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Spiral Structure Formation in Rotating Plasmas

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Abstract

A theory on spiral structure formation in rotating plasmas has been formulated and shown to reproduce the characteristic features of spiral structures observed in electron cyclotron resonance plasmas.

Keywords:
spiral structure, rotating plasma, centrifugal force instability, collisional drift wave instability

1. Introduction

Organized structures in plasmas have been a topic since they may not only give a deep insight into general aspects of self-organization in complex systems but also contribute to transport phenomena. In fact recently coherent structures have been observed in laboratories [1-4] and are subject to theoretical analysis for understanding underlying physics. Both stationary [3] and rotating [4] spiral structures are observed.

In this article we have developed a theory to explain the formation of spiral structures in magnetized plasmas and showed a good agreement of the results with observations in ECR plasmas whose characteristic features are (1) the direction of the arm stretching is reversed when the magnetic field is reversed, (2) the stationary structure is observed only in a narrow range of the background pressure, and (3) the spatial extent of the structure in the axial direction is of the order of the plasma radius.

2. Basic Equation

Equations for magnetized plasmas read

$$\frac{\partial n^{\alpha}}{\partial t} + \nabla(n^{\alpha}\boldsymbol{v}^{\alpha}) = Sn^{e}, \qquad (1)$$

$$\frac{\partial \boldsymbol{v}^{\alpha}}{\partial t} + \boldsymbol{v}^{\alpha}\cdot\nabla\boldsymbol{v}^{\alpha} = \frac{e^{\alpha}}{m^{\alpha}}(-\nabla\phi + \frac{1}{c}\boldsymbol{v}^{\alpha}\times\boldsymbol{B}) - \frac{1}{n}\nabla p_{\alpha} - \nu_{\alpha}\boldsymbol{v}^{\alpha}. \qquad (2)$$

where n^{α} and $\boldsymbol{v}^{\alpha}$ are the density and velocity of particle of species α (electron/ion), and ϕ and S are the potential and the source due to ionization, respectively. In the following ion temperature p_i is assumed zero.

A potential in magnetized plasmas in a cylindrical vessel is determined by the balance between ion cross field diffusion and electron axial transport since electrons are magnetized almost completely while ions can diffuse in radial direction. The space potential produced by the ion radial transport is short-circuited by the electron axial transport so that we have $\nabla(n_0^i\boldsymbol{v}_0^i) = \nabla(n_0^e\boldsymbol{v}_0^e) = Sn_0^e$ with the background density n_0^{α} and velocity $\boldsymbol{v}_0^{\alpha}$, which gives the relation between the density and potential. In the following we look for a potential profile for a given background density since the ambipolar equation is not easy to solve except for special cases. Invoking charge neutrality $n_0^e(r, z) = n_0^i(r, z) = n_0(r, z)$ the ambipolar equation gives the potential $\phi_0(r, z)$ which can be expressed in terms of the

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background density $n_0(r, z)$ with normalizations $\xi = r/r_d$ and $\eta = z/r_d$ where r_d is the plasma radius, as

$$\phi_0(\xi,\eta) = -\left(\frac{\Omega_i \nu_e}{\Omega_e \nu_i}\right)^2 \ln n_0(\xi,\eta) - \frac{\Omega_i^2}{\nu_i^2}\int_0^{\xi} d\xi' \frac{1}{\xi' n_0(\xi',\eta)} \int_0^{\xi'} d\xi''\xi'' \left\{\varepsilon n_0(\xi'',\eta) + \frac{\partial^2 n_0(\xi'',\eta)}{\partial \eta^2}\right\}, \qquad (3)$$

where $\varepsilon = r_d^2\left(\frac{\nu_e}{v_T^2} - \frac{\nu_i}{c_s^2}\right)S$, and we have assumed that $(\nu_e/\Omega_e)^2 \ll 1$.

The rotation frequencies of ion and electron azimuthal drift are given by

$$\omega_0^i \simeq \omega_E\left(1 - \frac{\omega_E}{\Omega_i}\right), \qquad \omega_0^e \simeq \omega_E - \omega_*, \qquad (4)$$

where ω_E and ω_* are the $\boldsymbol{E}\times\boldsymbol{B}$ rotation frequency and diamagnetic drift frequency, respectively defined by

$$\omega_E = \frac{C_s^2}{r_d^2 \Omega_i}\frac{1}{\xi}\frac{\partial \phi_0}{\partial \xi}, \qquad \omega_* = \frac{v_T^2}{r_d^2\Omega_e}\frac{1}{\xi}\frac{\partial \ln n_0}{\partial \xi}.$$

The difference of the ion and electron rotation frequencies is an origin of instabilities, a centrifugal instability and a dissipative drift wave instability which support spiral structures.

