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Fusion Plasma Self-Ignition by Nonlinear Dynamic Evolution

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Abstract

A new criterion for fusion plasma ignition in d-t tokamaks is established by incorporating reaction-thermal dynamics into the plasma state analysis. Considering continuous fueling we find that the separatrix established by the dynamic trajectorial evolution in the state space is the critical boundary which must be exceeded in order that the plasma evolves towards ignition without any auxiliary heating.

Keywords:

d-t tokamak, fusion burn dynamics, phase state trajectories, ignition

1. Introduction

The identification of state and parameter space regions capable of ignited fusion plasma operation [1-4] as well as controlling such operation regimes is evidently crucial if significant energy gains are to be realized from tokamak reactors. While fusion plasma ignition usually defines a state in which the alpha fusion power deposition in the plasma balances the several transport and radiation losses from the plasma, it was demonstrated in [4] that such stationary ignition criteria become meaningless when a fusion plasma state evolves to an envisaged operation regime by external and/or internal heating. For that the nonlinear dynamic approach of an operation point in the state space spanned by the plasma variables has to be considered [5].

2. Burn Dynamics Model

We consider a globally averaged formulation of the dynamic evolutions of density and temperature in an ITER-like d-t tokamak plasma. Referring to axisymmetry we take different parabolic profiles for the radial dependencies of the local state variables, i.e. of the ion density $n_i(r,t)$ and of the kinetic ion and electron temperature $T_i(r,t)$ and $T_e(r,t)$, respectively. Assuming charge neutrality, the electron density $n_e(r,t)$ is calculated from the density of fuel ions and that of impurities accounted for by a corresponding effective charge number, and from the alpha particle density $n_\alpha(r,t)$ which is found separately for each triple of $n_i(r,t)$, $T_i(r,t)$, $T_e(r,t)$ via a slowing down kinetic equation accounting for radial diffusion losses due to toroidal field ripples (TFR), toroidal Alfvén eigenmodes (TAE) and first orbit losses. Volume averaging (indicated in Eqs. (1)-(3) by an over-line) of the local particle and power balance equations for each plasma species yields the three determining global burn dynamics equations

$$\frac{\mathrm{d}n_i(t)}{\mathrm{d}t} = S(t) - \frac{1}{2}\overline{\langle\sigma v\rangle_{\mathrm{dt}}(T_i(r,t))\, n_i^2(r,t)} - \frac{n_i}{\tau_{\mathrm{pi}}}, \quad (1)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{3}{2}\overline{n_i(r,t)T_i(r,t)} = P_{\mathrm{ei}}(n_i, T_i, T_e) + P_{\alpha\mathrm{i}}(n_i, T_i, T_e) + P_{\mathrm{aux,i}}(\ldots, t) - \frac{3}{2}\frac{\overline{n_i(r,t)T_i(r,t)}}{\tau_{\mathrm{Ei}}}, \quad (2)$$

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$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\frac{3}{2}\overline{n_e(r,t)T_e(r,t)} &= -P_{ei}(n_i, T_i, T_e) + P_{\alpha e}(\dots) + P_{ohm}(\dots) \\ &\quad + P_{aux,e}(\dots, t) - P_{brems}(\dots) - P_{cycl}(\dots) \\ &\quad - \frac{3}{2}\frac{\overline{n_e(r,t)T_e(r,t)}}{\tau_{Ee}}, \end{aligned} \tag{3}$$

where $n_i(t) = n_i$ represents the volume average of the fuel ion density, $S(t)$ is the averaged fueling rate, and $\langle\sigma v\rangle_{dt}$ denotes the dt-fusion reactivity parameter, τ_{pi} the global particle confinement time of fuel ions and τ_{Ej} the global energy confinement time of species j = i,e. The several global power terms $P_j(\dots)$ are averages over the entire plasma volume of the specific local power terms accounting for energy exchange between electrons and ions (j = ei), for fusion alpha power transfer to ions and electrons (j = αi, αe), for Ohmic and auxiliary heating (j = Ohm, aux) and for local power leakage by bremsstrahlung and cyclotron radiation (j = brems, cycl) as in [5], and were finally expressed as a function of the averaged values n_i, T_i, T_e. Alpha power coupling was determined using a detailed kinetic calculation [6] indicating that co-action of TFR and TAE diffusion may synergisticly reduce internal fusion heating. The global confinement times were derived from thermal diffusivities [7] and checked with a semi-empirical scaling law based on a combination of Neo-Alcator and ITER-scaling [8] (with an H-mode enhancement factor 1.8). Further a soft β-limit was introduced by an empirical function enhancing the transport losses by 20% at approximately 80% of the critical β-limit taken for a Troyon factor $c_T = 3.5$. We note that the radial motion of the plasma possesses stabilizing effects in the dynamic evolution [9]. All calculations were subjected to the 1996 EDA parameter set for ITER.

