = 1000V, $I_b$ = 0.8mA, $n_e = 10 \times 10^{10}$cm$^{-3}$, $T_e$ = 4eV, where $I_b$ is the beam current. In both cases, $\omega_{ce}/2\pi$ =198MHz is fixed.

## 3. Kinetic Wave Equation and Transport Equation

Experimental results show that electron heating occurs at $\omega/\omega_{ce} \simeq 3/2$. This suggests that the electron heating is due to the sub-harmonic EC resonance. Here, we describe the kinetic wave equation and the transport equation for the sub-harmonic resonance.

We assume that the velocity distribution functions for bulk and beam electrons are the Maxwellian and shifted Maxwellian, respectively. The kinetic wave equation for the sub-harmonic resonance is presented as [9]:

$$\frac{\partial U_k}{\partial t} = 2\gamma_k U_k + \varepsilon_0 \omega_k \alpha_k \left| E_k \right|^2 U_k \tag{1}$$

$$\alpha_k = -\left(\frac{2}{n_e m_e v_{Te}^2}\right)\left(\frac{\partial \omega_k \varepsilon_k}{\partial \omega_k}\right)^{-1} \frac{4\sqrt{\pi}}{|k_{\parallel}| v_{Te}} \left(\frac{\omega_{pe}}{\omega_{ce}}\right)^4 \sum_{q,r}\left[A - \frac{BC}{D}\right]E$$

$$A = \int_0^\infty dte^{-t^2} J_p(\rho t) J_r(\rho t) J_{m-q}(\rho t) J_{m-r}(\rho t),$$

$$B = \int_0^\infty dte^{-t^2} J_q(\rho t) J_{m-q}(\rho t) J_m(2\rho t),$$

$$C = B(q \rightarrow r),$$

$$D = e^{-2\rho^2}\, \mathrm{I}_m(-2\rho^2)/2\,,$$

$$E = e^{-v_R^2}\,[(q-m/2)^2-1]^{-1}\,[(r-m/2)^2-1]^{-1}\,,$$

$$\varepsilon_k = 1 + \sum_{s=e,b} \frac{2\omega_{ps}}{k^2 v_{Ts}^2} \sum_{r=-\infty}^{\infty} e^{\lambda_s}\mathrm{I}_r(\lambda_s)[1+\zeta_{0s}Z_p(\zeta_{rs})]\,,$$

$$v_R = (2\omega_k - \omega_{ce})/2k_{\parallel}v_{Te}\,,$$

where $\rho = k_\perp v_{Te}/\omega_{ce}$, $\zeta_{re} = (\omega_k - r\omega_{ce})/k_\parallel v_\parallel$, $\zeta_{rb} = (\omega_k - k_\parallel v_\parallel - r\omega_{cb})/k_\parallel v_\parallel$, $\lambda_s = k_\perp^2 v_{Ts}^2/2\omega_{ce}$, $\omega_{pb}$ is the plasma frequency for beam electrons, $v_{Te}$ ($v_{Tb}$) is the thermal velocity of bulk (beam) electrons, $v_b = \sqrt{2\,eV_b/m_e}$, $J_q$ is the Bessel function of the q-th order, $I_r$ is the modified Bessel function of the r-th order, $E_k$ is the electric field of the wave ($\omega_k$, $k$), $U_k$ is the energy density of the wave, $Z_p$ is the plasma dispersion function, and the other notations are standard. The linear damping rate $\gamma_k$ is negligible small at $\omega_k/\omega_{ce} \simeq 3/2$. Transport equation indicating time evolutions of energy and momentum densities of bulk electrons are presented as:

$$\frac{\partial U_e}{\partial t} = -2\varepsilon_0\,\omega_k\alpha_k\,|E_k|^2\,U_k\,,$$
$$\frac{\partial P_{\parallel e}}{\partial t} = -2\varepsilon_0\,k_\parallel\alpha_k\,|E_k|^2\,U_k\,. \tag{2}$$

It is found from eq. (1) that explosive instability can occur when $\alpha_k > 0$. This is possible if a negative-energy wave ($\partial\omega_k\varepsilon_k/\partial\omega_k < 0$) is scattered from bulk electrons [9].

## 4. Discussion

We define the non-dimensional coupling coefficient as $\alpha_0 \equiv |\alpha_k| \times (n_e m_e v_{Te}^2/2)$. Since slow mode of the space charge wave is the negative-energy wave, $\alpha_k > 0$. On the other hand, $\alpha_k < 0$ for fast mode of the space charge wave.