3. Spiral Structure of Fluctuations

The fluctuating parts of the electron velocity $\tilde{\boldsymbol{v}}(\boldsymbol{r}, z)$ in a reference frame of normalized cylindrical coordinate (ξ, θ, η) which are expressed as

$$\tilde{\boldsymbol{v}}^e(\boldsymbol{r}, z) = \sum_{\ell}\begin{pmatrix} u_\ell^e(\xi,\eta) \\ v(\xi,\eta) \\ w_\ell^e(\xi,\eta)\end{pmatrix} e^{i\ell\theta},$$

are given from eq.(2) by

$$u_\ell^e = \frac{v_T^2}{r_d \Sigma_e(\omega)}\left[-\frac{i\ell(\Omega_e - 2\omega_0^e)}{\xi}\left(\phi_\ell - \frac{n_\ell^e}{n_0}\right) + \Gamma_e(\omega)\frac{\partial}{\partial\xi}\left(\phi_\ell - \frac{n_\ell^e}{n_0}\right)\right],$$

$$v = \frac{v_T^2}{r_d\Sigma_e(\omega)}\left[\left(\Omega_e - 2\omega_0^e - \xi\frac{d\omega_0^e}{d\xi}\right)\frac{\partial}{\partial\xi}\left(\phi_\ell - \frac{n_\ell^e}{n_0}\right) + \frac{i\ell\Gamma_e(\omega)}{\xi}\left(\phi_\ell - \frac{n_\ell^e}{n_0}\right)\right],$$

$$w_\ell^e = \frac{v_T^2}{r_d\Gamma_e(\omega)}\frac{\partial}{\partial\eta}\left(\phi_\ell - \frac{n_\ell^e}{n_0}\right),$$

where

$$\Gamma_e(\omega) = \nu_e - i(\omega - \ell\omega_0^e),$$

$$\Sigma_e(\omega) = (\Omega_e - 2\omega_0^e)\left(\Omega_e - 2\omega_0^e - \xi\frac{d\omega_0^e}{d\xi}\right) + \Gamma_e(\omega)^2.$$

Substituting these into the equation of electron continuity we have

$$[i(\omega - \ell\omega_0^e) + S]\frac{n_\ell^e}{n_0} = \frac{v_T^2}{r_d^2}\frac{\Gamma_e(\omega)}{\Sigma_e(\omega)}\left\{\frac{\partial^2}{\partial\xi^2}\left(\phi_\ell - \frac{n_\ell^e}{n_0}\right) + \left[\frac{1}{\xi} + \left(\frac{d}{d\xi}\ln\frac{n_0}{\Sigma_e(\omega)}\right)\right]\frac{\partial}{\partial\xi}\left(\phi_\ell - \frac{n_\ell^e}{n_0}\right) - \left[\frac{\ell^2}{\xi^2} + \frac{i\ell}{\xi}\frac{\Sigma_e(\omega)}{n_0\Gamma_e(\omega)}\left(\frac{d}{d\xi}\frac{n_0(\Omega_e - 2\omega_0^e)}{\Sigma_e(\omega)}\right)\right]\left(\phi_\ell - \frac{n_\ell^e}{n_0}\right)\right\} + \frac{v_T^2}{r_d^2 n_0}\frac{\partial}{\partial\eta}\frac{n_0}{\Gamma_e(\omega)}\frac{\partial}{\partial\eta}\left(\phi_\ell - \frac{n_\ell^e}{n_0}\right).$$

On the other hand the fluctuating parts of ion velocity are given in a similar way as before by

$$u_\ell^i = -\frac{C_s^2}{r_d\Sigma_i(\omega)}\left[-\frac{i\ell(\Omega_i + 2\omega_0^i)}{\xi}\left(\phi_\ell + \Gamma_i(\omega)\frac{\partial\phi_\ell}{\partial\xi}\right)\right],$$

$$v = \frac{C_s^2}{r_d\Sigma_i(\omega)}\left[\left(\Omega_i + 2\omega_0^i + \xi\frac{d\omega_0^i}{d\xi}\right)\frac{\partial\phi_\ell}{\partial\xi} - \frac{i\ell\Gamma_i(\omega)}{\xi}\phi_\ell\right],$$

$$w_\ell^i = -\frac{C_s^2}{r_d\Gamma_i(\omega)}\frac{\partial\phi_\ell}{\partial\eta},$$

where

$$\Gamma_i(\omega) = \nu_i - i(\omega - \ell\omega_0^i),$$

$$\Sigma_i(\omega) = (\Omega_i + 2\omega_0^i)\left(\Omega_i + 2\omega_0^i + \xi\frac{d\omega_0^i}{d\xi}\right) + \Gamma_i(\omega)^2.$$

