J. Plasma Fusion Res. SERIES, Vol. 2 (1999) 267–270

Numerical Simulation of Core-Localized Alpha-Driven Alfvén Eigenmodes in Tokamaks

GOLOBOROD'KO Victor Ya., BELIKOV Vladimir S., EDENSTRASSER Johann W.[1],
REZNIK Svyatoslav N., SCHOEPF Klaus F.[1]* and YAVORSKIJ Victor A.
Institute for Nuclear Research of the National Academy of Sciences of Ukraine, Kiev, Ukraine
[1]Institute for Theoretical Physics, University of Innsbruck, Innsbruck, Austria

(Received: 24 December 1998 / Accepted: 22 April 1999)

Abstract

The stability of core-localized toroidicity-induced alpha-driven Alfvén eigenmodes (TAE) is investigated numerically for tokamak equilibria with circular flux surfaces. It is demonstrated that the TAE mode growth rate is strongly affected by the tokamak magnetic field geometry (i.e. Shafranov shift of the magnetic axis) as well as by the ripple induced transport of high energetic alpha particles.

Keywords:

TAE modes, alpha particles, ripple transport

1. Introduction

The realization of thermonuclear tokamak plasma experiments invokes the interest on investigating the fusion product's influence on the ignited plasma. In particular, the destabilization of the toroidicity-induced Alfvén eigenmodes by energetic alpha particles has received great attention [1-8]. Alpha particle driven TAEs have been observed for the first time in DT plasmas on TFTR with the central safety factor elevated to $q(0) > 1$ [8]. The destabilization of TAE modes by circulating high energetic ions has also been proved experimentally [9,10] in neutral beam heated plasmas. However, the measured stability threshold was larger than the theoretically predicted one. In this context we note that many previous theoretical investigations of TAE mode destabilization were based on simplified approaches for fast fusion products behaviour in tokamak plasmas. Namely, the approximation of small or moderate radial excursions of fast particles was used as well as a model distribution function that does not take into account the anisotropy caused by the finite banana width effects. Recent theoretical studies investigated the role of finite banana width effects in the destabilization of TAEs [3,5] and demonstrated the importance of these effects.

This paper is a continuation of the investigations carried out in Refs. [4,7]. Its aim is to clarify the influence of the Shafranov shift of the magnetic axis and of the ripple induced transport of alphas on the core-localized alpha-driven TAE growth rate.

2. Method Used

Following the approach described in [5] the alpha driven TAE mode growth rate (γ) may be expressed in the form

$$\frac{\gamma}{\omega} = \frac{\mathrm{Im}\,\delta W_k(\xi_0^*, \xi_0, \omega_0)}{2\omega_0^2\,\delta K(\xi_0^*, \xi_0)} . \tag{1}$$

Here ζ is the plasma displacement, ω the TAE eigenfrequency and $\delta K = \int d^3x\rho|\zeta|^2$; the subscript 0 denotes the zeroth order of a small parameter related to weak kinetic effects. In order to derive an explicit expression for the kinetic integral δW_k one may employ

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Corresponding author's e-mail: klaus.schoepf@uibk.ac.at

the technique of Ref. [6] to obtain

$$\delta W_k = -\frac{2\pi^2 c}{e_a m_a^2} \sum_{\sigma} \sum_{p} \int \mathrm{d}P_{\varphi} \mathrm{d}E \mathrm{d}\mu \tau_b$$
$$\left(\omega \frac{\partial F_a}{\partial E} - n \frac{\partial F_a}{\partial P_{\varphi}} \right) \frac{|Y_{\mathrm{p}}|^2}{\Omega}, \tag{2}$$

where $\Omega = \omega + n\langle\dot{\varphi}\rangle + p\omega_b$ and P_{φ} denotes a longitudinal adiabatic invariant; further, E and μ are the particle energy and the magnetic moment, respectively, while ω_b and $\langle\dot{\varphi}\rangle$ represent the particle bounce frequency and the bounce-averaged frequency of the toroidal motion; σ indicates the sign of the longitudinal velocity at the point of maximum toroidal flux on the trajectory. As shown in Ref. [11], the longitudinal velocity sign chosen in this form permits to divide the phase space definition domain of the fast particle distribution function in an unique manner. As phase space coordinates [11] we use here the particle velocity V, the normalized adiabatic invariant $\lambda = \mu B_0/E$ and the square root of the maximal toroidal flux on the trajectory, R_m. The Fourier coefficients in Eq. (2) are defined by

$$Y_p = \sum_m \oint \frac{\mathrm{d}\tau}{\tau_b} A_m(r) \exp\left(-\mathrm{i}p\omega_b\tau + \mathrm{i}m\tilde{\vartheta} - \mathrm{i}n\tilde{\varphi}\right), \tag{3}$$

with $A_m(r)$ determined by the radial profile of the TAE mode; further is $\tilde{\varphi} = \varphi - \langle\dot{\varphi}\rangle\tau$ and $\tilde{\varphi} = \varphi - \langle\dot{\varphi}\rangle\tau$, where φ and θ are the toroidal and the poloidal angle, respectively.

