J. Plasma Fusion Res. SERIES, Vol. 2 (1999) 188–191

# Interpolation of Magnetic Surface Functions for an Axi-Symmetric Plasma

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(Received: 8 December 1998 / Accepted: 6 May 1999)

**Abstract**

Informations of the magnetic surface functions of magnetically confined plasma are indispensable for equilibrium, stability and transport analyses.

In this paper, in order to identify a realistic surface functions and compare those with ones which are introduced from Taylor's relaxation theory, we propose a code to interpolate these surface functions for an axi-symmetric plasma from experimentally measured data.

To confirm our code, we used the data which were analyzed from known functions given as a measured data. As a result, we have developed a code which can derive surface functions $I$ and $P$. Effects of measurement error on those functions are also examined.

**Keywords:**

magnetic surface function, relaxation theory, equilibrium analysis, Grad-Shafranov equation, reversed field pinch, least square method

## 1. Introduction

A reversed field pinch (RFP) has a property that plasma itself makes magnetic fields which are necessary to confine plasma. The self-organization phenomenon is explained by Taylor's relaxation theory.

From theoretical considerations or models of this relaxation processes, forms of the magnetic surface functions $I = rB_t$ and $P$, where $B_t$ is a toroidal magnetic field and $P$ is a kinetic pressure, are derived. Giving these surface functions, Grad-Shafranov equation can be solved for equilibrium, stability and transport analyses .

We are developing a code to interpolate magnetic surface functions from experimentally measured data. The merits of this code are as follows:

- We identify surface functions from experimental data and compare those with theoretical ones.
- We can analyze experimentally obtained equilibrium configurations.
- We can obtain the time history of the helicity ($K = \int_{V_p} \mathbf{A}\cdot\mathbf{B}\mathrm{d}v$) which is a crucial parameter of the relaxation theory.

As numerical method, we use the least-square method [1]. In §2 we explains equilibrium analysis code used in the least-square method. In §3 the algorithm of this code is described. In §4 and §5, numerical results are shown. In §6 the conclusions are summarized.

## 2. Equilibrium Analysis

In the cylindrical coordinate ($r$, $\phi$, $z$) (Fig. 1), MHD equilibrium of an axi-symmetric plasma is described by the Grad-Shafranov equation:

$$\nabla \cdot \frac{1}{r^2}\nabla\psi = -\frac{\mu_0}{r} J_\phi \tag{1}$$

$$J_\phi = \begin{cases} r\dfrac{\mathrm{d}P(\psi)}{\mathrm{d}\psi} + \dfrac{I(\psi)}{\mu_0 r}\dfrac{\mathrm{d}I(\psi)}{\mathrm{d}\psi} & \text{in Plasma } V_\mathrm{p} \\ J_\mathrm{c} & \text{in Vacuum } V_\mathrm{c} \end{cases} \tag{2}$$

$$\boldsymbol{B}=\frac{\nabla\psi}{r}\times\boldsymbol{e}_\phi+\frac{I}{r}\boldsymbol{e}_\phi \tag{3}$$

where $\boldsymbol{e}_\phi$ is an unit vector of $\phi$, $J_\phi$ is a current density following $\phi$ direction, $J_c$ is that of external field control coils.

The code 'EAFP' [2] can solve the Grad-Shafranov equation on conditions of arbitrary functions $I(\psi)$ and $P(\psi)$ and geometry of the limiter and conducting shell as a free boundary problem.

## 3. Algorithm for Least-Square Method

To use a least-square method code, we must establish forms of model functions and choose types of measured data.

For convenience the magnetic surface functions are simply shown:

$$I=f(\psi,a_1,a_2,\cdots) \tag{4}$$

$$P=g(\psi,b_1,b_2,\cdots) \tag{5}$$

where $a_i$, $b_i$ are unknown parameters and $f$, $g$ are functions of a model.

As for measured data, we use poloidal flux $\psi$ at some points of vacuum vessel surface, toroidal flux $\psi_\phi$, F-value, $\Theta$-value and $\beta_0$ of a $\beta$ value at a plasma center. An example of equilibrium which is analyzed by EAFP and positions of poloidal flux coils are shown in Fig. 2.