Substituting these into the ion continuity equation we have

$$[i(\omega - \ell\omega_0^i) + S]\frac{n_\ell^i}{n_0} = \frac{C_s^2\Gamma_i(\omega)}{r_d^2\Sigma_i(\omega)}\left\{\frac{\partial^2}{\partial\xi^2}\phi_\ell + \left[\frac{1}{\xi} + \left(\frac{d}{d\xi}\ln\frac{n_0}{\Sigma_i(\omega)}\right)\right]\frac{\partial\phi_\ell}{\partial\xi} - \left[\frac{\ell^2}{\xi^2} - \frac{i\ell}{\xi}\frac{\Sigma_i(\omega)}{n_0\Gamma_i(\omega)}\left(\frac{d}{d\xi}\frac{n_0(\Omega_i - 2\omega_0^i)}{\Sigma_i(\omega)}\right)\right]\phi_\ell\right\} - \frac{C_s^2}{r_d^2}\frac{1}{n_0}\frac{\partial}{\partial\eta}\left(\frac{n_0}{\Gamma_i(\omega)}\frac{\partial}{\partial\eta}\phi_\ell\right).$$

Invoking charge neutrality $n_\ell^e = n_\ell^i = n_\ell$, we have

$$\frac{\partial^2}{\partial\xi^2}\phi_\ell + [\frac{1}{\xi} + \frac{\mathrm{d}\ln n_0}{\mathrm{d}\xi}]\frac{\partial}{\partial\xi}\phi_\ell - \frac{\ell^2}{\xi^2}\phi_\ell$$
$$-\frac{\mathrm{i}\ell}{\xi}\frac{1}{\Gamma_\mathrm{i}(\omega)}\{(2\omega_0^\mathrm{i} + \xi\frac{\mathrm{d}\omega_0^\mathrm{i}}{\mathrm{d}\xi} + \frac{\ell\omega_\mathrm{E}^2}{\omega-\ell\omega_\mathrm{E}})$$
$$\frac{\mathrm{d}\ln n_0}{\mathrm{d}\xi} + 3\frac{\mathrm{d}\omega_0^\mathrm{i}}{\mathrm{d}\xi} + \xi\frac{\mathrm{d}^2\omega_0^\mathrm{i}}{\mathrm{d}\xi^2}\}\phi_\ell$$
$$+\frac{\Omega_\mathrm{i}^2}{\Gamma_\mathrm{i}^2(\omega)}\frac{\partial^2}{\partial\eta^2}\phi_\ell + \frac{\Omega_\mathrm{e}\Omega_\mathrm{i}}{\Gamma_\mathrm{e}(\omega)\Gamma_\mathrm{i}(\omega)}\frac{\partial^2}{\partial\eta^2}$$
$$[1+\frac{\ell\omega_*}{\omega-\ell\omega_\mathrm{E}}]\phi_\ell = 0. \qquad (5)$$

We have neglected terms of the order of or less than O $(\frac{\Omega_\mathrm{i}}{\Omega_\mathrm{e}})$ and used the following approximation which is valid when the scale length of the fluctuations is smaller than that of the background density,

$$\frac{n_\ell}{n_0} \simeq -\frac{v_\mathrm{T}^2}{r_\mathrm{d}^2\Omega_\mathrm{e}^2}\frac{\ell\Omega_\mathrm{e}}{\omega-\ell(\omega_0^\mathrm{e}+\omega_*)}\frac{1}{\xi}\frac{\mathrm{d}\ln n_0}{\mathrm{d}\xi}\phi_\ell =$$
$$-\frac{\ell\omega_*}{\omega-\ell\omega_\mathrm{E}}\phi_\ell. \qquad (6)$$

Equation (5) describes low frequency fluctuations subjected to both a flute mode instability due to centrifugal force acting on ions and a collisional drift wave instability, though modes related to spiral structures are decaying instead of propagating along the axial direction.

