J. Plasma Fusion Res. SERIES, Vol. 2 (1999) 177–179

Stability Analysis of Wave Propagation in 2-Fluid Dusty Plasma System

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(Received: 25 December 1998 / Accepted: 6 May 1999)

Abstract

The stability of electro-static waves in 2-fluid dusty plasma system is investigated. We study the fluid dynamics of the dust particles and ions with the electrons following the Boltzmann distribution. We find some branches in its linear dispersion that are approximated to the dust-ion-acoustic wave when $q_{d0} \neq 0$. (q_{d0} is the equilibrium charge on the dust particles.) We show the stability of these waves in 1 dimensional system,and demonstrate the growth of these waves.

Keywords:

dust, plasma, dispersion relation

1. Introduction

We study the corporate dynamics of dust particles and ions. We investigate the 2-fluid dusty plasma system,where 2 species fluid dynamics with the charge evolution process of the dust particles.In this paperwe show the results of the linear dispersion relationin the homogeneous dusty plasma.We find the acoustic-like mode,which is approximated to dust-ion-acoustic mode,of the linear dispersion relationin this system with homogeneous medium.

2. Linear Dispersion Relation

We assume that the dynamics of electrons behaves as the massless motion with Boltzmann distribution, and ions and dust particles as the fluid motion, because of the large mass ratio between electrons and ions (dust particles). In this paper we study 1-dimensional unmagnetized cold dusty plasma system for simplicity, and eliminate the decreament of n_i in our 2-fluid model from the condition that n_e, $n_i >> n_d$. We consider the variation of the charge of the dust particles as the collision current [1].

$$
\left\{
\begin{aligned}
&\frac{\partial}{\partial t} n_d + \frac{\partial}{\partial x}(n_d v_d) = 0\\
&\frac{\partial}{\partial t} v_d + v_d \frac{\partial}{\partial x} v_d = -\frac{q_d}{m_d}\frac{\partial \phi}{\partial x}\\
&\frac{\partial}{\partial t} q_d + v_d \frac{\partial}{\partial x} q_d = I_e(q_d, \phi) + I_e(q_d, n_i)\\
&\frac{\partial}{\partial t} n_i + \frac{\partial}{\partial x}(n_i v_i) = 0\\
&\frac{\partial}{\partial t} v_i + v_i \frac{\partial}{\partial x} v_i = -\frac{e}{m_i}\frac{\partial \phi}{\partial x}\\
&-\frac{\partial^2}{\partial x^2}\phi = 4\pi(-en_e(\phi) + en_i + q_d n_d),
\end{aligned}
\right.
$$

where

$$
\left\{
\begin{aligned}
I_e(q_d, \phi) &= -\left(\frac{8\pi T_e}{m_e}\right)^{\frac{1}{2}} R^2 e\, n_e(\phi) \exp\left(\frac{eq_d}{RT_e}\right)\\
I_i(q_d, n_i) &= \left(\frac{8\pi T_i}{m_i}\right)^{\frac{1}{2}} R^2 e\, n_i \left(1 - \frac{eq_d}{RT_i}\right)
\end{aligned}
\right.
$$

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($q_d < 0$). n_e, n_i, and n_d are the number densities of electrons, ions, and dust particles, v_i and v_d are the velocity of ions and dust particles, T_i and T_e are the temperature of ions and dust particles, respectively. q_d is the local average charge of dust particles, R is the size of dust grain.

We study the propagation of the electro-static waves in homogeneous medium.

i) $q_{d0} = 0$

In the case of $q_{d0} = 0$ (no dust charge in the equilibrium), the equilibrium conditions are $\frac{T_e}{m_e} = \frac{T_i}{m_i}$ and $n_{i0} = n_e^0$. However, from $m_i/m_e = 1836$, the first equation is not realistic in this plasma.

ii) $q_{d0} \neq 0$ ($q_{d0} < 0$)

In the case of $q_{d0} \neq 0$ (existence of the dust charge in the equilibrium), the equilibrium conditions are

$$n_{i0} = \left(\frac{T_e/m_e}{T_i/m_i}\right)^{1/2} \left(1 - \frac{eq_{d0}}{RT_i}\right)^{-1} \exp\left(\frac{eq_{d0}}{RT_e}\right) n_e^0$$

and

$$n_{d0} = \frac{e}{eq_{d0}} (n_e^0 - n_{i0})$$

under the condition that $n_{d0} > 0$. Figure 1 shows n_{i0}, n_{d0} for $n_e^0 = 10^{-6}$. When we treat this system under $n_e \approx n_i \gg n_d$, we may set $q_{d0}/e \approx 1700$.