First of all, to confirm our code we used the data $\psi$* which are obtained from the EAFP analysis as measured data. For this EAFP analysis we use magnetic surface functions [3] such as

$$I^*(\psi)=1+\lambda\left(\psi-1+\frac{(1-\psi)^{\alpha+1}}{\alpha+1}\right) \tag{6}$$

$$P^*(\psi)=1.5\beta_0\psi(1-\psi^2/3) \tag{7}$$

where $\psi$ is a normalized value of $rA_\theta$, $\lambda$, $\alpha$ and $\beta_0$ are assumed parameters. In this analysis, we use $\lambda = 1.2$, $\alpha = 4$ and $\beta_0 = 0.1$.

A flowchart of this code is shown in Fig. 3. Because of nonlinearity of this problem, we use Marquardt-method [4]. At first we establish initial value of unknown parameters. Using those parameters, EAFP calculates $\psi$, $\psi_\phi$, $F$, $\Theta$ and $\beta_0$. The unknown parameters are improved to minimize the functional:

$$\chi^2=\sum_{j=1}^{k}C_{1j}(\psi_j-\psi_j^*)^2+C_2(\psi_\phi-\psi_\phi^*)^2+C_3(F-F^*)^2+C_4(\Theta-\Theta^*)^2+C_5(\beta_0-\beta_0^*)^2 \tag{8}$$

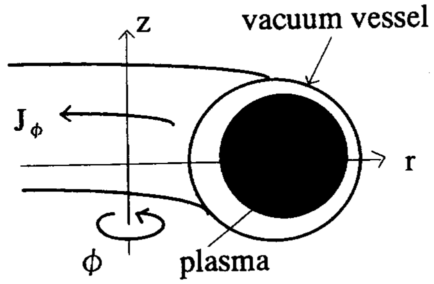


Fig. 1 The cylindrical coordinate in toroidal geometry.

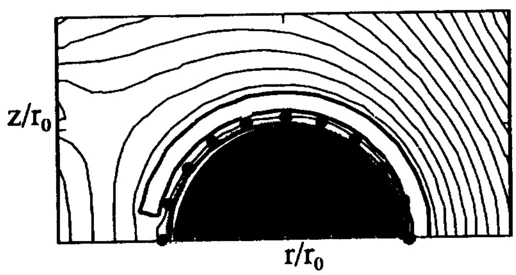


Fig. 2 A typical equilibrium by EAFP and arrangement of flux loops ●.

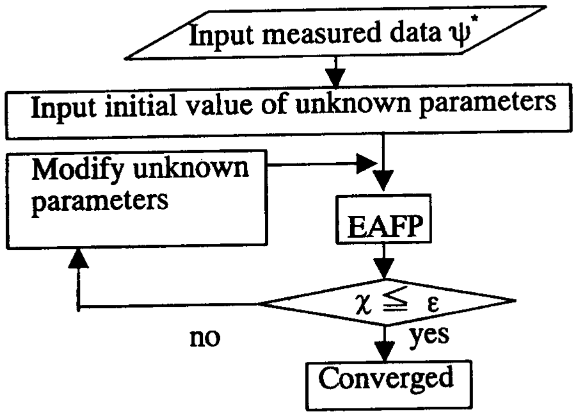


Fig. 3 Flowchart of least-square method.

where $k$ shows the number of flux loops and $C_{1j}$, $C_2$, $\cdots$, $C_5$ are weight coefficients which depend on a variance of data. In this analysis, we simply use reciprocals of squares of measured data for weight coefficients. Finally the unknown parameters and magnetic surface functions $I$ and $P$ are obtained by iterative calculations.

## 4. Interpolation

We analyzed using two forms of model functions as follow,

**Model 1:** In this analysis we analyzed using model functions $I_1$, $P_1$ for least-square whose interpolation functions are same to those of $I^*$ and $P^*$. The unknown parameters are $\lambda$ and $\beta_0$.

**Model 2:** We used model functions $I_2$, $P_2$ for least-square whose forms are different from $I^*$, $P^*$:

$$I_2(\psi) = 1 + \sum_{i=1}^{M} a_i (1-\psi)^i \tag{9}$$

$$P_2(\psi) = \sum_{i=1}^{N} b_i \psi^i \tag{10}$$

where $M$ = 5, $N$ = 3 and $a_i$, $b_i$ are unknown parameters. In this analysis, we append constraints on $P$ that $P$ has a maximum value and $dP/d\psi$ is zero at magnetic axis.

