J. Plasma Fusion Res. SERIES, Vol. 2 (1999) 291–294

HINT Computation with the Bootstrap Current

KANNO Ryutaro, NAKAJIMA Noriyoshi, HAYASHI Takaya,
YOKOYAMA Masayuki and OKAMOTO Masao
National Institute for Fusion Science, Toki 509-5292, Japan

(Received: 18 December 1998 / Accepted: 13 May 1999)

Abstract

The revised HINT computation for calculating a stellarator equilibrium with islands under the effects of the self-consistent bootstrap current is presented. To check the validity of the revised computation, it is applied to a helias equilibrium with islands.

Keywords:
HINT computation, 3-D MHD equilibrium, helias, island formation, bootstrap current

1. Introduction

To study three-dimensional (3-D) MHD equilibria with islands under the effects of the bootstrap current, the HINT code [1,2] is revised, which originally treats only currentless equilibria. The calculation is based on a time-dependent relaxation technique using small values of resistivity and viscosity. The original HINT code was applied to finite beta three-dimensional equilibria without a net toroidal current, because three-dimensional equilibria have the possibility of net current-free steady operation. The most important advantage of the HINT code is that it does not need to assume the existence of nested flux surfaces in an equilibrium. Thus, the HINT code enables us to investigate quantitatively; 1) the deterioration of magnetic surfaces due to the magnetic island formation and 2) the self-healing of magnetic islands in finite beta plasmas in three-dimensional currentless equilibria [3-5].

Although the three-dimensional equilibria have a possibility to exist under the currentless condition, the net toroidal current, namely, the bootstrap current can flow according to the experimental condition. Properties of a Large Helical Device (LHD) [6] equilibrium with the self-consistent bootstrap current are examined [7,8] and the effects of the net toroidal current on the Mercier stability are also investigated [9] under the assumption that nested good flux surfaces exist. The bootstrap current changes the rotational transform so drastically that the resonance condition for appearance of the magnetic island will be altered, which leads to the formation and/or self-healing of magnetic islands. To investigate such phenomena, the HINT code has been revised. In this paper, we propose the revised HINT computational method and apply it to some helias equilibrium with islands in order to check the validity of the new HINT code.

Corresponding author's e-mail: kanno@nifs.ac.jp

2. Modification of the HINT Computation

In the HINT code, an MHD equilibrium is obtained starting from an arbitrary nonequilibrium initial plasma and vacuum field configuration by means of a time-dependent relaxation method with small values of resistivity η and viscosity ν. Calculations are performed in the three steps.

The first step (A-step) is the relaxation process of the pressure along magnetic field lines. To speed up the relaxation of the pressure, we make an average of pressure, $\bar{p}$, along a field line, and interpolate a value of pressure at each grid point by using the averages $\bar{p}$ [3].

$$p \rightarrow \bar{p} = \frac{\int p\,\mathrm{d}\ell/B}{\int \mathrm{d}\ell/B}, \qquad (1)$$

where ℓ means the length along the field line.

In the second step (B-step), calculation of the bootstrap current is performed under conditions of a fixed pressure profile and a fixed magnetic field, if and only if closed flux surfaces are found. In the calculation of the bootstrap current, we employed the Boozer coordinates (ψ, θ, ζ), where ψ is the toroidal magnetic flux divided by 2π, and θ and ζ are the poloidal and toroidal angles, respectively. If the collisionality of electrons is same as one of ions, the bootstrap current is expressed in each collisionality limit as follows;

$$\left\langle \boldsymbol{j} \cdot \boldsymbol{B} \right\rangle_{\mathrm{bs}} = -G_{\mathrm{bs}} \left(L_1 \frac{\mathrm{d}p}{\mathrm{d}\psi} + L_2 n \frac{\mathrm{d}T}{\mathrm{d}\psi} \right), \qquad (2)$$

where $\langle\ \rangle$ means the flux surface average, n is the density and T is the temperature ($p = 2nT$; $n = n_{\mathrm{i}} = n_{\mathrm{e}}$, $T = T_{\mathrm{i}} = T_{\mathrm{e}}$, and n, $T \propto \sqrt{p}$ in this work). L_1 and L_2 are the transport coefficients, which are composed of the viscosity and the friction coefficients, and G_{bs} is the geometrical factor given in Refs. [7-10]. To evaluate the bootstrap current through Eq. (2), the spectra of the magnetic field strength in the Boozer coordinates are needed with some surface quantities [7-10]. To obtain the spectra of the magnetic field strength and surface quantities, we use the methods of Boozer [11] and Rome [12] with the magnetic field line tracing. To estimate the bootstrap current in various collisionality regimes, we use the connection formula derived in Ref. [9], instead of Eq. (2).