For any given choice of (V, λ, R_m, σ) one can calculate Y_p and perform all required orbit averaging by numerically integrating the system of drift equations of fast particle motion neglecting toroidal field ripples. It is assumed that ripples affect only the alpha particle transport but not the structure of the resonant mode.

The distribution function of alpha particles has been obtained by numerical solution of a 3D Fokker-Planck equation in the axisymmetric limit [11] and involving toroidal field ripple effects [12,13]. The alpha particle source profile was taken in the form

$$S_a = (1 - \frac{r^2}{a^2})^8, \tag{4}$$

where r and a are the flux surface and the plasma radius, respectively. The model profiles of the safety factor and of the Shafranov shift were chosen as

$$q = q(0) + \left[q(a) - q(0)\right] \frac{r}{a^2}, \tag{5}$$

and

$$\Delta = \Delta_0 (1 - \frac{r^2}{a^2}), \tag{6}$$

where $q(a)$ denotes the safety factor at the plasma edge and Δ_0 the Shafranov shift of the magnetic axis. Simulations were performed for the high energy alpha particles ($E > 0.25\ E_0$ with E_0 representing the birth energy of alphas) to exclude the influence of the low energy range on the TAE growth rate.

3. Results of Simulation

As a set of basic plasma parameters for the numerical calculations we choose typical parameters of the DT experiments in TFTR [14] taking $q(0) = 0.88$. Theoretical investigations [14] predicted that the core localized TAE modes with wave numbers $n = 5$ and $m = 4, 5$ may occur in these experiments. Numerical simulations were performed for a high ($\Delta_0 = 0.23\ a$) as well as for a low ($\Delta_0 = 0.10\ a$) Shafranov shift of the magnetic axis.

The calculational results for the dependence of the TAE growth rate on the plasma current for $\Delta_0 = 0.23\ a$ are presented in Fig.1. It is seen in the axisymmetric approximation that the mode growth rate increases as the plasma current is enhanced. This is due to the reduction of width of alpha particle orbits associated with a plasma current increase. This effect is in good qualitative agreement with theoretical predictions [3,5] and previous simplified modeling [4]. Calculating the alpha particle distribution function in the rippled magnetic field, we took into account stochastic

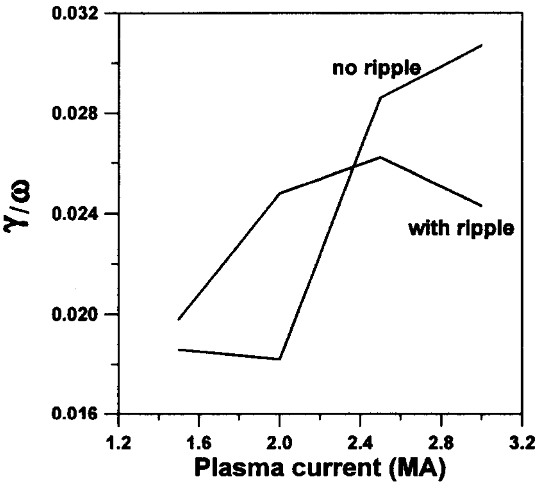

Fig. 1 TAE mode growth rate vs plasma current for a Shafranov shift $\Delta_0 = 0.23\ a$ of the magnetic axis.

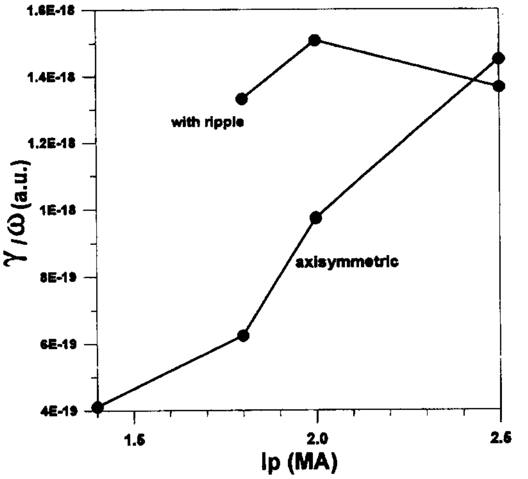

Fig. 2 TAE mode growth rate vs plasma current for the real TFTR geometry [7].