Assuming the axial dependence of the potential as

$$\phi_\ell(\xi,\eta) = \phi_\ell(\xi)e^{-\kappa\eta},$$

and putting

$$\phi_\ell(\xi) = \frac{\psi_\ell(\xi)}{\sqrt{n_0(\xi)}},$$

we can transform eq.(5) to

$$\frac{\mathrm{d}^2\psi_\ell}{\mathrm{d}\xi^2} + \frac{1}{\xi}\frac{\mathrm{d}\psi_\ell}{\mathrm{d}\xi} + [\beta^2(\xi) - \frac{\ell^2}{\xi^2}]\psi_\ell = 0, \qquad (7)$$

where

$$\beta^2(\xi) \simeq \begin{cases} \dfrac{\kappa^2\Omega_\mathrm{e}\Omega_\mathrm{i}}{\Gamma_\mathrm{e}(\omega)\Gamma_\mathrm{i}(\omega)}\dfrac{\omega-\ell\omega_0^\mathrm{e}}{\omega-\ell\omega_\mathrm{E}} & \text{for} \quad \kappa\sim 1, \\ -\dfrac{\mathrm{i}\ell}{\xi}\dfrac{1}{\Gamma_\mathrm{i}(\omega)}(2\omega_0^\mathrm{i} + \xi\dfrac{\mathrm{d}\omega_0^\mathrm{i}}{\mathrm{d}\xi} + \dfrac{\ell\omega_\mathrm{E}^2}{\omega-\ell\omega_\mathrm{E}})\dfrac{\mathrm{d}\ln n_0}{\mathrm{d}\xi} & \text{for} \quad \kappa\sim 0, \end{cases} \qquad (8)$$

The main contribution comes from the collisional drift wave instability for $\kappa\sim 1$ and from the gravitational instability for $\kappa\sim 0$. For $\kappa\sim 1$ the axial extent of the plasma is of the same order of the plasma radius, which is the situation of the ECR plasma in which spiral structures are observed. Then the solution is approximated in the case of weak ξ-dependence of the zero-th order quantities by

$$\psi_\ell(\xi) \simeq \mathrm{J}_\ell(\beta\xi), \qquad (9)$$

where J_ℓ is the Bessel function of the first kind. The real part of the argument of the Bessel function should be positive to give a convergent behavior while the imaginary part is responsible for a spiral structure which comes from $\Gamma_\mathrm{e}(\omega)$ and $\Gamma_\mathrm{i}(\omega)$ with $\omega = \omega_r + \mathrm{i}\gamma$.

Multiplying ψ_ℓ^* to eq.(7) and integrating the resultant equation from the center to the edge of the plasma under the boundary condition

$$\psi_\ell(0) = \psi_\ell(1) = 0,$$

we have

$$\int_0^1 \xi\{|\frac{\mathrm{d}\psi_\ell}{\mathrm{d}\xi}|^2 + \frac{\ell^2}{\xi^2}|\psi_\ell|^2 - \beta^2(\xi)|\psi_\ell|^2\}\mathrm{d}\xi = 0. \qquad (10)$$

From the imaginary part of this equation we have for $\kappa\sim 1$

$$\omega_r \propto \ell\int\omega_0^\mathrm{e}\xi|\psi_\ell(\xi)|^2\mathrm{d}\xi = \ell<\omega_0^\mathrm{e}>,$$

which guarantees that the real part of $\beta^2(\xi)$ is positive and the imaginary part is proportional to Ω_i, indicating the winding direction of spiral arms is reversed when the magnetic field is reversed. Furthermore a stationary spiral structure is formed when a so called Rayleigh condition

$$\omega_r \simeq \ell<\omega_0^\mathrm{e}> = 0,$$

can be satisfied, implying there is a radial point where $\omega_0^\mathrm{e} = 0$ and electrons cannot locally cancel the charge separation produced by ions which rotate with $\omega_0^\mathrm{i}\sim\omega_\mathrm{E}$. For a wide range of parameters, however, spiral structures are rotating.

For $\kappa\sim 0$ we cannot have a stationary spiral structure since when $<\omega_\mathrm{E}> = 0$, the real part of $\beta^2(\xi)$ also becomes zero and ψ_ℓ is expressed by a linear combination of $\xi^{-\ell}$ and ξ^ℓ, that is, ψ_ℓ is divergent.