3. Trajectorial Ignition Dynamics in the Plasma State Space

Solving the set of Eqs. (1)-(3) which can be finally arranged into the form of the three coupled, nonlinear, first-order differential equations $\dot{n}_i = f(n_i, T_i, T_e)$, $\dot{T}_i = g(n_i, T_i, T_e)$ and $\dot{T}_e = h(n_i, T_i, T_e)$, allows for an illustrative demonstration of the nonlinear dynamic evolution by following the vector flows in the plasma state space spanned by the dynamic variables n_i, T_i, T_e. We show such a flow field in Fig. 1 for constant fueling and no auxiliary heating. The solid curves represent trajectories starting from states with $T_i = T_e$, whereas the dotted lines start at $T_i < T_e$. Since there is no principal qualitative change in the trajectorial flow, cases with $T_i > T_e$ are not displayed. Two stable fixed points attracting the trajectories become evident, which, according to their definition ($\dot{n}_i = 0$, $\dot{T}_i = 0$, $\dot{T}_e = 0$), represent states of ignition. The one at low temperature is classified by stability analysis as a stable node and represents Ohmic ignition. The second is a stable focus and suggests a preferred steady-state operation at $n_i = .98 \times 10^{20}$ m^{-3}, $T_i = 11.45$ keV and $T_e = 12.30$ keV. Further an unstable saddle point is identified by its repelling impact. It is perceived from the trajectorial flow pattern that there exists a surface separating the basin of attraction of the stable focus from that of the Ohmic ignition node. Trajectories starting from a state $T_i \neq T_e$ are seen to be quickly drawn towards the $T_i = T_e$-plane due to the small electron-ion energy exchange time $\tau_{ei} << \tau_E < \tau_p$. Hence the fusion dynamic variations are, in most cases, insignificant during temperature equilibration and the trajectorial investigation for t >> τ_{ei} may be reduced to a more intelligible 2-dimensional dynamic in the n_i-T state plane with $T = (T_i + T_e)/2$ being the average plasma temperature.

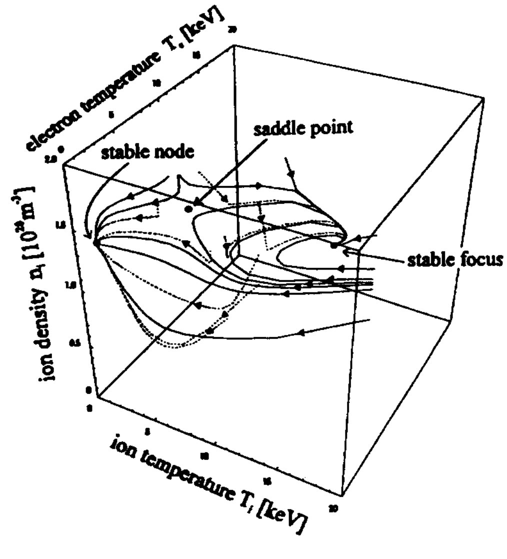

Fig. 1 Dynamic evolution of a d-t tokamak fusion plasma in the 3-dimensional state space. Depending on initial conditions, the system will evolve along the displayed trajectories if no auxiliary heating is supplied ($c_T = 3.5$, $P_{aux} = 0$, $S = 10^{19}$ m^{-3}·s^{-1}).

The ignition dynamic of a dt-fusion plasma in the n_i-T state plane yields the trajectories and separatrix of Fig. 2. Evidently, a low density region from which the stable ignited focus is attained dynamically exists outside the stationary zero-power ignition contour. Plasma states in this region will evolve – without any auxiliary heating or power control – to, and remain at, the stable high temperature attractor. This constitutes the process of 'dynamic self-ignition'. So the ignition criterion for an ITER-like fusion plasma is not merely achieving a plasma state within the stationary zero-power contour, but rather the separatrix of Fig. 2 must be exceeded for the system to evolve on its own towards conventional fusion ignition.

The trajectorial dynamics are modified by altered plasma dimensions and confinement regimes, and most significantly, by a locally and temporally varying external power supply and/or fueling rate. The location and nature of the ignited attractor as well as the form of the separatrix have been shown elsewhere [4,5] to vary strongly with the fuel injection rate. It appears even possible to bend the right hand tail of the separatrix such that it ends in the saddlepoint thus enclosing the stable focus. Such a fueling regime (S = 1.3 × 10^{18} m^{-3}·s^{-1}) renders the transgression of the separatrix, and subsequent self-ignition, at minimum requirements for plasma density and temperature. The corresponding case of minimum auxiliary heating until the onset of dynamic ignition is displayed in Fig. 3. After a sole Ohmic heating phase over 20 s the plasma is heated by neutral beam injection (NBI) delivering 226 MJ into the core. This brings the plasma into a state slightly beyond the separatrix where auxiliary heating can be turned off and the process of self-ignition begins. For NBI we assumed here the injection of 500 keV deuterium atoms and hence beam-target fusion reactions had to be considered as well. The heating route presented in Fig. 3, though providing a clear idea of the minimum energy path, may not be desirable for practical reactor operation because the dynamic approach of the stable focus is seen to go along with large density and temperature changes over a long time period. A more applicable path towards ignition avoiding such fluctuations is proposed in Fig. 4. After an initial Ohmic heating phase the energy from 9 MW ion cyclotron heating impels the plasma directly to the same stable operation point as in Fig. 3. Upon terminating ICRH only insignificant variations of the plasma parameters are observed.