The calculation scheme of the bootstrap current in the island regions is briefly described as follows. Even if an equilibrium contains islands or stochastic regions, we can construct the magnetic coordinates (ψ, θ, ζ) by using the method of the quasi-magnetic coordinates [13]. The profile of pressure in the island becomes flat, when the numerical equilibrium sufficiently achieves the relaxation state, because the islands are generated by bifurcating from a rational surface. Thus, the gradients of pressure and temperature, $\mathrm{d}p/\mathrm{d}\psi$ and $\mathrm{d}T/\mathrm{d}\psi$, become zero in the islands, and the value of the bootstrap current is zero, by assuming that the formulation of the bootstrap current, Eq. (2), is also appropriate in the quasi-magnetic coordinates. Note that the bootstrap current also becomes zero in the stochastic region outside the outermost surface, because $p = 0$ in this region. To numerically distinguish sufficiently large islands from negligibly small islands (or rational surfaces), we trace a field line started from a fixed point (an O point of the island) and calculate the residue, $\mathscr{R}$, of Cary and Hanson [14]. If the residue satisfies the condition $1 > \mathscr{R} > \varepsilon_R$, the island is decided to be numerically visible, where ε_R is a small parameter, $1 \gg \varepsilon_R \geq 0$, and defines the size of the negligibly small islands. In order to decide the boundary of islands, contour lines of the pressure can be used. We define that the island region is given by the condition $p^{(+)}_{\mathrm{island}} > p > p^{(-)}_{\mathrm{island}}$, where $p^{(\pm)}_{\mathrm{island}} = p_{\mathrm{island}} \pm \varepsilon_p$, p_{island} is the pressure at an O point of the island, and ε_p is a small parameter with $1 \gg \varepsilon_p/p_{\mathrm{island}} > 0$ and gives the width of the island region. Note that in progress of the relaxation process the profile of pressure in islands is not exactly flat. The bootstrap current is given as zero in the island region with the condition $p^{(+)}_{\mathrm{island}} > p > p^{(-)}_{\mathrm{island}}$, while the current in all plasma regions except islands is calculated by the connection formula based on Eq. (2).

The third step (C-step) is the relaxation process of the magnetic field under a fixed pressure profile [1,2,15]. To calculate the MHD equilibrium with the net toroidal current, we revise the Faraday equation in the C-step, according to Refs. [16,17]. If there are closed magnetic surfaces in the plasma region, the Faraday equation can be generalized as

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} = \nabla \times \left(\boldsymbol{v} \times \boldsymbol{B} - \eta \left\{ \boldsymbol{j} - \boldsymbol{B} \frac{\left\langle \boldsymbol{j} \cdot \boldsymbol{B} \right\rangle_{\mathrm{bs}}}{\left\langle B^2 \right\rangle} \right\} \right), \qquad (3)$$

where $\boldsymbol{B} \langle \boldsymbol{j} \cdot \boldsymbol{B} \rangle_{\mathrm{bs}} / \langle B^2 \rangle = 0$ in the island region and regions in which closed magnetic surfaces do not exist.

Solving numerically the above relaxation equations in the revised HINT computation, we find a 3-D MHD equilibrium with the bootstrap current in a helical system.

3. Results of the Revised HINT Computation and Discussions

To check the validity of the revised HINT computation, the revised code is applied to a helias equilibrium with the field period of $M = 5$ and the magnetic field strength of $B_0 = 4$ T at the magnetic axis, where there is no island in the vacuum magnetic field structure.

For the central beta value $\beta_0 = 8.5\%$, the magnetic field structure of the helias equilibrium is shown in Fig. 1 under the currentless condition, and the island chain with the rotational transform of $\iota/2\pi = 5/6$ is generated.