Figure 2 shows the coefficient $\alpha_0$ versus $\omega/\omega_{ce}$ for $V_b$ = 1500V and $V_b$ = 1000V, respectively. Parameters for numerical calculation are chosen from the experimental condition. In the case of $V_b$ = 1500V, the coefficients for both modes become maximum at $\omega/\omega_{ce} \simeq 1.48$. In the case of $V_b$ = 1000V, the coefficients for both modes become maximum at $\omega/\omega_{ce} \simeq 1.45$, and these increase again below $\omega/\omega_{ce} \simeq 1.38$ because $|\partial\omega_k\varepsilon_k/\partial\omega_k|$ reaches to zero.

First, we discuss the case of $V_b$ = 1500V. Experimental results show that electron density decreases at the same time that electron heating occurs. This is similar to the electron heating due to the explosive instability via non-linear Landau damping of space charge wave and Trivelpiece-Gould (TG) mode in an electron beam-plasma system [10]. However, in the previous experiments [10], heating occurs in the lower range ($\omega/\omega_{ce}$ = 1.1 – 1.45), and $\Delta T_e/T_{e0}$ has some peaks. This is because TG mode with various frequencies can be scattered and it heats electrons. In the present experiments, heating occurs only around $\omega/\omega_{ce} \simeq 1.5$. This is qualitatively consistent with the coupling coefficient of the sub-harmonic resonance. Therefore, the cause of the present electron heating is not non-linear Landau damping of space charge wave and TG mode, but the sub-harmonic EC resonance.

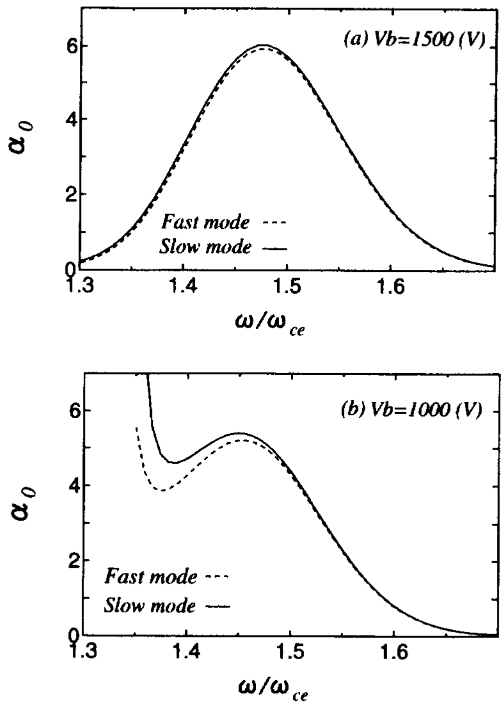


Fig. 2 (a) The non-dimensional coupling coefficient $\alpha_0$ versus $\omega/\omega_{ce}$ for $V_b$ = 1500V. Parameters correspond to the experimental condition ($\omega_{ce}/2\pi$ =198GHz, $I_b$ = 1.7mA, $n_e = 6\times10^{10}$cm$^3$, $T_e$ = 7eV). (b) The non-dimensional coupling coefficient $\alpha_0$ versus $\omega/\omega_{ce}$ for $V_b$ = 1000V. Parameters correspond to the experimental condition ($\omega_{ce}/2\pi$ =198MHz, $I_b$ = 0.8mA, $n_e = 10\times10^{10}$cm$^3$, $T_e$ = 4eV). In both cases, we assume that electron beam is cold (i.e. $T_b$ = 0).

Next, we discuss the case of $V_b$ = 1000V. Experimental results show that the electron temperature increases and the electron density decreases by applying the external RF signal in the range of $\omega/\omega_{ce} \simeq 1.4$–$1.6$. The coefficient $\alpha_0$ increase with $\omega/\omega_{ce}$ beyond $\omega/\omega_{ce}$ = 1.4 and becomes maximum at $\omega/\omega_{ce}$ = 1.45. Both $\Delta T_e$ and $\alpha_0$ show similar dependence in $\omega/\omega_{ce} > 1.4$. In $\omega/\omega_{ce} < 1.4$, $\Delta T_e$ and $\alpha_0$ shows different behavior. Further studies are needed in this region.

The second harmonic component of the space charge waves can be coupled with electrons via the linear third harmonic resonance. In order to investigate this linear effect, we apply the RF signal with the frequency $\omega/\omega_{ce} = 3$ and the small power $P$ = 0.1W to the electrode of the gun. However, no increase in electron temperature is measured.

## 5. Summary

In launching a space charge wave, electron heating occurs in an electron beam-plasma system at $\omega \simeq \frac{3}{2}\omega_{ce}$ on $V_b$ = 1500V and 1000V. Experimental results in both cases are compared with the numerically calculated nonlinear coupling coefficient for the sub-harmonic EC resonance. The experimental results and the coefficient show qualitative agreement. This suggests that the electron heating is due to the sub-harmonic EC resonance.

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