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Current Loop Coalescence in Dust Plasmas

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Abstract

The weakly ionized plasmas that occur in protostellar disks and in the cores of molecular clouds generally have a dust component. We consider the effects of the presence of charged dust on magnetic reconnection process. We investigate the dynamics of two current loops coalescence, for both cases; partial and complete magnetic reconnection. It is shown that the field-free gas produced during the two loops coalescence with complete magnetic reconnection would be able to initiate star formation.

Keywords:
dust plasma, magnetic reconnection, coalescence of two loops, star formation

1. Introduction

The weakly ionized plasmas that occur in protostellar disks and in the cores of molecular clouds generally have a dust component. The ionization fraction of molecular clouds is typically only $\approx 10^{-7}$, and the dust may contribute $\approx 1\%$ of the mass of the cloud.

It has been shown that the ion-neutral drift leading to ambipolar diffusion can lead to the steepening of the magnetic field profile and to the formation of singularities in the current density of hydromagnetic fluctuations [1,2] and Alfvén waves [3,4]. Recently Bulanov and Sakai (1998) [5] investigated in datails magnetic reconnection process in weakly ionized plasmas.

In the present paper we consider the effects of the presence of charged dust on magnetic reconnection process. We investigate the dynamics of two current loops coalescence, for both cases; partial and complete magnetic reconnection. It is shown that the field-free gas produced during the two loops coalescence with complete magnetic reconnection would be able to initiate star formation.

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2. Basic Equations

We consider molecular clouds consisting of neutral atomic and molecular species, the ionized atomic and molecular species, the electrons, and negatively charged dust grains. We start a 4-fluid model of the plasma, which employs the fluid momentum equations for plasma ions (singly charged), neutral molecules, charged dust grains and electrons:

$$\rho_{\rm i}\left(\frac{\partial \boldsymbol{v}_{\rm i}}{\partial t}+\boldsymbol{v}_{\rm i}\cdot\nabla\boldsymbol{v}_{\rm i}\right)=-\nabla p_{\rm i}+n_{\rm i}e(\boldsymbol{E}+\boldsymbol{v}_{\rm i}\times\boldsymbol{B})-\rho_{\rm i}\nu_{\rm in}(\boldsymbol{v}_{\rm i}-\boldsymbol{v}_{\rm n})-\rho_{\rm i}\nu_{\rm id}(\boldsymbol{v}_{\rm i}-\boldsymbol{v}_{\rm d}), \quad (1)$$

$$\rho_{\rm n}\left(\frac{\partial \boldsymbol{v}_{\rm n}}{\partial t}+\boldsymbol{v}_{\rm n}\cdot\nabla\boldsymbol{v}_{\rm n}\right)=-\nabla p_{\rm n}-\rho_{\rm n}\nu_{\rm ni}(\boldsymbol{v}_{\rm n}-\boldsymbol{v}_{\rm i})-\rho_{\rm n}\nu_{\rm nd}(\boldsymbol{v}_{\rm n}-\boldsymbol{v}_{\rm d}), \quad (2)$$

$$\rho_d\left(\frac{\partial \boldsymbol{v}_d}{\partial t}+\boldsymbol{v}_d\cdot\nabla\boldsymbol{v}_d\right)=Z_d n_d e(\boldsymbol{E}+\boldsymbol{v}_d\times\boldsymbol{B}) -\rho_d\nu_{dn}(\boldsymbol{v}_d-\boldsymbol{v}_n) -\rho_d\nu_{di}(\boldsymbol{v}_d-\boldsymbol{v}_i), \quad (3)$$

$$0=-n_e e(\boldsymbol{E}+\boldsymbol{v}_e\times\boldsymbol{B})-\rho_e\nu_{en}(\boldsymbol{v}_e-\boldsymbol{v}_n) -\rho_e\nu_{ed}(\boldsymbol{v}_e-\boldsymbol{v}_d)-\rho_e\nu_{ei}(\boldsymbol{v}_e-\boldsymbol{v}_i), \quad (4)$$

where $\boldsymbol{E}$ is the wave electric field, m_s is the species mass, ρ_s is the species mass density and $\boldsymbol{v}_s$ is the species velocity. p_i and p_n are the ion thermal and neutral thermal pressures, and ν_{st} is the collision frequency of a particle of species s with the particles of species t. We have neglected electron inertia in (4), momentum transfer to ions from electrons in (1) and to dust grains from electrons in (3), and the dust thermal pressure gradient in (3).