We derive the following linear dispersion relation with the Fourier mode exp (i ($kx - \omega t$)),

$$\begin{aligned} 0 = \omega^2 [& -\mathrm{i}\,(k^2 + k_{De}^2)\,\omega^3 + \{ C_1 (k^2 + k_{De}^2) \\ & + 4\pi n_{d0}\, C_2 \}\, \omega^2 + \mathrm{i}\,(\omega_{pd}^2 + \omega_{pi}^2) k^2 \omega \\ & - \{ (\omega_{pd}^2 + \omega_{pi}^2)\, C_1 + \frac{n_{d0}}{e}\, \omega_{pi}^2\, C_3 \} k^2 \,] \end{aligned}$$

where

$$C_1 = -\frac{e}{RT_e}\left(1 + \frac{T_e}{T_i(1 - \frac{eq_{d0}}{RT_e})}\right) I_{e0}\,,$$

$$C_2 = -\frac{e}{T_e} I_{e0}\,, \quad C_3 = -\frac{1}{n_{i0}} I_{e0}\,,$$

$$I_{e0} = -\left(\frac{8\pi T_e}{m_e}\right)^{\frac{1}{2}} R^2 e\, n_e^0 \exp\left(\frac{eq_{d0}}{RT_e}\right)$$

and

$$\omega_{pd} = \left(\frac{4\pi n_{d0} q_{d0}}{m_e}\right)^{1/2}.$$

Figure 2 shows the frequency of these electro-static waves when $q_{d0}/e = -1783$. These are approximated to

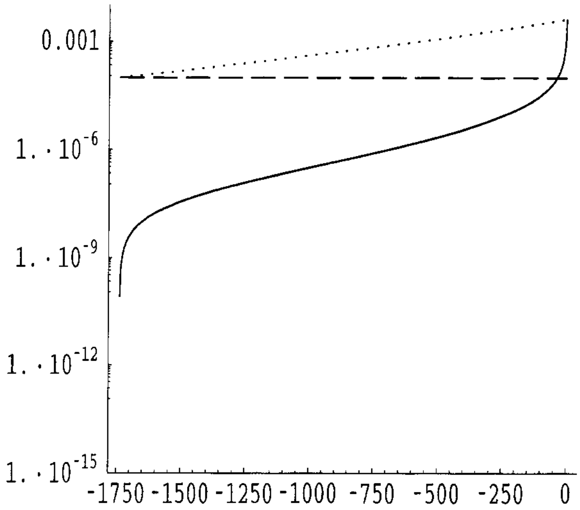

Fig. 1 n_{i0}, n_{d0} versus q_{d0}/e when $n_{e0} = 10^{-4}$, R = 10^{-4}, m_{d0} = 10^{-15}. Solid line describe the n_d, dotted line describe n_i, and hashed line describe n_e.

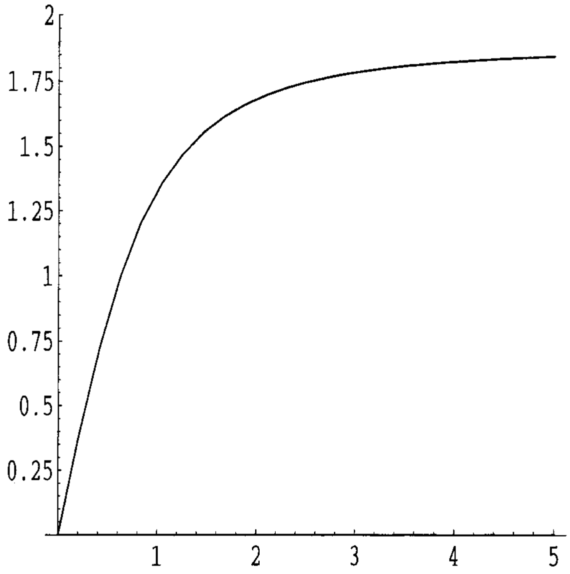

Fig. 2 Real frequency ω, normalized to ω_e, versus wavenumber k, normalized to k_{De}, when $q_{d0}/e = -1738$.

the *dust-ion-acoustic* mode $\omega_{DIA} = \left(\frac{k^2}{k^2 + k_{De}^2}\right)^{1/2} (\omega_{pd}^2 + \omega_{pi}^2)^{1/2}$ [2]. Figure 3 shows the growth rate of this mode. This mode is slowly growing for $k/k_{De} > 1.8$.

3. Conclusions

In this paper we have presented the 2-fluid dusty plasma system for the condition that n_e, $n_i \gg n_d$.

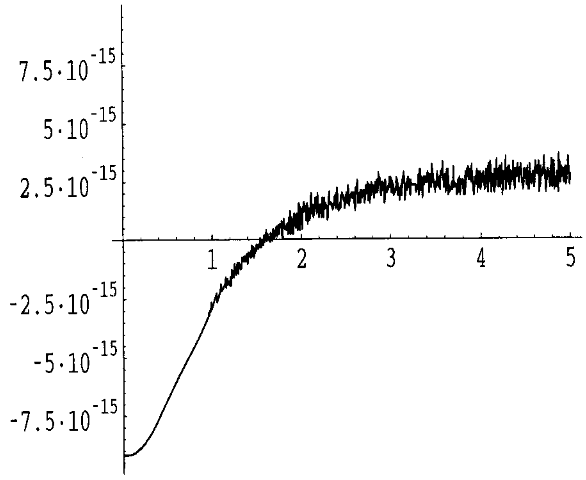

Fig. 3 Growth rate, normalized to ω_e, versus k/k_{De} when $q_{d0}/e = -1738$.

Treating the dynamics of ions as the fluid motion, we can analyze the wave propagation that the mode is based on that of the plasma without dust particles. We mainly consider the case $q_{d0} < 0$. We find the modified ion-acoustic mode, which is approximated to the dust-ion-acoustic mode. For the case that k is much larger than k_{De}, electrons distribute homogeneously in space. When the density of ions locally decreases, the negative charge of dust particles increases, and it makes the electro-static attractive force between ions and dust particles larger. Then the electro-static waves grow so that the fluctuations of the density of ions and the charge of dust particles are in the out-of-phase.

References

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