In the following we compare our results with the experimental data in the ECR plasmas [3]. A stationary spiral structure is shown in Fig.1 for which the background density is chosen as $n_0(\xi) = \exp(-a\xi^2 - b\xi^4)$, ($a = 3/2$ and $b = 1$) and is compared with the

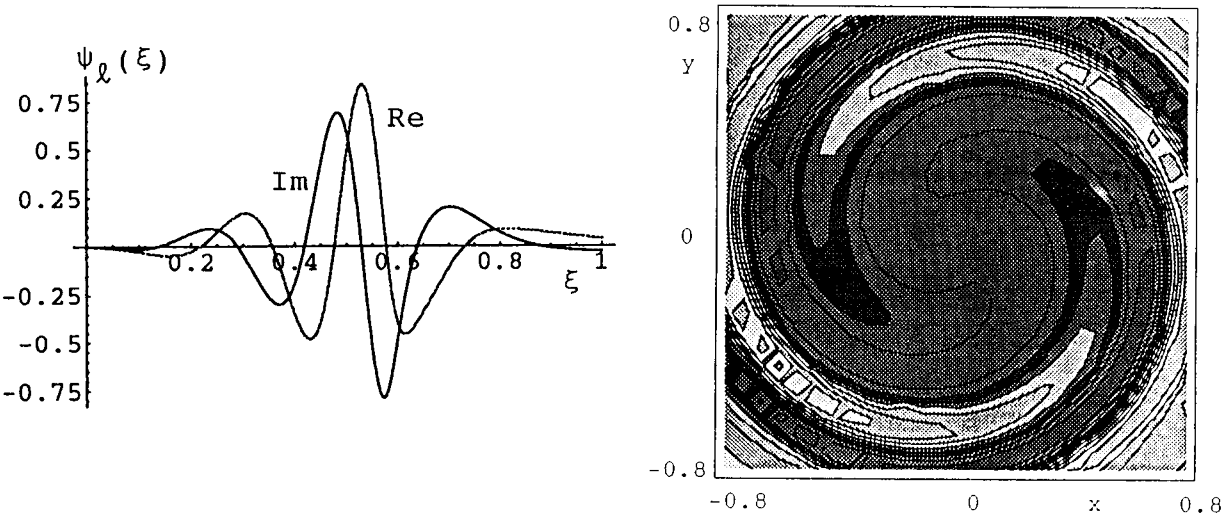

Fig. 1 A spiral structure: the radial profile of the real and imaginary parts of eigenfunction ψ_ℓ (ξ) (left) and the contour of an eigenfunction (right).

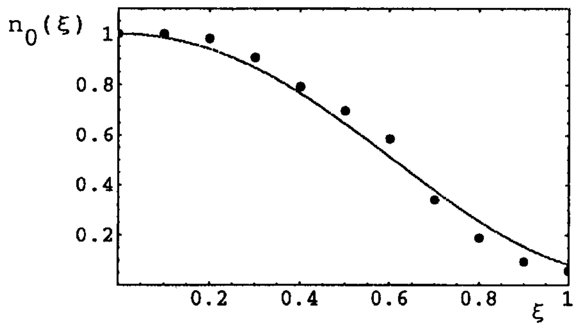

Fig. 2 A density profile given by $n_0(\xi)$ = exp $(-a\xi^2 - b\xi^4)$ with a = 3/2 and b =1 (a solid line) and the observed data (a dotted line).

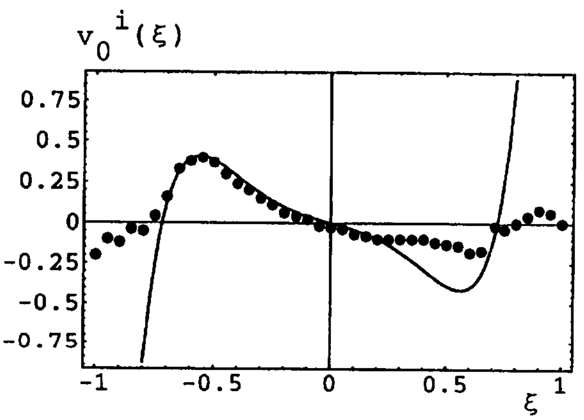

Fig. 3 An ion drift velocity: theory (a solid line) and observation (a dotted line).

experimental data in Fig.2. The corresponding ion azimuthal drift velocity is compared with the observed one in Fig.3.

4. Discussion

In this paper we have developed a theory on spiral structure formation in rotating plasmas and reproduced the characteristic features of spiral structures observed in electron cyclotron resonance plasmas[3]. The energy stored in a velocity shear is released to give an instability which drives a spiral structure. Under the special condition that there is a radial point where ω_0^ξ = 0, a spiral structure becomes stationary which has been observed in the experiment [3]. Beside stationary spiral structures there can also be excited rotating spiral structures for monotonically decreasing potential structures.

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