The parameter $\delta = n_e/n_i < 1$ measures the charge imbalance of the electrons and ions in the plasma, with the remainder of the negative charge residing on the dust particles, so that the total system is charge neutral.

$$-en_e+en_i-Z_d e n_d=0. \quad (5)$$

A typical value of δ for molecular clouds is $\delta = 1 - 10^{-4}$. The charge on each dust grain is assumed constant, and for simplicity we also assume that δ is constant, even though n_n and thus n_i are variable. The neutral mass density obeys the continuity equation

$$\frac{\partial \rho_n}{\partial t}+\nabla\cdot(\rho_n\boldsymbol{v}_n)=0. \quad (6)$$

To complete the system of equations, Maxwell's equations ignoring the displacement current are used, with the conduction current density given by

$$\boldsymbol{j}=e(n_i\boldsymbol{v}_i-n_e\boldsymbol{v}_e-n_d Z_d\boldsymbol{v}_d). \quad (7)$$

where equilibrium charge neutrality is expressed by (5).

Equations (4) and (7) lead to the following generalized Ohm's law:

$$\boldsymbol{E}+\left(\frac{\boldsymbol{v}_i}{\delta}-\frac{(1-\delta)}{\delta}\boldsymbol{v}_d\right)\times\boldsymbol{B}=\frac{\boldsymbol{j}\times\boldsymbol{B}}{n_e e}-\frac{m_e}{e}\nu_{en}(\boldsymbol{v}_e-\boldsymbol{v}_n) -\frac{m_e}{e}\nu_{ed}(\boldsymbol{v}_e-\boldsymbol{v}_d)-\frac{m_e}{e}\nu_{ei}(\boldsymbol{v}_e-\boldsymbol{v}_i). \quad (8)$$

We now use the strong coupling approximation (Suzuki and Sakai 1996) [3], whereby the ion inertia term (the left hand side) and the ion thermal pressure term are neglected in (1), leaving a balance between the remaining terms. At this point it is useful to normalize the magnetic field by a reference field B_0, and define the Alfvén speed based on the field B_0 and the ion density: $v_A = B_0/(\mu_0\rho_i)$. Eq. (1) may then be written, using (8) and Faraday's law neglecting the displacement current,

$$\frac{v_A^2}{\delta}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}=\Omega_m(\boldsymbol{v}_i-\boldsymbol{v}_d)\times\boldsymbol{B} +\nu_{in}(\boldsymbol{v}_i-\boldsymbol{v}_n)+\nu_{id}(\boldsymbol{v}_i-\boldsymbol{v}_d), \quad (9)$$

where $\Omega_m = \Omega_i(1-\delta)/\delta$ and Ω_i is the ion cyclotron frequency, $\Omega_i = B_0 e/m_i$. The presence of dust introduces the first and third terms on the rhs of (9).

For strongly coupled dust grains and neutral gas, such that $\boldsymbol{v}_d = \boldsymbol{v}_n$, the ion equation of motion gives the following expression for the ion-neutral relative drift velocity:

$$\boldsymbol{v}_D=\boldsymbol{v}_i-\boldsymbol{v}_n=F\frac{v_A^2}{\nu_i\delta}[(\nabla\times\boldsymbol{B})\times\boldsymbol{B} -(R/\rho)((\nabla\times\boldsymbol{B})\times\boldsymbol{B})\times\boldsymbol{B}], \quad (10)$$

where $\nu_i = \nu_{in} + \nu_{id}$, $R = \Omega_m\rho_n/\nu_i\rho_0$, $B^2 = B_x^2 + B_y^2 + B_z^2$ and $F = 1/(1 + R^2B^2/\rho^2)$. For a small dust number density, $\nu_{id} << \nu_{in}$, so we approximate $\nu_i = \nu_{in}$. Summing the momentum conservation equations (1,2,3) of the ions, neutrals and dust, the equation of motion for the neutral velocity is obtained. We normalize the density by ρ_0, defining $\rho = (\rho_n + \rho_d)/\rho_0$, and the pressure by p_0. The velocity is normalized by the Alfvén velocity based on ρ_0, $V_A = B_0/(\mu_0\rho_0)^{1/2}$, and space and time are normalized by L_0 and $\tau_A = L_0/V_A$. The result is