As a result using Model 1, we obtained parameters which were equal to the exact parameters $\lambda = 1.2$ and $\beta_0 = 0.1$. Next, the result using Model 2 is shown. Next, Fig. 4 shows $I^*$, $P^*$ and $I_2$, $P_2$ at the converged point using Model 2. And we could get surface functions which equal to those of exact functions.

When we analyze a practical data, models of $I$ and $P$ are unknown indeed. One solution of this problem is to use higher order power series as model functions, but the use of large value of $M$ and $N$ requires many measurement coils and yields a problem of orthogonality between unknown parameters [4]. So it is necessary to analyze using many types of model functions, in case of analysis using real data. And using the constraints which relates $P$, if exact model is not chosen, we can obtain surface functions of a certain degree of approximation.

## 5. The Effects of Measurement Errors

Adding random error to measured data, we investigate these effects.

Figure 5 shows poloidal flux which are exact data, measured data with 5% error and converged result. Figure 6 shows exact functions $I^*$, $P^*$ and $I_2$, $P_2$ which are converged ones. It was found that the maximum difference between exact solution $I^*$ and converged result $I_2$ was within the amount of artificial errors. But the difference between $P^*$ and $P_2$ often becomes much larger. Therefore, to obtain confident function $P$, it is necessary to use measured data which relate to pressure at some points in plasma.

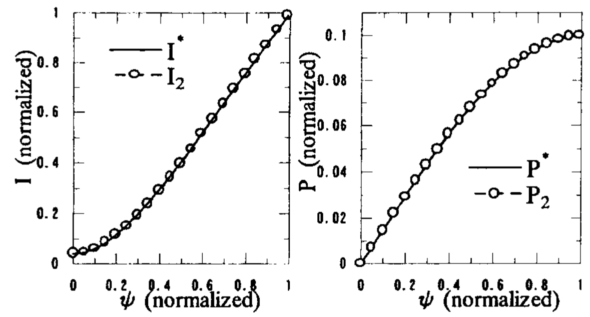


Fig. 4 Profiles of measured data $I^*$, $P^*$ and converged $I_2$, $P_2$ without an artificial noise.

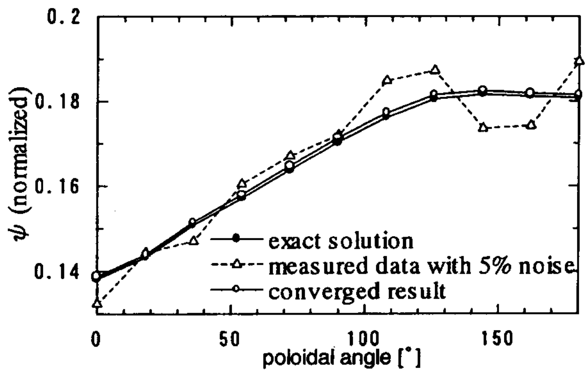


Fig. 5 Poloidal flux in noise-free case, with 5% noise and at the converged result.

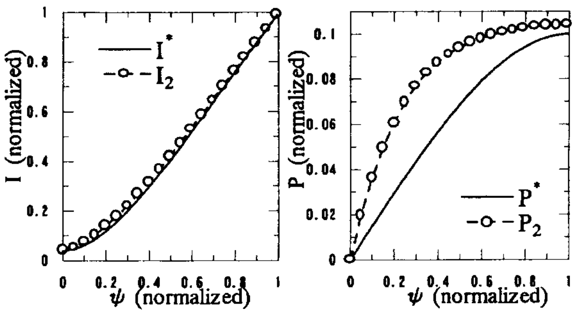


Fig. 6 Profiles of $I^*$, $P^*$ and $I_2$, $P_2$ provided 5% noise.

## 6. Conclusions

From analysis using Model 1, it is checked that if the forms of surface functions are equal to those of reference functions, we can obtain original surface functions. As the result of analysis using Model 2, it is concluded that if we select model functions which can approximate $I^*$ and $P^*$ sufficiently, we can obtain the surface functions. And it was found that in order to get the suitable model, it is necessary to analyze using many forms of model functions.

From analysis of measured data with error, it is found that function $I$ is obtained accurately and $P$ is easily influenced by measured error.

In the future, we plan to analyze magnetic surface functions of experimentally obtained data using many types of function models.

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