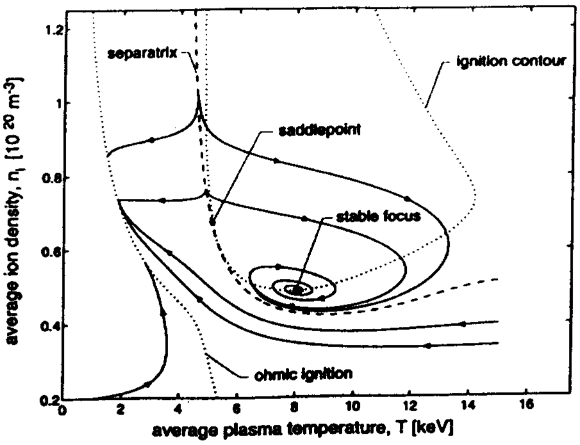

Fig. 2 Trajectorial dynamic of an ITER-like fusion plasma (Troyon factor c_T = 3.5, P_{aux} = 0) in the n_i-T state plane for a constant fueling rate S = 1.4 × 10^{18} m^{-3}·s^{-1} resulting in a stable focus.

If stable operation is preferred at higher values of n_i and T, there is no need for further auxiliary heating: one rather increases slightly the fueling rate such that the newly formed separatrix lies somewhat higher but

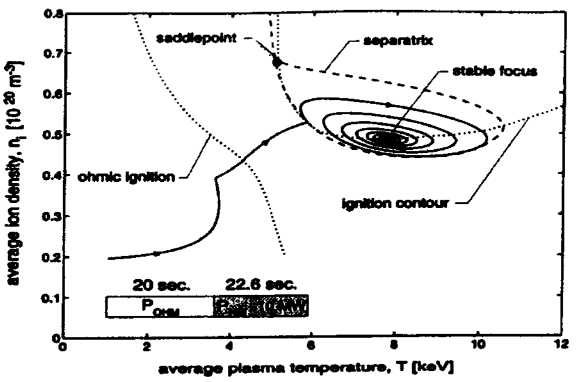

Fig. 3 Ignition path associated with minimum auxiliary heating energy. The d-t plasma (c_T = 3.5) is considered to be fueled at a constant rate S = 1.3 × 10^{18} m^{-3}·s^{-1}.

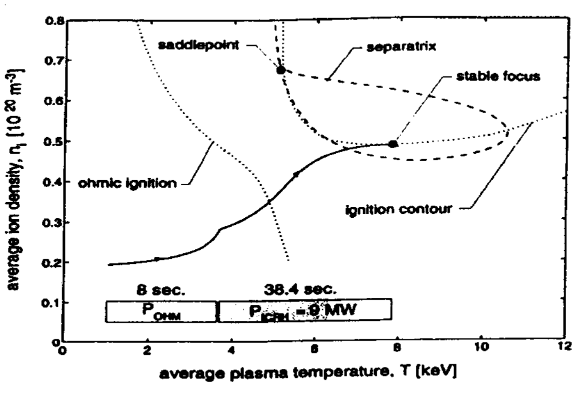

Fig. 4 Preferable path to ignited stable operation of a tokamak d-t plasma (c_T = 3.5, S = 1.3 × 10^{18} m^{-3}·s^{-1}) at low fusion power.

still below the old operation point. Upon that the plasma will dynamically evolve by itself into an ignited state determined by the new attractor appearing at larger n_i and higher T [5]. Appropriate repetition of this procedure can drive the fusion plasma towards any stable equilibrium point without auxiliary heating.

4. Conclusions

Our fusion burn dynamics study has instigated a novel realization of fusion plasma ignition. We conclude that the criterion for ignited operation of a continuously fueled fusion plasma is the crossing of the separatrix in the n_i-T state plane or, correspondingly, the transgression of a separatrix-surface in the n_i-T_i-T_e state space – a consequence of the nonlinear dynamics of the system. A region of 'dynamic self-ignition' outside the positive power balance state space was identified. Employing the fueling rate as a control parameter, the attracting ignited operation point could be managed to appear at fairly moderate plasma conditions in ITER-like fusion devices.

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