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t}+\boldsymbol{v}\cdot\nabla\boldsymbol{v}\right)=-\beta\nabla p+(\nabla\times\boldsymbol{B})\times\boldsymbol{B}. \quad (11)$$

The magnetic induction equation gives, using (8) with the collisional electron momentum transfer terms neglected, and neglecting the Hall term,

$$\frac{\partial \boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{v}\times\boldsymbol{B})+A_D\nabla\times\left[\frac{F}{\rho}\left(((\nabla\times\boldsymbol{B})\times\boldsymbol{B})\times\boldsymbol{B}+(R/\rho)B^2(\nabla\times\boldsymbol{B})\times\boldsymbol{B}\right)\right], \quad (12)$$

where $\mathbf{B} = [\boldsymbol{B}]$ and $A_D = \rho_n/\nu_i\tau_A\rho_i$.

3. Simulation Results

We use a 3-D simulation code of the above equations in which the numerical scheme is the modified 2-step Lax-Wendroff method. The system sizes are $0 \le x = y \le 6\pi L$, and $0 \le z \le 0.5\pi L$ in the x, y and z directions, respectively. The mesh points are $N_x =$

200, $N_y = 200$ and $N_z = 10$ in the x, y and z directions, respectively. We used free boundary conditions (first derivatives of all physical quantities are continuous) for the x and y directions, while periodic boundary condition is used in the z direction. We take an initial each current loop, which is placed along the z direction to satisfy a force-free condition. Other parameters used here are $\beta = 0.01$ and $A_D = 0.5$.

We show the simulation results of both cases for symmetric two current loops collision, where partial or complete reconnection occurs. Fig. 1 shows four snapshots at $t = 8.1\ \tau_A$: (a) density distribution overlapped with velocity field $(v_x - v_y)$, (b) magnetic filed B_z with vector field plots of $(B_x - B_y)$, (c) velocity v_z with vector plots of $(v_x - v_y)$, and temperature distribution. As seen in Figs.1(a) and (b), during two loops coalescence the density enhancement is about twice of the initial state and relatively weak. There appears strong wave emissions of fast magnetosonic waves from the reconnection region, which are propagating upward the incoming flows. There occurs strong plasma heating near the wave fronts as seen in Fig.1 (d).

Fig.2 shows the simulation results for complete magnetic reconnection case at $t = 6.3\ \tau_A$. As seen in Fig.2 (a) there occurs strong density accumulation with about 13 times larger than the initial value near the region where the magnetic fields are almost free, due to complete magnetic reconnection. The field-free gas produced during the two loops coalescence with complete magnetic reconnection could be able to initiate star formation.

4. Conclusion

We investigated the effects of the presence of charged dust on magnetic reconnection process for both