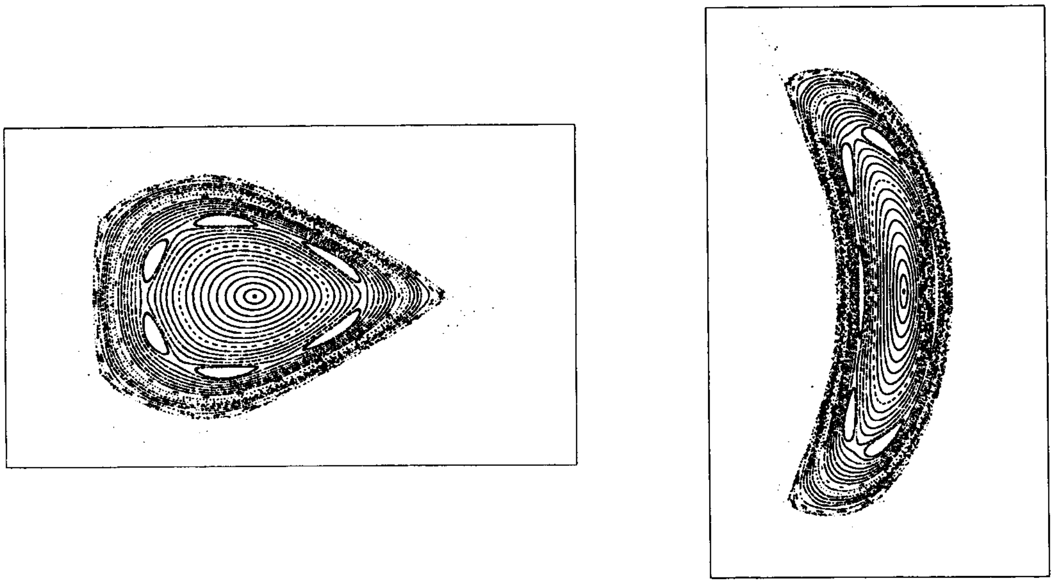

Fig. 1 Poincare plots of magnetic field lines for a currentless helias equilibrium. Here, the value of the magnetic field strength at the magnetic axis is given as $B_0 = 4$ T.

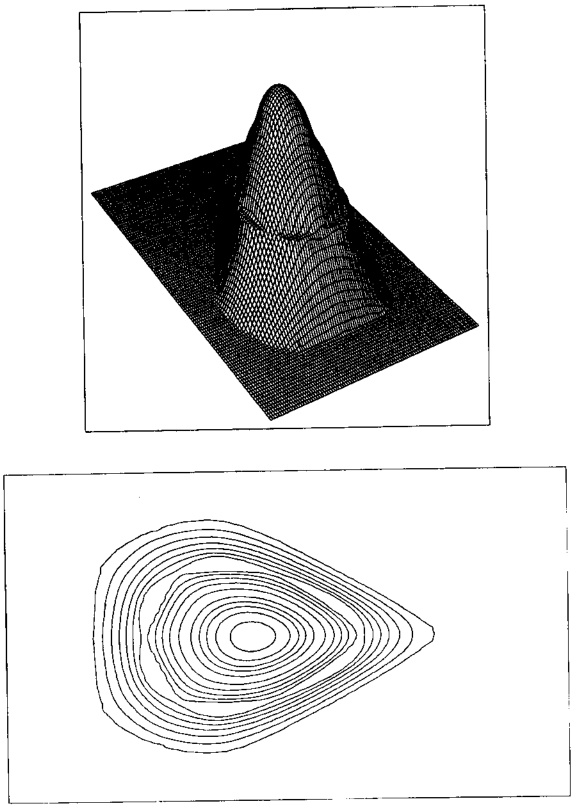

Fig. 2 The pressure profile of the currentless helias equilibrium.

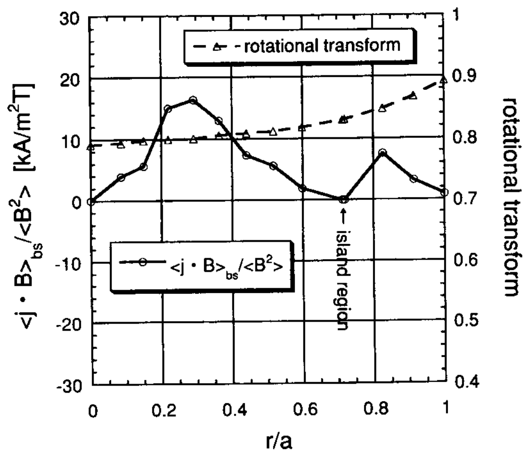

Fig. 3 Profiles of the bootstrap current and the rotational transform are given as functions of the miner radius, $r/a \propto \sqrt{\psi}$. $T_0 = 1.8$ keV and $n_0 = 7.8 \times 10^{20}$ 1/m^3 are given.