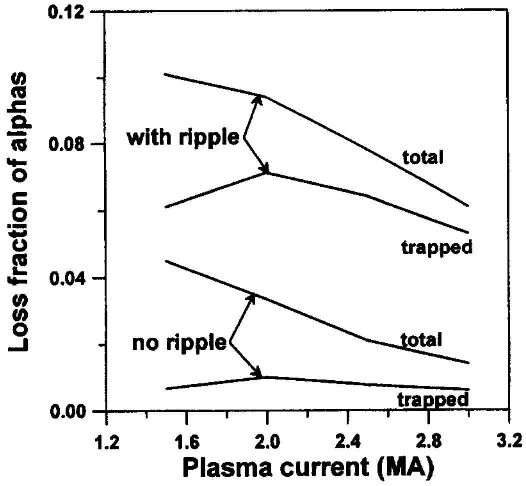

Fig. 3 Loss fraction of alphas vs plasma current for $\Delta_0 = 0.23\ a$.

diffusion, collisional ripple diffusion and an additional loss cone in the origin of local magnetic wells. In Fig.1 one can see that, at moderate plasma currents, the TAE growth rate in the rippled magnetic field may exceed the one in the axisymmetric limit. The same effect was observed in [7] where the TAE growth rate was simulated for real TFTR geometry (Fig.2) with ($\Delta_0 \cong 0.23\ a$. It might be a result of ripple induced stochastic radial diffusion that makes the alpha particle distribution function more inhomogeneous in comparison with the axisymmetric approximation. Some discrepancies of the results presented in Fig.1 and Fig.2 are due to the gap between the plasma and the first wall that was not considered in the present paper. Radial diffusion, on the other hand, results in an enhanced loss of trapped alphas as shown in Fig.3 as a function of the plasma current. Note that the ripple induced degradation of the population of trapped alphas reduces their influence on the mode destabilization. Calculational results for the case "with ripple" presented in Fig.1 and Fig.2 demonstrate that alpha ripple transport may cause the decrease of the TAE growth rate.

The TAE growth rate at low and moderate plasma currents is found to be very sensitive to the Shafranov shift of the magnetic axis. The results of the numerical calculations for the case $\Delta_0 = 0.1\ a$ are displayed in Fig.4. The total alpha loss fraction as well as the loss fraction of trapped alphas exhibit a slight dependence on Δ_0 due to the absence of the gap between the plasma and the first wall in the model used. Thus, comparing the results in Fig.1 and Fig.4 we see that, only for high plasma currents, γ is practically independent of Δ_0 as has been pointed out in [7]. At lower plasma currents, Δ_0 essentially affects the mode resonant structure that determines mainly the dependence of γ on I_p. We conclude from Fig.4 that, for low Δ_0, ripple induced transport and enhanced trapped particle loss result in a decrease of the TAE growth rate when compared to the rate in the axisymmetric limit.

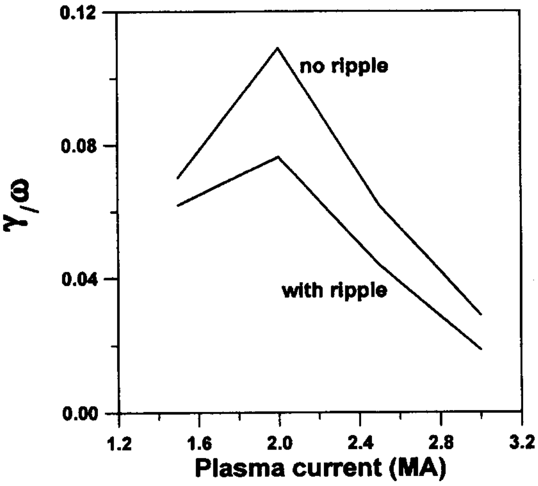

Fig. 4 TAE mode growth rate vs plasma current for $\Delta_0 = 0.10\ a$.

4. Conclusions

Our numerical simulation has demonstrated the stabilization effect of alpha particle finite orbit width on the TAE growth rate. While, at high plasma currents, γ

is affected only insignificantly by the Shafranov shift of the magnetic axis, the contrary is seen for the cases of low I_p. Ripple induced transport ordinarily decreases γ, however, in the case of high Shafranov shift and low I_p it may increase the TAE growth rate due to the additional anisotropy of the alpha distribution function. Also the q-profile plays an important role in TAE destabilization by fast alphas [8]. Numerical modeling of alpha driven TAEs for $q(0) > 1$ is suggested as a next step investigation.

Acknowledgments

This work was partially supported by the Austrian ÖAW-Euratom-Association, Project P8 and Grant #2.5.2/8 of the Ministry of Science and Technologies of Ukraine and the theme 0197U015786 of the National Academy of Sciences of Ukraine.

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