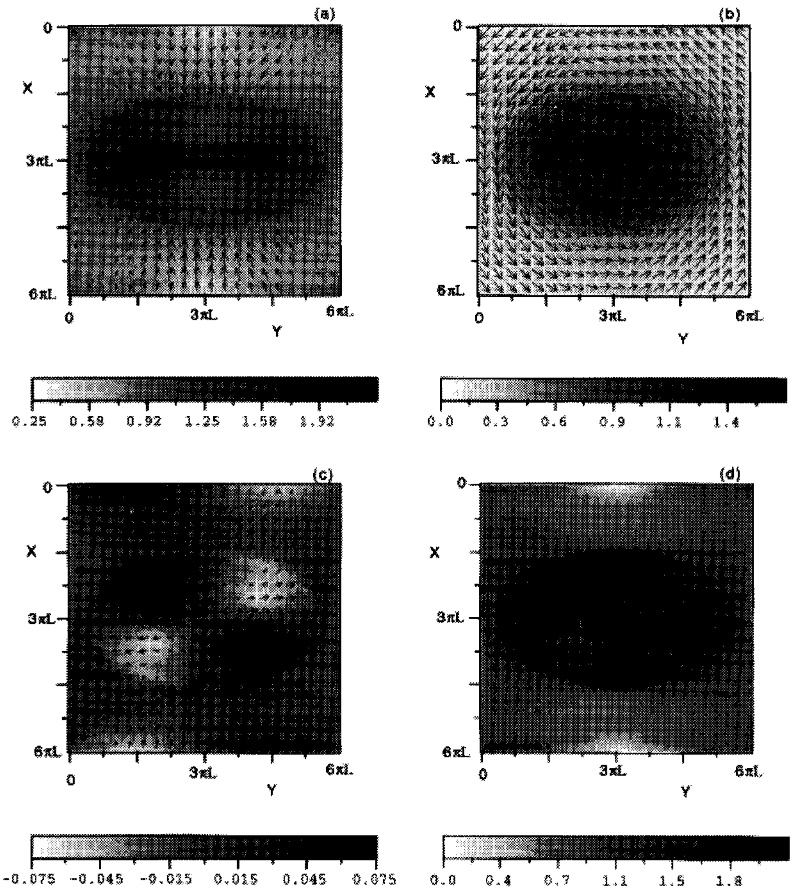

Fig. 1 (a) Density distribution overlapped with velocity field $(v_x - v_y)$, (b) magnetic filed B_z with vector field plots of $(B_x - B_y)$, (c) velocity v_z with vector plots of $(v_x - v_y)$, and (d) temperature distribution at $t = 8.1\ \tau_A$ for partial magnetic reconnection.

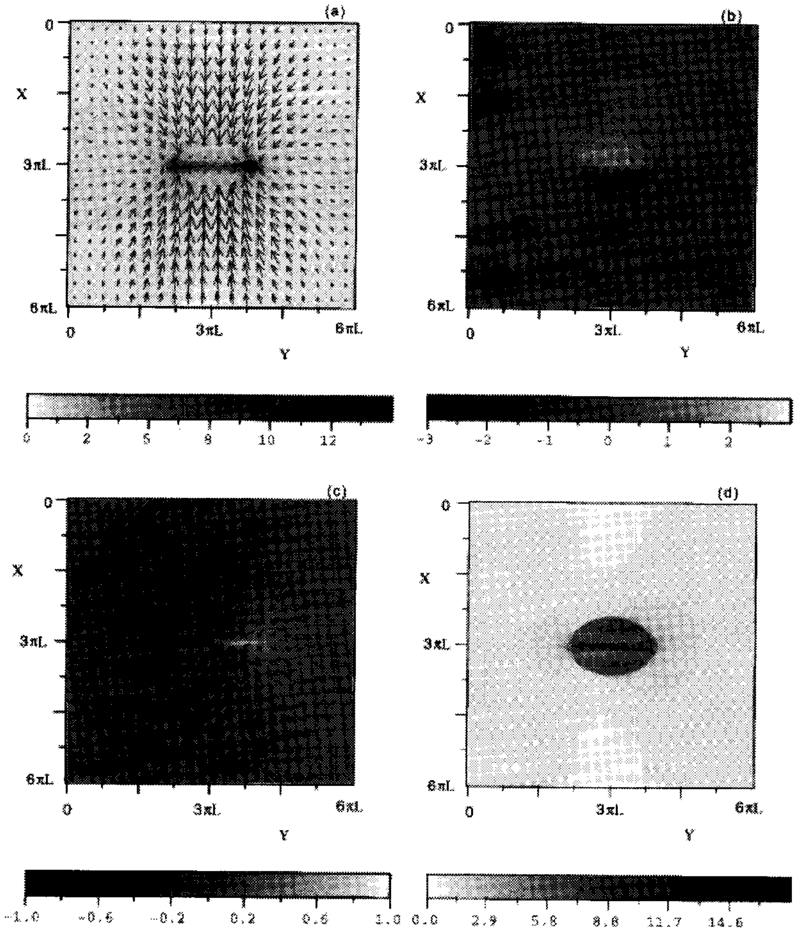

Fig. 2 (a) Density distribution overlapped with velocity field $(v_x - v_y)$, (b) magnetic filed B_z with vector field plots of $(B_x - B_y)$, (c) velocity v_z with vector plots of $(v_x - v_y)$, and (d) temperature distribution at $t = 6.3\ \tau_A$ for complete magnetic reconnection.

complete and partial magnetic reconnection during two current loops coalescence. We showed that the field-free gas produced during the two loops coalescence with complete magnetic reconnection could be able to initiate star formation.

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