From this figure we easily conclude that there exist closed magnetic surfaces in the equilibrium, and the magnetic coordinates can be constructed. The pressure profile becomes flat in the island region as shown in Fig. 2. In this region the bootstrap current should be zero, according to the idea of the B-step. If the bootstrap current is calculated actually by using the equilibrium quantities for this case, the profile of the net current is described with the rotational transform in Fig. 3. Here we define in Fig. 3 that the temperature and the density

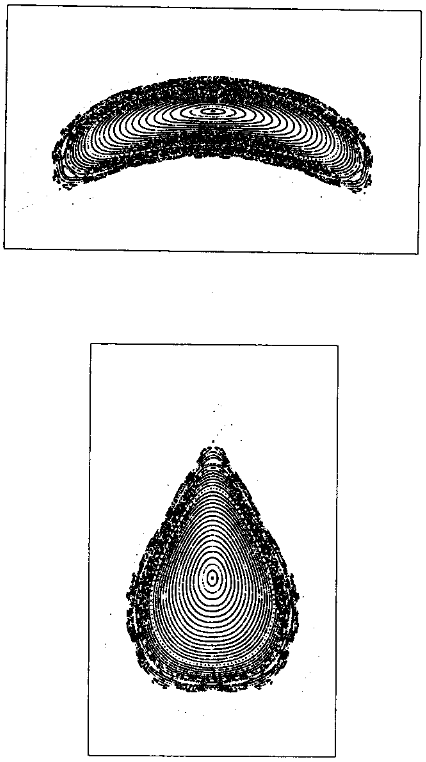

Fig. 4 Poincare plots of magnetic field lines for the helias equilibrium with the self-consistent bootstrap current of about 7 kA.

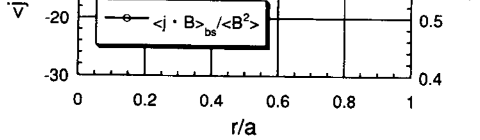

Fig. 5 Profiles of the bootstrap current and the rotational transform for the final result. T_0 = 2 keV and n_0 = 6.8 × 10^{20} 1/m^3 are defined.

at the magnetic axis, T_0 and n_0, are equal to 1.8 keV and 7.8 × 10^{20} 1/m^3, respectively. The zero net current appears in the island region with $\iota/2\pi$ = 5/6 and the total value of the net current is estimated to be about 4 kA for this case.

Continuing the revised HINT computation, we have the final result as shown in Fig. 4. The islands at $\iota/2\pi$ = 5/6 are suppressed by the effects of the self-consistent bootstrap current. Profiles of the bootstrap current and the rotational transform are given in Fig. 5. The net current makes a rapid ascent of the rotational transform at $\iota/2\pi$ = 5/6 as compared with the currentless case, and as a result the island region is healed and reduced to a rational surface.

We continue to study the effects of the self-consistent bootstrap current in order to understand the physical mechanism of the island suppression and especially focus on the relation to the 'self-healing'.

References

[1] T. Hayashi, Theory of Fusion Plasmas, CRPP Association EURATOM, Lausanne, p.11 (1989).

[2] K. Harafuji, T. Hayashi and T. Sato, J. Comput. Phys. **81**, 169 (1989).

[3] T. Hayashi, A. Takei and T. Sato, Phys. Fluids B **4**, 1539 (1992).

[4] T. Hayashi, T. Sato, P. Merkel, J. Nührenberg and U. Schwenn, Phys. Plasmas **1**, 3262 (1994).

[5] T. Hayashi, T. Sato, H.J. Gardner and J.D. Meiss Phys. Plasmas **2**, 752 (1995).

[6] A. Iiyoshi, M. Fujiwara, O. Motojima, N. Ohyabu and K. Yamazaki, Fusion Technol. **17**, 169 (1990).

[7] N. Nakajima and M. Okamoto, J. Phys. Soc. Jpn. **61**, 833 (1992).

[8] K. Watanabe, N. Nakajima, M. Okamoto, Y. Nakamura and M. Wakatani, Nucl. Fusion **32**, 1499 (1992).

[9] K.Y. Watanebe, N. Nakajima, M. Okamoto, K. Yamazaki, Y. Nakamura and M. Wakatani, Nucl. Fusion **35**, 335 (1995).

[10] K. Ichiguchi, N. Nakajima and M. Okamoto, Nucl. Fusion **37**, 1109 (1997).

[11] A.H. Boozer, Phys. Fluids **25**, 520 (1982).

[12] J.A. Rome, J. Comput. Phys. **82**, 348 (1989).

[13] A. Reiman and H. Greenside, Comput. Phys. Commun. **43**, 157 (1986).

[14] J.R. Cary and J.D. Hanson, Phys. Fluids **29**, 2464 (1986).

[15] W. Park, D.A. Monticello, H. Strauss and J. Manickam, Phys. Fluids **29**, 1171 (1986).

[16] R. Kanno, N. Nakajima, T. Hayashi and M. Okamoto, J. Plasma Fusion Res. SERIES **1**, 448 (1998).

[17] R. Kanno, N. Nakajima, T. Hayashi and M. Okamoto, J. Plasma Phys. **61**, 213 (1999).